Minimum time control of a two DOF robotic arm with noised measurements

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Abstract: This paper presents a new approach for minimum time control dynamics of a two links manipulator robot in the case of noised outputs. Briefly, this technique consists of linearizing a nonlinear dynamic model of the robot by using a feedback linearization control. Once, the linear model has been obtained, a minimum time control with constraints, using the Pontryagin Minimum Principle will be developed. Here, the objective is to control the arm robot from an initial configuration to the final configuration in minimum time. The state variables are estimated by a Kalman-Luenberger observer. In order to show the efficiency of the proposed method, some simulation results are given.

Keywords: Robotic arm, Minimum time control, Non-linear control, Kalman-Luenberger observer.

1. INTRODUCTION

The development of production and to facilitate the difficult and repetitive tasks for human motivated the increasing interest in arm robotics [1-2]. The mechanical structure of this class of robots is complex (articulated rigid-body) which makes the task of control more difficult. Many approaches are available in the literature to control of a robotics arms. In [3], a feedback controller is developed to improve the robust performance under structural and parametric uncertainty disturbance in electro-hydraulic servo system (EHSS) for a 2-DOF robotic arm. In [4], a robust control of a robotic arm is developed, with taking into account the friction in the robot model. In [5], a coordinated fuzzy control is developed for robotic arms with actuator hysteresis and motion constraints. Also, the adaptive control scheme is introduced to reduce the harmful effects from unknown nonlinearities. In [6], a solution to the inverse kinematics problem of a three-link planar manipulator, needed for generating desired trajectories in the Cartesian space (2D) is found by using a feed-forward neural network. In [7], a hybrid controller for three-degrees-of-freedom (3-DOF) robotic manipulators is presented. The proposed controller comprises of an independent joint controller, designed in the configuration space, and a sliding mode controller that enforces desired dynamics for the tracking error projections to the Frenet-Serret frame. In [8], a novel methodology for motion specification and robust reactive execution, for an industrial robotic manipulators is developed. Traditional trajectory generation techniques and optimisation-based control strategies are merged into a unified framework for simultaneous motion planning and control. In [9], a kinematic modeling and control of a robot arm using unit dual quaternions is proposed. In [10], a new approach to tracking control of a six degrees of freedom (6-DOF) robotic arm is developed.

Increasing the production and minimizing the cost associated to the duration, require us to determine a minimum time control for this class of robot. In this work, we are interested in the minimum time control of a two links robot arm. Many approaches are available in the literature to study this area. In [11], a convex optimization approach is developed for time-optimal path-constrained trajectory planning of robot systems. In [12] a minimum time control of the Acrobot is proposed. The principle of this approach is to use a direct search algorithm for finding an optimal trajectory for the Acrobot. In [13], a time-optimal control of robotic manipulators along specified paths is presented. Here the dynamics of the robot is ignored and when the optimal trajectory is found, a feedback control is used to follow it. In [14], a minimum-time control of robotic manipulators with geometric path constraints is developed. Here the robot control algorithms are divided into two stages, namely, path or trajectory planning and path tracking (or path control). This division has been adopted mainly as a means of alleviating difficulties in dealing with complex, coupled manipulator dynamics. In [15], a time optimal control of a robotic manipulator modelled with actuator dynamics is presented. Here, the system dynamic equations with the inclusion of actuator dynamics are derived using Pontryagin Maximum Principle (PMP) which results in a nonlinear two-point boundary value problem.

Different to the above cited work, our paper proposes a novel approach for controlling a two links arm robot. This approach involves determining a minimum time control dynamics of manipulator robot with two degrees of freedom (DOF). The dynamic model of this robot is nonlinear, so a feedback linearization control is applied to the robot dynamic model to make it linear. Next, based on the obtained linear model, a minimum time control with constraints, using the Pontryagin Minimum Principle is developed.

This paper is organized as follows: in the second section, the description of the manipulator robot with two DOF here considered and its dynamic model are given. In the third section, the control approach for controlling the robot from an initial configuration to the final configuration in minimum time is presented. In the fourth section, a Kalman-Luenberger observer is introduced. Simulation results are presented in the fifth section.

2. DYNAMIC MODEL

A plan robot with two degrees of freedom that is treated in this work is presented in Fig. 1 where θ_i, L_i and M_i $\{i=1,2\}$ are respectively the joint, length and the mass of the first link (i=1) and the second link (i=2). The gravitational acceleration is denoted by g.

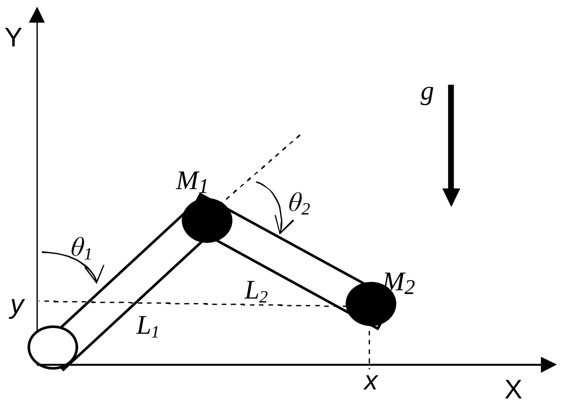

Fig.1 Two link robot arm

The calculation of the dynamic model of this robot is based on the kinetic and potential energies. These last are computed using the direct geometric model (DGM) given by the following formula:

$$\begin{cases} x = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \\ y = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \end{cases} \tag{1}$$

Using the Euler-Lagrange method, the dynamic model of a robotic arm with two degrees of freedom (DOF) is given by the following formula [16]:

$$\begin{cases} M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) = \tau \\ Y = \theta \end{cases} \tag{2}$$

where:

- $\theta = [\theta_1 \quad \theta_2]^{\mathrm{T}}$ is a vector of joints variables,
- $\tau = [\tau_1 \quad \tau_2]^{\mathrm{T}}$ is a vector of torques (control inputs),
- Y is the output vector,

- $G(\theta)=\begin{bmatrix}-(M_1+M_2)gL_1\sin(\theta_1)-M_2gL_2\sin(\theta_1+\theta_2)\\ -M_2gL_2\sin(\theta_1+\theta_2)\end{bmatrix}$ is a vector of gravity torques,

- $C(\theta,\dot{\theta})=\begin{bmatrix}-M_2L_1L_2\left(2\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_1^2\right)\sin(\theta_2)\\ -M_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_2)\end{bmatrix}$ represents the vector of Coriolis and centrifugal forces,

$M(\theta)=\begin{bmatrix}D_1 & D_2\\ D_3 & D_4\end{bmatrix}$ is the inertia matrix with:

$$\begin{aligned}D_1&=(M_1+M_2)L_1^2+M_2L_2^2+2M_2L_1L_2\cos(\theta_2)\\ D_2&=M_2L_2^2+M_2L_1L_2\cos(\theta_2)\\ D_3&=D_2\\ D_4&=M_2L_2^2\end{aligned}$$

3. CONTROL DESIGN

In this section, a minimum time control of a robotic arm with two DOF is developed. For that, we consider the nonlinear dynamic model given by (2). First, we determine a feedback linearization control to make the model (2) linear. Once the linear model has been obtained, an optimal control will be designed in the second step.

Feedback linearization control

The main idea of this technique is to transform the nonlinear dynamics of the system to a completely or partially linear such that linear control approaches can be applied to stabilize it [17-18]. Here, the control approach with feedback linearization is developed for a dynamic model (2) of the two-link robot arm. So, we differentiate the output *Y* until the control input *τ* appears. In our case, the control input *τ* appears in the second derivative of the output *Y*. This implies that the relative degree *r* is equal to two. The second derivative of *Y* is given by the following formula

$$\ddot{Y}=\ddot{\theta}=M(\theta)^{-1}\left(-C(\theta,\dot{\theta})-G(\theta)+\tau\right)=v \qquad (3)$$

where $v=[v_1 \quad v_2]^{T}$ is a synthetic control vector. And finally, from (3) we get the feedback linearization control

$$\tau=M(\theta)v+C(\theta,\dot{\theta})+G(\theta) \qquad (4)$$

Applying the control law given by (4) to the nonlinear system (2), the dynamic model of the manipulator robot with two DOF becomes a linear system like a double integrator as follows:

$$\frac{Y(s)}{v(s)}=\frac{1}{s^2} \qquad (5)$$

The relative degree *r* is equal to two. This means that by using the control law (4), we obtain a complete linearization of the nonlinear system (2) and we get a linear system for each joint variable:

$$\frac{\theta_1(s)}{v_1(s)}=\frac{1}{s^2} \text{ and } \frac{\theta_2(s)}{v_2(s)}=\frac{1}{s^2} \qquad (6)$$

Now, the linearization of the nonlinear system was done. So, we can develop a minimum time control for the two-link robot arm which will be the object of the next subsection.

Minimum time control

In the case of a robot arm with two DOF and after application of the feedback linearization (4) to the nonlinear system (2), we obtain the following two decoupled linear systems

$$\begin{cases} \ddot{\theta}_1 = v_1 \\ \ddot{\theta}_2 = v_2 \end{cases} \tag{7}$$

Let us define the error e_i between the actual angle θ_i and the desired angle θ_{id} as

$$e_i = \theta_i - \theta_{id} \quad i = 1,2 \tag{8}$$

The desired angle θ_{id} $\{i = 1,2\}$ is constant. Differentiating the equation (8) twice, we obtain.

$$\ddot{e}_i = \ddot{\theta}_i = v_i \quad i = 1,2 \tag{9}$$

Considering a single decoupled linear system

$$\ddot{e}_1 = v_1 \tag{10}$$

The state space representation of the system (10) is given by the following formula:

$$\begin{cases} \dot{X} = AX + Bv_1 \\ Y = CX \end{cases} \tag{11}$$

where:

- $X = [x_1 \quad x_2]^T = [e_1 \quad \dot{e}_1]^T \in \mathbb{R}^2$ is a state vector
- $Y \in \mathbb{R}$ is the output vector
- v_1 is a synthetic control
- $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = [1 \quad 0]$

Therefore the system (11) can be rewritten as a first order differential system

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = v_1(t) \\ Y(t) = x_1(t) \end{cases} \tag{12}$$

We assume that boundedness of synthetic control

$$v_1(t) \le 1; \quad t \in [t_0, t_f] \tag{13}$$

with t_0 the initial time and t_f the final time.
In all the paper, the asterisk symbol '*' means the optimal value. Here, the problem is to find an optimal control $v_1^*(t) \in \mathbb{R}$ which satisfies the constraint (13) and transfer the system (12) from the initial state $X(t_0)$ to the final state $X(t_f)=0$ in minimum time. To solve this problem, we will follow the following steps:

Step 1:

The performance criterion is defined as

$$J = \int_{t_0}^{t_f} dt = t_f - t_0 \tag{14}$$

where t_0 is fixed and t_f is free.

Step 2:

We form the Hamiltonian *H* for the problem described by the system (12) and the performance index (14). The Hamiltonian is given by the following formula:

$$H\left(X(t), v_1(t), \lambda(t)\right) = 1 + \lambda^T\left(\dot{X}(t)\right) \tag{15}$$

where $\lambda(t) \in \mathbb{R}^n$ is a vector of costate variables

Introducing (11) into (15), the Hamiltonian becomes:

$$H\left(X(t), v_1(t), \lambda(t)\right) = 1 + \lambda_1(t)x_2(t) + \lambda_2(t)v_1(t) \tag{16}$$

Step 3:

Now, we minimize the Hamiltonian (16), using the Pontryagin's minimum principle, we obtain:

$$\begin{aligned} H\left(X^*(t), v_1^*(t), \lambda^*(t)\right) &\le H\left(X^*(t), v_1(t), \lambda^*(t)\right) \\ &= \min_{|v_1| \le 1} H\left(X^*(t), v_1(t), \lambda^*(t)\right) \end{aligned} \tag{17}$$

Substituting (16) in the inequality (17), we obtain:

$$\begin{aligned} &1 + \lambda_1^*(t)x_2^*(t) + \lambda_2^*(t)v_1^*(t) \le 1 + \lambda_1^*(t)x_2^*(t) + \lambda_2^*(t)v_1(t) \\ &\text{hence} \\ &\lambda_2^*(t)v_1^*(t) \le \lambda_2^*(t)v_1(t) \ = \min_{|v_1(t) \le 1|}\left(\lambda_2^*(t)v_1(t)\right) \end{aligned} \tag{18}$$

The optimal control $v_1^*(t)$ is obtained from (18) as follows:

- if $\lambda_2(t) > 0$ the optimal control $v_1^*(t)$ must be the smallest value of the admissible control (-1) in order to: $\min_{|v_1(t) \le 1|}\left(\lambda_2^*(t)v_1(t)\right) = -\lambda_2^*(t) = -\left|\lambda_2^*(t)\right|$
- if $\lambda_2(t) < 0$ the optimal control $v_1^*(t)$ must be the greatest value of the admissible control (+1) in order to: $\min_{|v_1(t) \le 1|}\left(\lambda_2^*(t)v_1(t)\right) = \lambda_2^*(t) = -\left|\lambda_2^*(t)\right|$

So, from the two points mentioned above, the optimal control is:

$$v_1^*(t) = \begin{cases} +1 & if \quad \lambda_2^*(t) < 0 \\ -1 & if \quad \lambda_2^*(t) > 0 \\ 0 & if \quad \lambda_2^*(t) = 0 \end{cases} \tag{19}$$

then the optimal control (19) can be rewritten as:

$$v_1^*(t) = -\operatorname{sgn}\left(\lambda_1^*(t)\right) \tag{20}$$

where 'sgn' is the sign function.

Step 4:

The equations of the costates variables are given by the following formula:

$$\begin{cases} \dfrac{d\lambda_1^*(t)}{dt} = -\dfrac{\partial H}{\partial x_1^*(t)} = 0 \\ \dfrac{d\lambda_2^*(t)}{dt} = -\dfrac{\partial H}{\partial x_2^*(t)} = \lambda_1^*(t) \end{cases} \tag{21}$$

Solving the systems (12) and (21), the minimum time control (20) can be rewritten in terms of (x_1, x_2) as follows [19-20]:

$$v_1^*(t) = -\text{sgn}\left(x_1 + \frac{1}{2}x_2|x_2|\right) \tag{22}$$

4. FILTERING BY KALMAN-LUENBERGER OBSERVER

Generally, if the measurement of the full state vector is not available an observer is added to the control structure to derive an output feedback law. Here, we consider that the measurements of the end-effector robot arm position are very noisy. So, a Kalman-Luenberger observer is introduced:

$$\begin{cases} \dot{\hat{X}} = A\hat{X} + Bv_1 - K\left(Y - \hat{Y}\right) \\ \hat{Y} = C\hat{X} \end{cases} \tag{23}$$

where *K* is a vector of the observer gains, $\hat{X}$ is the estimated state vector and $\hat{Y}$ is the estimated output vector.

The dynamics of the estimation error $e(t) = X(t) - \hat{X}(t)$ are given by:

$$\dot{e}(t) = (A + KC)e(t) \tag{24}$$

The estimation error (24) converges to zero if a matrix $(A + KC)$ has an eigenvalues with a real part strictly negative.

5. SIMULATION RESULTS

In order to show the efficiency of the proposed approach, some simulation results are given. For simulation purpose, we assume that the mass and the length of the first and the second links of the robot arm are $M_{i=(1,2)} = 1(\text{kg})$ and $L_{i=(1,2)} = 1(\text{m})$ respectively. The initial and the desired orientations of the first and the second links of the robot arm are $\theta_1(0) = -\pi/3$, $\theta_2(0) = \pi/2$, $\theta_1\text{d} = \pi/2$ and $\theta_2\text{d} = -\pi/2$ respectively. The states are estimate by a Kalman-Luenberger observer. A uniform noise signal in the range of ±0.03 rad for the orientations has been applied.

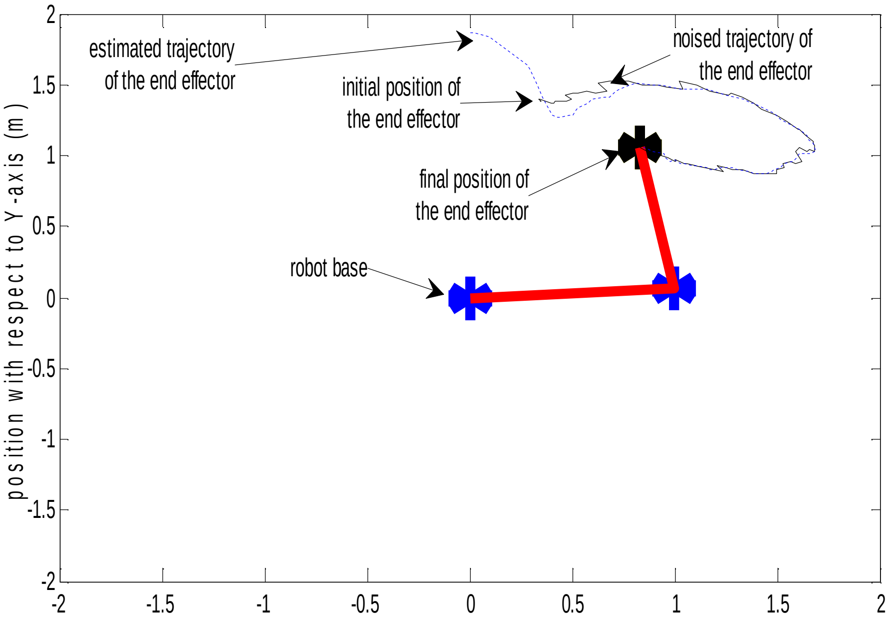

Fig.2 Final situation: the robot end effector reaches its objective point.

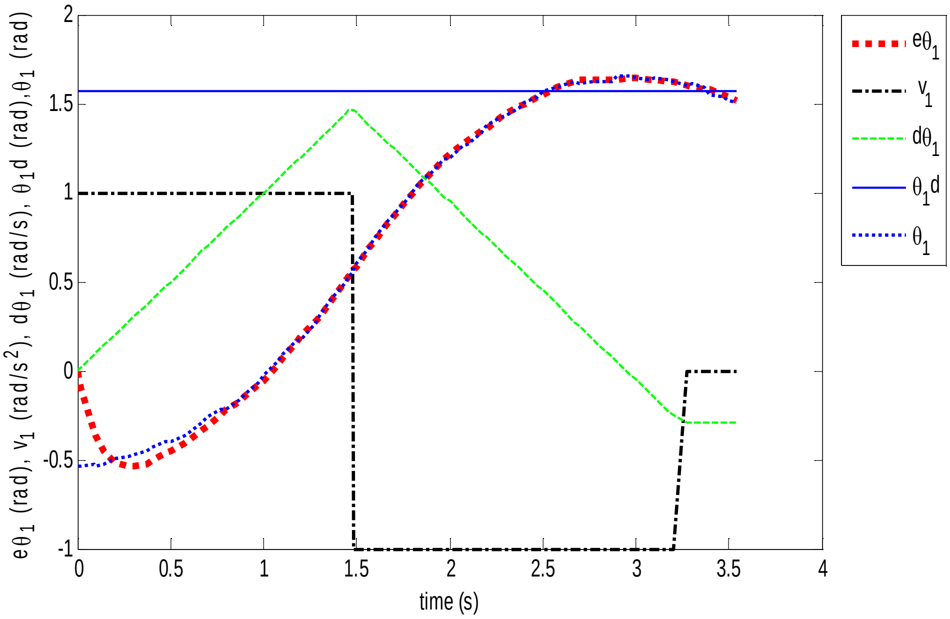

Fig.3 Estimated orientation, synthetic control, angular velocity, the desired orientation and the real orientation of the first link of the robot

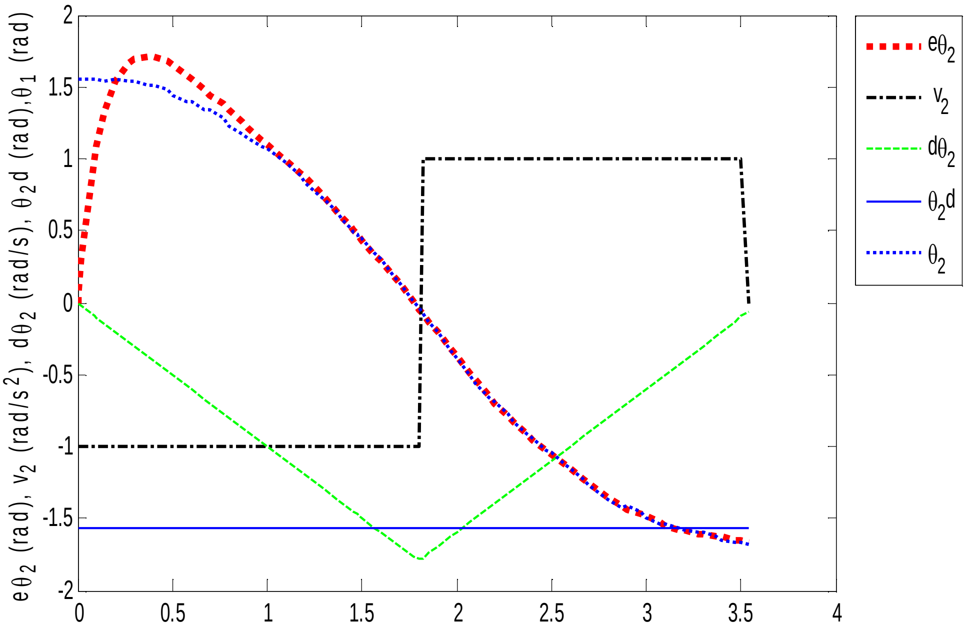

Fig.4 Estimated orientation, synthetic control, angular velocity, the desired orientation and the real orientation of the second link of the robot

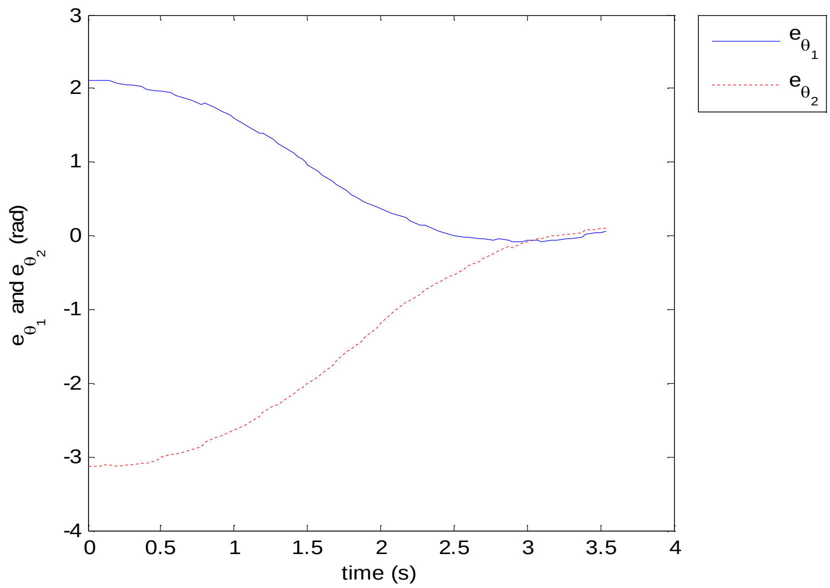

Fig.5 Angles errors

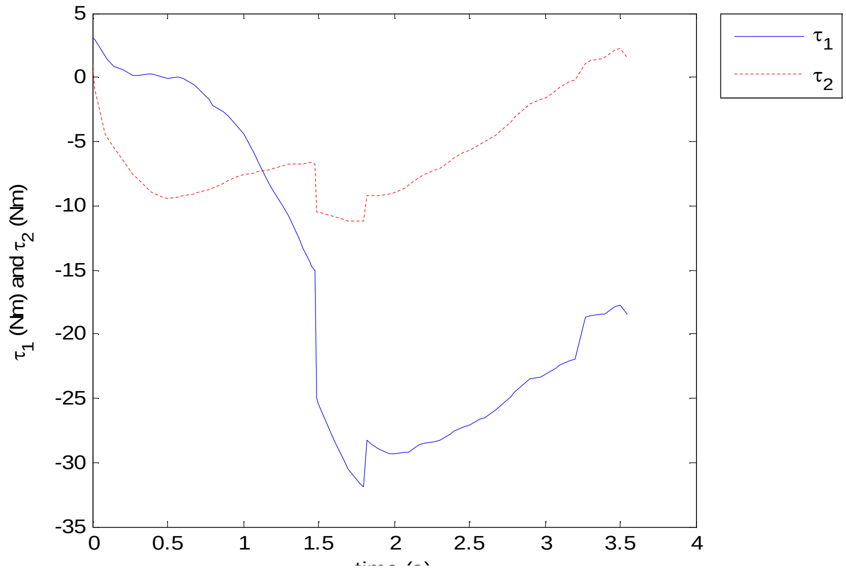

Fig.6 Robot torques

Fig.2 represents the covered estimated and real trajectory of the end effector of the robot arm and when it reaches its objective. We can see, the estimated trajectory converges to the actual trajectory with noise reduction. Fig.3 represents the settings of the first link of the robot arm. The angle θ_1 (dotted line) started from $\theta_1(0)=-\pi/3$ and reaches the desired angle $\theta_1 \mathrm{d}=\pi/2$ (solid line) at time $t_f=3.23(\mathrm{s})$. Also, we can see the angular velocity $d\theta_1$ (dashed line) and the synthetic control v_1 (dashed-dot line) which performed one jumps between +1 and -1 in the time interval $t\in[0,2.8](\mathrm{s})$. The dark dotted line curve presents the estimated angle ($e\theta_1$). In Fig.4, we present the same settings as in Fig.3 but for the second link of the robot arm. Here, we can see the angle θ_2 (dotted line) started from $\theta_2(0)=\pi/2$ and reaches the desired angle $\theta_2 \mathrm{d}=-\pi/2$ (solid line) at time $t_f=3.55(\mathrm{s})$. The angular velocity $d\theta_2$ (dashed line) and the synthetic control v_2 (dashed-dot line) are depicted. As in Fig.3, the synthetic control v_2 performed one jumps between +1 and -1 in the time interval $t\in[0,3.55](\mathrm{s})$. The dark dotted line curve presents the estimated angle ($e\theta_2$). The Fig.5 represents the convergence of the angles errors of the two links robot arm towards zero using the proposed approach of control. The robot torques present the feedback linearization control given by the equation (4) and they are depicted in the Fig.6.

6. CONCLUSION

The present article proposes a minimum time control approach for a robotic arm with two degrees of freedom in the case of the presence of the noise on the output. This technique consists of linearizing a nonlinear dynamic model of the robot by using a feedback linearization control. Next, based on the obtained linear model a minimum time control with constraints, using the Pontryagin Minimum Principle, has been developed. The state variables are estimated by a Kalman-Luenberger observer. Using Matlab, the obtained simulation results show the efficiency of the proposed approach. Future work is aimed to develop this approach for a manipulator robot of six DOF.

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