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Tolerance Synthesis of Delta-like Parallel Robots Using a Nonlinear Optimisation Method

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Abstract: Robotic systems require high accuracy in manipulating objects. Positioning errors are influenced by geometric tolerances and various sources. This paper introduces a new technique based on the interior-point algorithm optimisation method to allocate tolerances to the geometric parameters of a robot. This method consists of three steps. First, a method for modelling the kinematic problem as well as the geometric errors must be used. The Denavit–Hartenberg rule is the most suitable method for this modelling case. Then, a mathematical model for tolerance allocation is developed and used as a nonlinear multivariable optimisation problem. Finally, the "interior-point" algorithm is used to solve this optimisation problem. The accuracy and efficiency of the proposed method, in determining the tolerance allocations for a Delta parallel robot, is illustrated via calculation and simulation results. The values of the dimensional tolerances found are optimal. As a result, these values always keep the accuracy less than or equal to the imposed value.

Keywords: Delta parallel robot; modelling; tolerance synthesis; nonlinear optimisation

1. Introduction

Robots are often employed in many industrial and medical applications to position or orient an object. Typical examples of such applications include high-speed manufacturing processes, such as welding, drilling, assembling, etc., where the precision, repeatability, and stability of operations are required. Robot end-effector positioning deviations can be caused by different sources, such as joint clearances, manufacturing and assembly errors, measurement and control errors, elastic deformations of structural components, etc. [1,2]. Designing a robotic system often begins with selecting the length of segments, mass, and inertia of the links to meet performance specifications. In most cases, designing a robot that satisfies the desired performance requirement is quite complex. This is due to the nonlinearity and the coupled relationship between the actuators and the end-effector and the presence of uncertainties in the geometric, kinematic, and dynamic parameters. Therefore, selecting an optimal tolerance of design parameters becomes a challenging task.

An extensive review of the methodologies to obtain the robust design of products that have low performance variation, caused by the variations in control factors, was presented by Rout [3]. Wang et al. [4] studied the impact of manufacturing tolerances on the accuracy of a Stewart platform. Brahmia et al. [5,6] developed a new approach for the robust design of mechanisms based on error sensitivity analysis. Lian [7] investigated the parametric sensitivity of a 5-DoF parallel manipulator, in terms of mass and stiffness performance, using the response surface approach and performance reliability. Deterministic approaches based on matrix numerical analysis were utilised for mechanism tolerance calculations and sensitivity analysis, where the robustness problem is frequently referred to as "Conditioning" [8,9]. Caro et al. [10] studied the impact of dimensions and angular variations on the position of an Orthoglide's end-effector at three degrees of freedom of translation. In that study, two sensitivity indices were used: one for position sensitivity and the other one for orientation sensitivity.

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Because of the random nature of the joint clearances, Jeong Kim et al. [11] developed a stochastic technique to determine the reliability of the open-loop mechanism. With the premise that all kinematic parameters are normally distributed random variables, they proposed a stochastic model of links with dimensional tolerances and revolute joints with clearances. Based on an advanced first-order second-moment method, the kinematic reliability for positioning and orientation repeatability was then computed analytically. Rout and Mittal [12] employed an experimental design strategy to reduce performance variability by employing a probabilistic approach for the selection tolerance specification of robot kinematic and dynamic parameters. The performance metric, i.e., signal-to-noise ratio, was used to validate Monte Carlo simulations. Rao and Bhatti [13] presented manipulator reliability to represent its kinematic and dynamic performance using a probabilistic technique, where the manipulator reliability was defined as the probability of an end-effector pose to fall within a certain range of the desired pose.

Many researchers consider tolerance allocation to be an optimisation problem in which part-tolerance values are used as control variables and machining costs are used as the objective function to be minimised. However, robust tolerance design of mechanisms has been discussed in depth by creating multiobjective optimisation models [14–16]. Parkinson [17] described an application in which system tolerances were chosen using an optimisation approach. Goldsztejn et al. [18] proposed a local uniqueness hypothesis that would allow them to safely compute the pose error upper bounds, using a parametric version of Kantorovich's theorem and certified nonlinear global optimisation. To determine optimal parameter tolerance, Rout [19] used a genetic algorithm, differential evolution, and optimisation techniques. A local uniqueness hypothesis for reliably obtaining pose error upper limits using nonlinear optimisation was proposed in [20]. Trang et al. [21] introduced a technique using the generalised reduced gradient algorithm optimisation to allocate tolerances into robot parts. By using the Denavit–Hartenberg (DH) rule for modelling the kinematic problem in that study, a mathematical model for tolerance allocation was developed and used in the nonlinear multivariable optimisation problem. The generalised reduced gradient method was then used to solve this optimisation problem.

This paper proposes a new approach to determine tolerances of linear and angular dimensions. The allocation of tolerances is represented as an optimisation problem, and a new mathematical model is established and solved using the ”interior-point” algorithm. The modelling method and the allocation of tolerances for a Delta parallel robot case study are established using the proposed technique, and the results show that the method can be used in designing the optimal tolerance values of the parts of the mechanism and can be generalised for other types of robots.

The rest of this paper is organised as follows: In Section 2, the modelling of the Delta parallel robot is detailed. In Section 3, a model of geometric error is introduced. A new tolerance synthesis method is proposed in Section 4. Finally, the article ends with some concluding remarks in Section 5.

2. Materials and Methods

2.1. Modelling the Delta Parallel Robot

2.1.1. Geometric Description

A Delta parallel robot (cf. Figure 1) with 3 degrees of freedom (DOFs) has been the subject of several studies in the literature [22–25]. It was developed by Clavel (and others) in Switzerland at EPFL [26]. A Delta parallel robot was explored as an example of a high-speed application (e.g., pick and place). This type of robot has exceptional performance in terms of speed (up to 10 m/s) and acceleration (up to 20 g), and it has achieved significant commercial success.

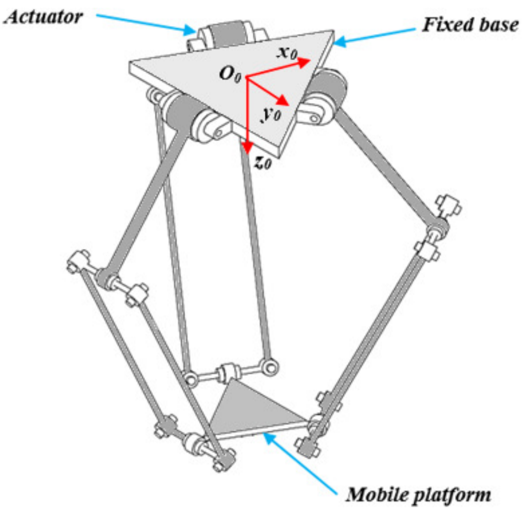

Figure 1. CAD view of Delta parallel robot.

Because a Delta parallel robot is symmetrical, only one of the three legs can be considered, as illustrated in Figure 2. The following coordinate systems can be established:

$O_0x_0y_0z_0$: The reference frame system also denoted by R_0, where O_0 is the centre of an equilateral triangle; defined by A_i $(i = 1, 2, 3)$; x_0 is along OA_1; and z_0 is orthogonal to the base plane determined by O_0A_i $(i = 1, 2, 3)$.

$A_ix_iy_iz_i$: The coordinate system denoted by R_i, where x_i is along OA_i.

A_i are the centres of the actuated links between the arms and the base $(i = 1, 2, 3)$, C_i are the centres of the cardan links between the arms and the parallelograms, B_i are the centres of the cardan links between the parallelograms and the end-effector (centre of mobile platform), O_0 is the barycentre of the points A_i, and P is the barycentre of the points B_i.

The dimensional parameters of the Delta parallel robot are the following lengths: $La_i = 260$ mm for the arms, $Lb_i = 480$ mm for the front arms, and the difference between the radii, $Ra = 194$ mm and $Rb = 30$ mm, is denoted by R.

θ_i is the angle between the ith arm and the plan of the fixed base, i.e., between x_0 and x_{0i} around the axis z_0. Its value is equal to $\frac{2\pi(i-1)}{3}$; $i = 1, 2, 3$.

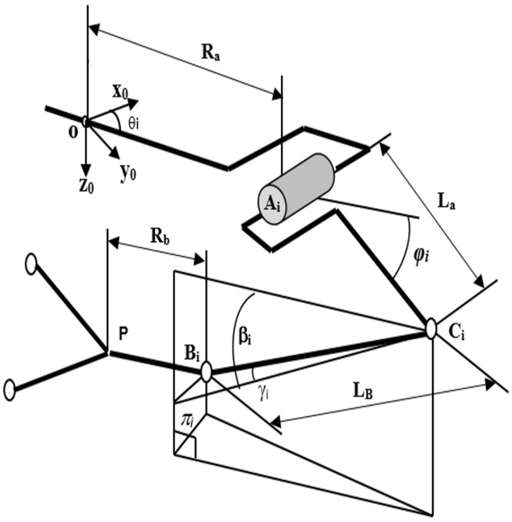

Figure 2. Illustration of the main geometric parameters of a Delta parallel robot.

2.1.2. Kinematic Equations

To parametrise the Delta parallel robot (using the Khalil–Kleinfiger parameterisation [27] for the robot with complicated chains), it is sufficient to open the kinematic closed loops at the mobile platform while taking into account the limitations imposed by the other chains.

Illustrated in Figure 3, the Khalil–Kleinfiger parameterisation rule is used to formulate the kinematic equation of each subchain, and the geometric parameters are listed in Table 1.

The forward kinematic equation of the ith chain, in terms of its Khalil–Kleinfiger parameters, is given by [27]:

$$P_i(q_a, q_p, v) = {}^{0}_{A}T_i \; {}^{A}_{1}T_i \; {}^{1}_{2}T_i \; {}^{2}_{3}T_i \; {}^{3}_{4}T_i \; {}^{4}_{5}T_i \; {}^{5}_{E}T_i \times [0\;0\;0\;1]^T, \\ (i = 1, 2, 3) \tag{1}$$

which can also be rewritten as:

$$P_i(q_a, q_p, v) = {}^{0}_{A}T_i \; {}^{A}_{1}T_i \; {}^{1}_{2}T_i \; {}^{2}_{6}T_i \; {}^{6}_{7}T_i \; {}^{7}_{5}T_i \; {}^{5}_{E}T_i \times [0\;0\;0\;1]^T, \\ (i = 1, 2, 3) \tag{2}$$

$$\begin{aligned} &q_a = q_1 i, q_p = \{q_{2i}, q_{3i}, q_{4i}, q_{5i}, q_{6i}, q_{7i}\}, \\ &q_v = \{La_i,\ Lb_i,\ Ra,\ Rb,\ d_i,\ \theta_i,\ \alpha_{1i},\ \alpha_{3i},\ \alpha_{5i},\ \alpha_{6i},\ \alpha_E\}, \\ &q_{1i} = \varphi_i, q_{3i} = \gamma_i \end{aligned}$$

where P_i is the mobile platform position and ${}^{j-1}_{j}T_i$, $(j = 1, \ldots, 5)$ $(i = 1,\ 2,\ 3)$ is the homogeneous transformation matrix (HTM) between the jth and $j + 1$th coordinate frame in the ith subchain. ${}^{5}_{E}T_i$ is the HTM between the end-effector and the 5th coordinate frame. The variables q_a, q_p, and q_v represent the robot's active joint variables, passive joint variables, and parametric variables, respectively. Each chain from the base frame to the mobile platform can be considered as a serial robot.

The elementary matrices to describe a kinematic chain i are as follows:

$${}^{0}_{A}T_i = \begin{bmatrix} cos(\theta_i) & -sin(\theta_i) & 0 & 0 \\ sin(\theta_i) & cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{A}_{1}T_i = \begin{bmatrix} cos(q_{1i}) & -sin(q_{1i}) & 0 & R_a \\ 0 & 0 & -1 & 0 \\ sin(q_{1i}) & cos(q_{1i}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{1}_{2}T_i = \begin{bmatrix} cos(q_{2i}) & -sin(q_{2i}) & 0 & 0 \\ sin(q_{2i}) & cos(q_{2i}) & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{2}_{3}T_i = \begin{bmatrix} cos(q_{3i}) & -sin(q_{3i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin(q_{3i}) & -cos(q_{3i}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{3}_{4}T_i = \begin{bmatrix} cos(q_{4i}) & -sin(q_{4i}) & 0 & Lb \\ sin(q_{4i}) & cos(q_{4i}) & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{4}_{5}T_i = \begin{bmatrix} cos(q_{5i}) & -sin(q_{5i}) & 0 & 0 \\ 0 & 0 & -1 & d \\ sin(q_{5i}) & cos(q_{5i}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{2}_{6}T_i = \begin{bmatrix} cos(q_{6i}) & -sin(q_{6i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin(q_{6i}) & -cos(q_{6i}) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^{6}_{7}T_i = \begin{bmatrix} cos(q_{7i}) & -sin(q_{7i}) & 0 & Lb \\ sin(q_{7i}) & cos(q_{7i}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{5}_{E}T_i = \begin{bmatrix} 1 & 0 & 0 & Rb \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

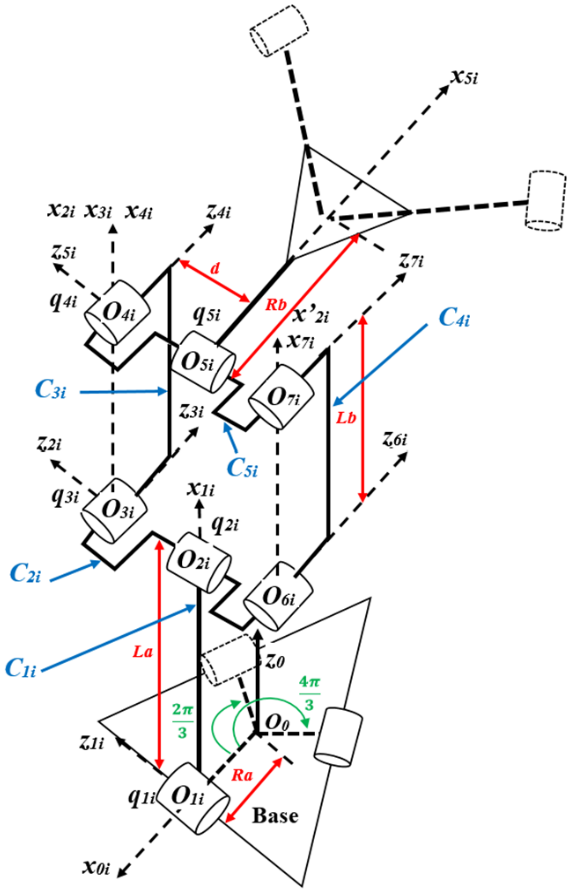

Figure 3. Khalil–Kleinfiger parameterisation of the Delta parallel robot.

Table 1. Khalil–Kleinfiger parameters of the Delta parallel robot.

i	$a(j)$	μ_j	γ_j	b_j	ε_j	d_j	q_j	r_j
1	0	1	0	0	$\pi/2$	R_a	q_1	0
2	1	0	0	$2d$	0	L_a	q_2	d
3	2	0	0	0	$-\pi/2$	0	q_3	0
4	3	0	0	0	0	L_b	q_4	0
5	4	0	0	0	$\pi/2$	0	q_5	$-d$
5	7	0	0	0	$\pi/2$	0	q_5	$-d$
6	2	0	0	0	$-\pi/2$	0	q_6	0
7	6	0	0	0	0	L_b	q_7	0
E		0	0	0	$\pi/2$	R_b	0	0

Indeed, by considering a restriction on the ends of the three serial subchains, the forward kinematics of the parallel mechanism can be determined. The end points of the three subchains, P_i $(Px_i,\ Py_i,\ Pz_i)$, for $i = 1, 2, 3$, are the same and are represented by the expression:

$$\begin{bmatrix} x - Px_i \\ y - Py_i \\ z - Pz_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

To calculate the values of the active q_a and passive q_p joint variables in a given position (x, y, z), we solved the inverse kinematic model (3).

For each chain, there are seven joint variables and four rotation angles in each subchain, each with the same value $(q_{3i} = q_{6i}, q_{4i} = q_{7i} = -q_{3i})$. Using the three constraint equations (in (3)), it is possible to calculate the value of the three variables. If the position (x, y, z) of the platform is known, the resolution of the inverse geometric model is possible.

2.2. Modelling of Geometric Errors

This section presents an error model that includes robot kinematic parameter errors and active and passive joint errors. Existing parallel-mechanism kinematic error models are mostly based on the Khalil–Kleinfiger parameters model [27]. This model concerns the presentation of manufacturing and assembly errors, such as variations in the lengths of the links and perpendicularity errors. The Khalil–Kleinfiger rule defines the transformation matrix, which includes the error sources, and forward kinematics can be solved by multiplying these transformation matrices. The error model between successive coordinates is represented in Equation (4) with twelve error components, including four joint error components, seven kinematic error components (related to kinematic parameters), and one control error parameter. In HTM form, the transformation matrix that contains the aforementioned errors, ${}^{j-1}_{j}T_{ei}$, $(j = 1, \ldots, 5)$ $(i = 1, 2, 3)$, may be described as follows:

$$\begin{aligned} {}^{j-1}_{j}T_{ei} = Rot(x_j, \alpha_j + \Delta\alpha_j)\ \ Trans(x_j, d_j + \Delta d_j) \\ Rot(z_j, q_j + \Delta q_j) Trans(z_j, r_j + \Delta r_j) \end{aligned} \tag{4}$$

with *Rot* and *Trans* representing the homogeneous matrices of rotation and translation, respectively.

By connecting the Khalil–Kleinfiger parameter matrix from the base frame to the mobile platform, the relationship between this last one and the kinematic error parameters, including active and passive joints, can be determined. Because the mechanism is composed of several subchains, the kinematic constraint equation may be determined at the end position of each subchain, with errors.

The error model's mobile platform position, Pe_i, may be represented as a function of the error parameters [28] as follows:

$$P_{ei} = P_i + \Delta P_i = f(q_a + \Delta q_a, q_p + \Delta q_p, q_v + \Delta q_v) \tag{5}$$

$$\Delta q_a = \{\Delta q_{1i}\}, \Delta q_p = \{\Delta q_{2i}, \Delta q_{3i}, \Delta q_{4i}, \Delta q_{5i}, \Delta q_{6i}, \Delta q_{7i}\}$$
$$\Delta q_v = \{\Delta La_i, \Delta Lb_i, \Delta d_i, \Delta Ra, \Delta Rb, \Delta\theta_i, \Delta\alpha_{1i}, \Delta\alpha_{3i}, \Delta\alpha_{5i}, \Delta\alpha_{6i}, \Delta\alpha_E\}, (i = 1, 2, 3).$$

f represents the forward kinematics. The ith chain's tool tip is expressed as $P_{ei}(x_i, y_i, z_i)$ at the platform centre position and each serial subchain may be defined using the Khalil–Kleinfiger parameters. The geometric parameter errors of the Delta parallel robot are summarised in Table 2.

The forward kinematic equation of the ith chain is expressed by [29]:

$$\begin{aligned} &P_{ei} = {}^{0}_{A}T_{ei}\ {}^{A}_{1}T_{ei}\ {}^{1}_{2}T_{ei}\ {}^{2}_{3}T_{ei}\ {}^{3}_{4}T_{ei}\ {}^{4}_{5}T_{ei}\ {}^{5}_{E}T_{ei} \times [0\ 0\ 0\ 1]^T, \\ &(i = 1, 2, 3) \end{aligned} \tag{6}$$

This clearly highlights that the end points of the three chains, $P_{ei}(x_i, y_i, z_i)$ $(i = 1, 2, 3)$, are identical.

From the systems in Equations (5) and (6), we can reduce the following relationships:

$$\begin{cases} P_{xei} = P_{xi} + \Delta P_{xi} \\ P_{yei} = P_{yi} + \Delta P_{yi} \\ P_{zei} = P_{zi} + \Delta P_{zi} \end{cases} \tag{7}$$

Squaring the two sides of (7) leads to:

$$\begin{cases} (P_{xei} - P_{xi} - \Delta P_{xi})^2 = 0 \\ (P_{yei} - P_{yi} - \Delta P_{yi})^2 = 0 \\ (P_{zei} - P_{zi} - \Delta P_{zi})^2 = 0 \end{cases} \tag{8}$$

The sum of the two sides of (8) yields:

$$(P_{xei} - P_{xi} - \Delta P_{xi})^2 + (P_{yei} - P_{yi} - \Delta P_{yi})^2 + (P_{zei} - P_{zi} - \Delta P_{zi})^2 = 0 \tag{9}$$

It is clear that Equation (9) is always ≥ 0, so its minimum is 0.
Denoting the objective function for the left-hand side of Equation (9) by F leads to:

$$F = (P_{xei} - P_{xi} - \Delta P_{xi})^2 + (P_{yei} - P_{yi} - \Delta P_{yi})^2 + (P_{zei} - P_{zi} - \Delta P_{zi})^2 \tag{10}$$

Table 2. Khalil–Kleinfiger parameters for error modelling of the Delta parallel robot.

i	$a(j)$	μ_j	γ_j	b_j	ε_j	d_j	q_j	r_j
1	0	1	0	0	$\pi/2 + \Delta\alpha_1$	$R_a + \Delta R_a$	$q_1 + \Delta q_1$	0
2	1	0	0	$2(d + \Delta d)$	0	$L_a + \Delta L_a$	$q_2 + \Delta q_2$	$d + \Delta d$
3	2	0	0	0	$-(\pi/2 + \Delta\alpha_3)$	0	$q_3 + \Delta q_3$	0
4	3	0	0	0	0	$L_b + \Delta L_b$	$q_4 + \Delta q_4$	0
5	4	0	0	0	$\pi/2 + \Delta\alpha_5$	0	$q_5 + \Delta q_5$	$-(d + \Delta d)$
5	7	0	0	0	$\pi/2 + \Delta\alpha_5$	0	$q_5 + \Delta q_5$	$-(d + \Delta d)$
6	2	0	0	0	$-(\pi/2 + \Delta\alpha_6)$	0	$q_6 + \Delta q_6$	0
7	6	0	0	0	0	$L_b + \Delta Lb$	$q_7 + \Delta q_7$	0
E		0	0	0	$\pi/2$	$R_b + \Delta Rb$	0	0

2.3. Proposed Tolerance Synthesis Method

To calculate geometric tolerances, we propose a new approach for the synthesis of tolerance mechanisms. In this approach, we are interested in calculating the geometric tolerances of the mechanisms with a calculation based on the resolution of the nonlinear system of the inverse geometric model, along with errors defined using the Khalil–Kleinfiger method and intended for the robots with a parallel structure.
To calculate geometric tolerances using this approach, we propose the following three steps:

1. Calculate the values of the active joints q_{1i} and passive joints $(q_{2i}, q_{3i}, q_{4i}, q_{5i}, q_{6i}, q_{7i})$ for different configurations (x_k, y_k, z_k), $k = 1, \ldots, m$ by solving the inverse kinematic model Equation (3), leading to:

$$Q_k = (q_{1i}, q_{2i}, q_{3i}, q_{4i}, q_{5i}, q_{6i}, q_{7i})^{(k)} \tag{11}$$

2. Formulate the following optimisation problem to find the geometric tolerances of the robot:

$$Min\ F(\Delta L_i, \Delta\phi_j), i = 1, \ldots, 11; j = 12, \ldots, 47 \tag{12}$$

Subject to:

$$\begin{cases} \Delta l_{imax} \geq |\Delta L_i| \geq \Delta l_{imin} \\ \Delta\varphi_{jmax} \geq |\Delta\phi_j| \geq \Delta\varphi_{jmin} \end{cases}$$

where ΔL_i are the tolerance intervals of the ith linear parameter; $\Delta\phi_j$ are the tolerance intervals of the jth angular parameter; and Δl_{imin}, $\Delta\varphi_{jmin}$, Δl_{imax}, and $\Delta\varphi_{jmax}$ are the minimum and maximum dimensional tolerances tolerated for L_i and ϕ_j, respectively. This nonlinear optimisation problem can be solved using several methods, such as the ”interior-point” method, SQP (sequential quadratic programming), active set, GA (genetic algorithm), etc.

3. The tolerance of the dimensional parameters is chosen after solving the optimisation problem (12) for different configurations of the robot. Figure 4 illustrates the preferred tolerance range of the different dimensional parameters.

For a regular workspace of the robot, we distinguish several representative points, in which the resolution of the optimisation problem is possible. The tolerance range of the *i*th dimensional parameter that changes around the nominal value can be calculated based on small movements of the end-effector. The set of calculated tolerance values, which have negative values (corresponding to dimensions smaller than the nominal dimensions), are defined by the vector $y_{neg} = (y_{neg1}, y_{neg2}, \ldots, y_{negn})$. The set of calculated tolerance values, which have positive values (corresponding to dimensions higher than the nominal dimensions), are defined by the vector $y_{pos} = (y_{pos1}, y_{pos2}, \ldots, y_{posn})$. In order to keep the tolerance of the end-effector within the range of admissible values, the tolerance of the *i*th dimensional parameter should be chosen as follows: the minimum tolerance is equal to the maximum value of the vector $y_{neg}(y_{neg\_max})$ and the maximum tolerance is equal to the minimum value of the vector $y_{pos}(y_{pos\_min})$.

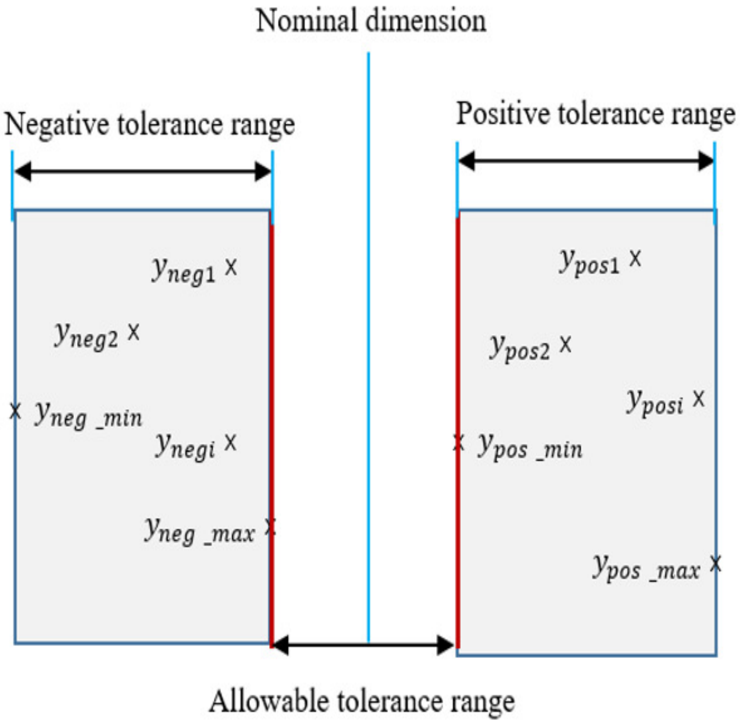

Figure 4. Choice of the tolerance of the dimensional parameters.

3. Results

3.1. Tolerance Synthesis of the Delta Parallel Robot

In this section, both linear and angular geometric tolerances of the Delta parallel robot are calculated using the proposed tolerance synthesis method. The passive and active joints are gathered in the vector Q_k (Equation (11)), which are calculated for fifteen different postures of the robot (cf. illustration of Figure 5). These postures are located inside a regular workspace in the form of a cylinder [30].

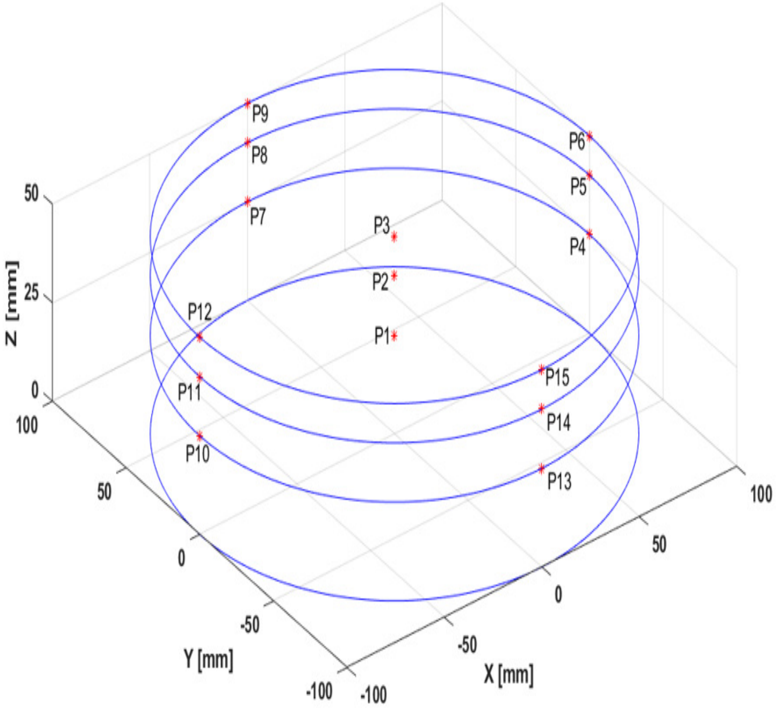

Figure 5. Representation of the P_i points in the regular workspace of the Delta parallel robot.

The following optimisation problem is then used to calculate geometric tolerances:

$$Min\ F(\Delta L_i, \Delta\phi_j), i = 1, \ldots, 11; j = 12, \ldots, 47 \tag{13}$$

Subject to:

$$\begin{cases} -0.015\text{ mm} \geq |\Delta L_i| \geq 0.015\text{ mm} \\ -0.015^\circ \geq |\Delta\phi_j| \geq 0.015^\circ \end{cases}$$

The parameters of the linear dimensions of the robot are: the fixed base and the moving platform are equiangular triangles with lengths $O_0O_{1i} = Ra = 194$ mm and $O_{5i}P = Rb = 30$ mm, respectively, and the nominal dimensions of the parallelepipeds are $La_i = 260$ mm, $Lb_i = 480$ mm, and $d_i = 20$ mm.

The tolerances of the angular parameters are grouped in the vector $\Delta\phi = \{\Delta q_{1i}, \Delta q_{2i}, \Delta q_{3i}, \Delta q_{4i}, \Delta q_{5i}, \Delta q_{6i}, \Delta q_{7i}, \Delta\theta_i, \Delta\alpha_{1i}, \Delta\alpha_{3i}, \Delta\alpha_{5i}, \Delta\alpha_{6i}\}$, $i = 1, \ldots, 3$. The solution of the formulated optimisation problem is calculated using MATLAB function fmincon, and the “interior-point” method is used.

The tolerances of the linear parameters are grouped in the vector $\Delta L = \{\Delta La_i, \Delta Lb_i, \Delta d_i, \Delta Ra, \Delta Rb\}$, $i = 1, \ldots, 3$.

The solution of the optimisation problem is calculated using the Matlab function $''fmincon''$ and the ”interior-point” method is used. The maximum position error imposed is equal to 0.2 mm. The optimisation calculation was carried out for the points $P1$, $P3$, $P6$, and $P9$, shown in Figure 5. For each posture, six tests were performed, and the results of the calculations are summarised in Tables A1 and A2 (see Appendix A).

By applying the tolerance range shown in Figure 4, the tolerance range of each dimensional parameter is computed, and they are shown in Table 3. It is worth noting that linear parameters are presented in mm and angular parameters are presented in degrees.

Table 3. Resulting dimensional tolerances of the Delta parallel robot.

Parameter	Tolerance min	Tolerance max	Parameter	Tolerance min	Tolerance max
ΔRa	-0.0097	0.0096	Δq_{12}	-0.0012	0.0010
ΔRb	-0.0096	0.0097	Δq_{22}	-0.0023	0.0010
Δd_1	-0.0111	0.0111	Δq_{32}	-0.0016	0.0010
ΔLa_1	-0.0098	0.0096	Δq_{42}	-0.0011	0.0010
ΔLb_1	-0.0096	0.0097	Δq_{52}	-0.0023	0.0010
$\Delta \alpha_{11}$	-0.0109	0.0109	Δq_{62}	-0.0016	0.0010
$\Delta \alpha_{31}$	-0.0101	0.0100	Δq_{72}	-0.0015	0.0014
$\Delta \alpha_{51}$	-0.0098	0.0095	$\Delta \theta_2$	-0.0012	0.0011
$\Delta \alpha_{61}$	-0.0111	0.0111	Δd_3	-0.0017	0.0020
Δq_{11}	-0.0098	0.0098	ΔLa_3	-0.0025	0.0016
Δq_{21}	-0.0096	0.0097	ΔLb_3	-0.0017	0.0012
Δq_{31}	-0.0026	0.0010	$\Delta \alpha_{13}$	-0.0017	0.0019
Δq_{41}	-0.0057	0.0010	$\Delta \alpha_{23}$	-0.0024	0.0016
Δq_{51}	-0.0068	0.0010	$\Delta \alpha_{53}$	-0.0011	0.0011
Δq_{61}	-0.0057	0.0010	$\Delta \alpha_{63}$	-0.0015	0.0012
Δq_{71}	-0.0021	0.0017	Δq_{13}	-0.0013	0.0020
$\Delta \theta_1$	-0.0071	0.0022	Δq_{23}	-0.0016	0.0016
Δd_2	-0.0032	0.0016	Δq_{33}	-0.0011	0.0015
ΔLa_2	-0.0070	0.0022	Δq_{43}	-0.0013	0.0020
ΔLb_2	-0.0020	0.0013	Δq_{53}	-0.0016	0.0016
$\Delta \alpha_{12}$	-0.0057	0.0055	Δq_{63}	-0.0055	0.0058
$\Delta \alpha_{32}$	-0.0043	0.0038	Δq_{73}	-0.0049	0.0049
$\Delta \alpha_{52}$	-0.0057	0.0055	$\Delta \theta_3$	-0.0051	0.0051
$\Delta \alpha_{62}$	-0.0011	0.0011			

3.2. Flow Diagram

The flow diagram (Figure 6) shows the algorithm for calculating geometric tolerances.

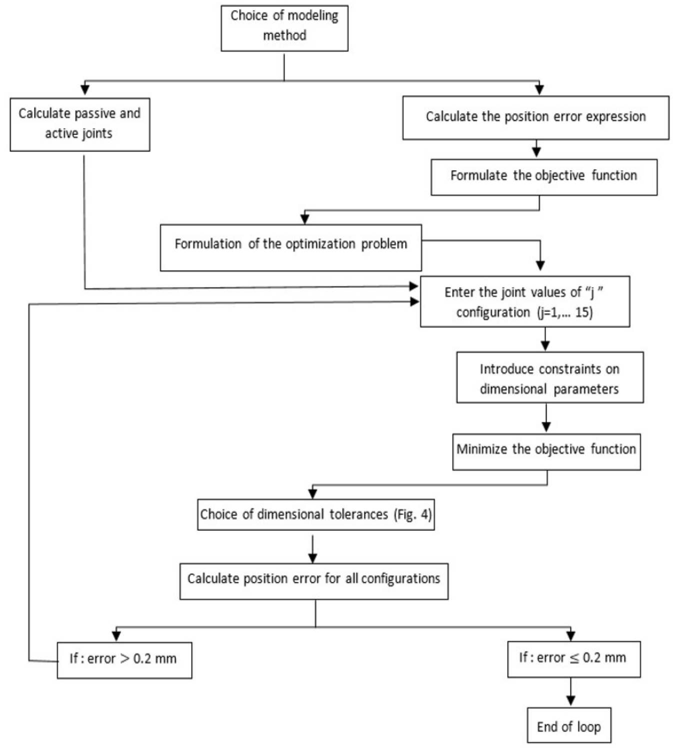

Figure 6. Design flow diagram.

4. Discussion

4.1. Verification of the Accuracy of the Proposed Method

In the robot design technique, the manufacturing tolerances of the geometric parameters are calculated to meet the requirements cited in the specifications, such as precision, manufacturing cost, etc. If precision is required (as in this study), the position error of the end-effector should not exceed their imposed value, regardless of the robot posture. However, to validate our design method, we performed a calculation of the position error for different robot postures (points $P1$, $P2$, . . ., $P15$) and different values of dimensional tolerances.

Figure 7 shows the calculation of the position error for the following tolerance values:

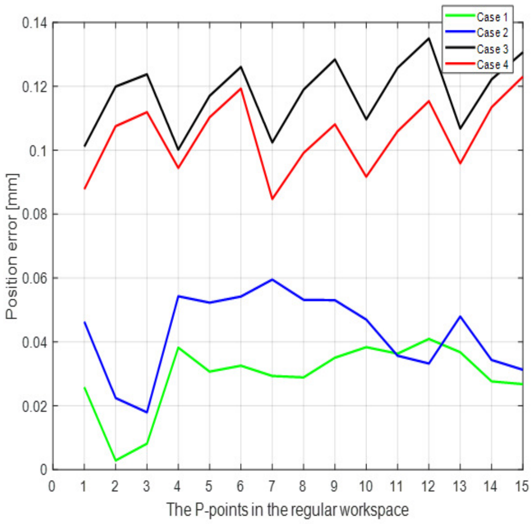

Figure 7. Variation in the position error of the 15 robot postures as a function of the variation in the geometrical tolerances.

1. Case 1: The geometric tolerance of each parameter is equal to the upper limit of its tolerance interval (the green curve in Figure 7);
2. Case 2: The geometric tolerance of each parameter is equal to the lower limit of its tolerance interval (the blue curve in Figure 7);
3. Case 3: The geometric tolerance of each linear parameter is equal to the upper limit of its tolerance interval, while the geometric tolerance of each angular parameter is equal to the lower limit of its tolerance interval (the black curve in Figure 7);
4. Case 4: The geometric tolerance of each linear parameter is equal to the lower limit of its tolerance interval, while the geometric tolerance of each angular parameter is equal to the upper limit of its tolerance interval (the red curve in Figure 7).

From Figure 7, it can be clearly seen that the value of the position error is between 0.0029 mm and 0.1350 mm; it is always lower than the imposed value (0.2 mm), regardless of the value of the tolerance of the geometric parameters. Moreover, although the optimisation calculation was only performed for points $P1$, $P3$, $P6$, and $P9$, the results obtained remain valid for all the other points (cf. Figure 7). This means that the calculated tolerances of the geometrical parameters satisfy the required accuracy of the initial design.

4.2. Effect of Geometric Tolerances on Robot Repeatability

The accuracy of a robot's design and manufacturing is an interesting factor that affects the repeatability of the robot. Repeatability is an important measure of a robot's performance because it determines how accurately the robot can perform repetitive tasks. High repeatability means that the robot is more likely to produce parts or products of consistent quality. Our design method allows us to obtain high-accuracy robots. This is obtained by calculating geometric tolerances according to a given accuracy (required value) of the robot's end-effector. In this way, the repeatability of the robot can be improved.

4.3. Impact of the Proposed Method on the Robust Design of the Delta Robot

Increasing the robot's accuracy makes it possible to obtain tight values for the robot's dimensional parameters, which influences the robot's manufacturing costs. However, a robust design is required. To achieve a robust design, we can add to the optimisation problem (Equation (13)) other constraints on the dimensions for which the robot has great sensitivity. Tolerance intervals of these dimensions must be tight, while all other tolerance intervals must be wide. In this case, we can obtain a robust mechanism that meets the requirements mentioned in the specifications (precision in our case) with a minimum cost.

5. Conclusions

In this study, a position error modelling procedure and tolerance synthesis of the Delta parallel robot are proposed. In the position error modelling part, Khalil–Kleinfiger parameterisation was used, and the geometric errors were introduced into the homogeneous transformation matrices. The direct geometric model equations are used to calculate geometric errors. Concerning the tolerance synthesis part, a three-step sequential method was proposed to calculate the geometric tolerances of the Delta parallel robot. The mathematical model used does not require the calculation of the inverse of the Jacobian matrix. Therefore, the convergence is faster in this case.

The calculation and simulation results show the efficiency and accuracy of the proposed method for determining linear and angular geometric tolerances for the Delta parallel robot. The obtained dimensional tolerance values are optimal. Therefore, these values maintain accuracy at a level that is lower than or equal to the imposed value (0.2 mm).

A verification calculation of the position error for different values of geometric tolerances was carried out. The verification results show that the value of the position error is always less than the value of the accuracy, which means that the proposed method is very effective for the design of mechanism geometric tolerances.

In addition, the proposed design method allows us to obtain high-accuracy robots. This is obtained by calculating geometric tolerances according to a given accuracy (required value) of the robot's end-effector. In this way, the repeatability of the robot can be improved.

When high accuracy is required, it is necessary to achieve a robust design. In this case, other constraints on the dimensional parameters must be added to the optimisation problem (Equation (13)). To determine these constraints, a sensitivity analysis must be performed. The robust design allows us to obtain a mechanism that meets the requirements mentioned in the specifications at a minimum cost.

The proposed method presents an interesting advantage in the number of calculations of geometric tolerances. As mentioned in Section 4.2, among the fifteen robot configurations, only four are needed to calculate all the geometric tolerances, which means that the calculation algorithm quickly converges to the desired solution.

The proposed method allows for the calculation of all types of linear and angular errors in a robot and can be generalised to more complex types of serial and parallel robots. In future works, we may consider the study of orientation tolerances (parallelism, perpendicularity, and inclination) to quantify their impact on the robot's accuracy and validate the calculation on a real robot. We can improve the proposed tolerance method and achieve a robust design based on the accuracy criterion. We are also interested in exploring other methods of calculating geometric tolerances, especially statistical methods.

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Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Table A1. Extracted results of measured tolerances of the Delta robot. Points P1 and P3.

	Point: $P1$ (mm) $P_x = 0, P_y = 0, P_z = 250$						Point: $P3$ (mm) $P_x = 0, P_y = 0, P_z = 500$					
Parameter	1	2	3	4	5	6	1	2	3	4	5	6
$P_x + \Delta P_x$	0.2	0	−0.2	0	0	0	0.2	0	−0.2	0	0	0
$P_y + \Delta P_y$	0	0.2	0	−0.2	0	0	0	0.2	0	−0.2	0	0
$P_z + \Delta P_z$	250	250	250	250	250.2	249.8	500	500	500	500	500.2	499.8
ΔRa	0.0095	−0.00973	0.00997	−0.01266	0.01388	−0.01388	0.01218	−0.01196	0.01199	0.01015	−0.01007	0.00998
ΔRb	−0.00958	0.01151	0.01104	−0.00979	0.00967	−0.00972	−0.01226	0.01202	0.01289	−0.01196	0.01006	−0.01000
Δd_1	−0.01389	0.01181	0.01173	0.01185	0.01169	−0.01176	−0.01256	0.01233	−0.01375	0.01203	0.01194	−0.01188
ΔLa_1	0.00958	−0.01035	−0.01181	0.01050	−0.00982	0.00985	0.01259	−0.01234	0.01249	0.01231	0.01250	−0.01247
ΔLb_1	−0.00958	0.00971	0.01183	−0.00987	0.00966	−0.00971	0.01243	−0.01233	−0.01174	−0.01130	0.01002	−0.00995
$\Delta\alpha_{11}$	0.00105	−0.01013	−0.00345	0.00409	−0.00521	0.00476	0.00968	−0.01034	−0.00308	0.00304	−0.00499	0.00400
$\Delta\alpha_{31}$	0.00104	0.00919	0.00792	0.00765	−0.00808	0.00812	0.00786	0.00775	0.00820	0.00750	−0.00798	0.00804
$\Delta\alpha_{51}$	0.00104	0.01026	0.00799	0.00745	−0.00831	0.00838	0.00786	0.00760	−0.00717	0.00715	−0.00810	0.00821
$\Delta\alpha_{61}$	0.00104	0.00919	0.00792	0.00765	−0.00808	0.00812	0.00787	0.00773	0.00822	0.00747	−0.00797	0.00803
Δq_{11}	−0.00574	0.00291	−0.01047	0.01241	0.00113	−0.00113	−0.00931	0.00920	0.00820	0.01087	0.00115	−0.00113
Δq_{21}	0.00104	−0.00122	0.00318	−0.00825	−0.00622	0.00622	0.00517	−0.00744	−0.00728	−0.00910	−0.00547	0.00539
Δq_{31}	0.00104	−0.00244	0.00232	−0.00330	−0.00231	0.00222	0.00530	−0.00437	0.00289	−0.00291	−0.00346	0.00301
Δq_{41}	0.00104	−0.00351	0.00721	0.00573	−0.00175	0.00165	0.00768	−0.00777	0.00750	0.00788	−0.00158	0.00207
Δq_{51}	0.00104	−0.00187	0.00713	0.00448	−0.00154	0.00148	0.00748	−0.00801	0.00797	0.00719	−0.00107	0.00156
Δq_{61}	0.00104	−0.00244	0.00232	−0.00330	−0.00231	0.00221	0.00531	−0.00438	0.00290	−0.00291	−0.00346	0.00299
Δq_{71}	0.00104	−0.00351	0.00721	0.00573	−0.00175	0.00165	0.00769	−0.00779	0.00752	0.00786	−0.00157	0.00206
$\Delta\theta_1$	0.01300	0.00784	0.00786	0.00784	0.00783	0.00785	0.00788	0.00787	0.00788	0.00779	0.00784	0.00785
Δd_2	0.01200	−0.01202	0.01244	−0.01203	0.01184	−0.01193	0.01244	−0.01232	0.01220	−0.01177	0.01201	−0.01196
ΔLa_2	−0.01036	0.01355	0.00999	−0.01012	−0.01093	0.01099	−0.01219	0.01234	0.01235	−0.01166	0.01212	−0.01205
ΔLb_2	0.00988	0.00958	−0.00978	0.00952	0.01034	−0.01039	−0.01226	0.01208	0.01178	−0.01023	0.01061	−0.01052
$\Delta\alpha_{12}$	0.00497	−0.00304	0.00192	−0.00271	−0.00951	0.00952	0.00207	−0.00288	0.00166	−0.00208	−0.00806	0.00797
$\Delta\alpha_{32}$	0.01046	0.00781	0.00225	0.01000	−0.00795	0.00797	0.00724	0.00812	0.00679	0.00843	−0.00784	0.00785
$\Delta\alpha_{52}$	0.01146	0.00778	0.00163	−0.00323	−0.00806	0.00808	0.00658	0.00836	0.00573	−0.00870	−0.00782	0.00784
$\Delta\alpha_{62}$	0.01017	0.00781	0.00225	0.01000	−0.00795	0.00797	0.00724	0.00812	0.00679	0.00845	−0.00784	0.00785
Δq_{12}	0.00236	−0.01283	−0.00606	0.00457	0.00195	−0.00193	0.01005	−0.01004	−0.01039	0.01299	0.00143	−0.00149
Δq_{22}	−0.00123	0.00337	0.00109	−0.00118	−0.00596	0.00595	−0.00788	0.00552	0.00612	−0.00758	−0.00483	0.00488
Δq_{32}	−0.00194	0.00302	−0.00208	0.00226	−0.00320	0.00321	−0.00208	0.00289	−0.00166	0.00196	−0.00510	0.00506
Δq_{42}	−0.00252	0.01156	0.00159	−0.00340	−0.00512	0.00505	−0.00736	0.00754	0.00900	−0.00970	−0.00516	0.00510
Δq_{52}	−0.00180	0.01205	0.00124	−0.00170	−0.00452	0.00443	−0.00824	0.00777	0.00806	−0.01048	−0.00340	0.00348
Δq_{62}	−0.00200	0.00303	−0.00209	0.00226	−0.00320	0.00321	−0.00208	0.00288	−0.00166	0.00194	−0.00510	0.00505
Δq_{72}	−0.00244	0.01156	0.00159	−0.00340	−0.00512	0.00505	−0.00736	0.00754	0.00900	−0.00971	−0.00516	0.00510
$\Delta\theta_2$	0.00787	0.00783	0.00784	0.00784	0.00783	0.00785	0.00787	0.00788	0.00788	0.00791	0.00785	0.00785
Δd_3	−0.01211	0.01196	0.01193	−0.01162	0.01169	−0.01176	−0.01244	0.01234	0.01297	−0.01201	0.01194	−0.01188
ΔLa_3	−0.01121	0.01050	−0.01119	0.01019	−0.00982	0.00985	−0.01375	0.01239	−0.01244	0.01209	0.01250	−0.01248
ΔLb_3	0.01037	−0.00979	0.01045	−0.00959	0.00966	−0.00971	−0.01192	0.01167	−0.01153	0.01186	0.01003	−0.00995
$\Delta\alpha_{13}$	−0.00382	0.00347	−0.00880	0.00743	−0.00521	0.00476	−0.00307	0.00261	0.00131	−0.00198	−0.00428	0.00354
$\Delta\alpha_{23}$	0.00948	0.00549	0.00870	0.00565	−0.00808	0.00812	0.00819	0.00739	0.00580	0.00830	−0.00797	0.00802
$\Delta\alpha_{53}$	−0.00430	0.00384	0.00938	0.00388	−0.00831	0.00838	−0.00721	0.00690	0.00397	0.00873	−0.00809	0.00819
$\Delta\alpha_{63}$	0.00945	0.00549	0.00870	0.00565	−0.00808	0.00812	0.00819	0.00739	0.00580	0.00832	−0.00797	0.00803
Δq_{13}	0.00608	−0.00782	0.00501	−0.00836	0.00113	−0.00113	0.01243	−0.01231	0.01186	−0.00863	0.00115	−0.00113
Δq_{23}	−0.00149	0.00125	−0.00160	0.00121	−0.00622	0.00622	−0.00827	0.00683	−0.00837	0.00489	−0.00547	0.00539
Δq_{33}	0.00339	−0.00304	−0.00395	0.00446	−0.00231	0.00222	0.00290	−0.00251	−0.00131	0.00197	−0.00339	0.00301
Δq_{43}	−0.00467	0.00421	−0.00517	0.00366	−0.00175	0.00165	−0.00884	0.00893	−0.00620	0.00689	−0.00156	0.00206
Δq_{53}	−0.00274	0.00204	−0.00332	0.00189	−0.00154	0.00148	−0.00920	0.00910	−0.00899	0.00726	−0.00112	0.00156
Δq_{63}	0.00339	−0.00304	−0.00395	0.00444	−0.00231	0.00221	0.00290	−0.00251	−0.00131	0.00198	−0.00339	0.00299
Δq_{73}	−0.00464	0.00421	−0.00517	0.00366	−0.00175	0.00165	−0.00884	0.00893	−0.00620	0.00690	−0.00156	0.00206
$\Delta\theta_3$	.00785	0.00784	0.00786	0.00784	0.00783	0.00785	0.00788	0.00788	0.00787	0.00767	0.00784	0.00785

Table A2. Extracted results of measured tolerances of the Delta robot. Points *P*6 and *P*9.

	Point : *P*6 (mm) $P_x = 100, P_y = 0, P_z = 500$						Point : *P*9 (mm) $P_x = 0, P_y = 100, P_z = 500$					
Parameter	1	2	3	4	5	6	1	2	3	4	5	6
$P_x + \Delta P_x$	100.2	100	−99.8	100	100	100	0.2	0	−0.2	0	0	0
$P_y + \Delta P_y$	0	0.2	0	−0.2	0	0	0	100.2	100	−99.8	100	100
$P_z + \Delta P_z$	500	500	500	500	500.2	499.8	500	500	500	500	500.2	499.8
ΔRa	0.00998	−0.01034	0.01182	−0.01211	0.01183	−0.01183	0.01200	−0.01106	0.01285	−0.01316	−0.01164	0.01164
ΔRb	−0.00998	0.01057	−0.01183	0.01194	0.01089	−0.01090	−0.01217	0.01111	−0.01269	0.01296	−0.01294	0.01294
Δd_1	−0.01176	0.01138	−0.01246	0.01228	0.01108	−0.01107	−0.01243	0.01161	−0.01240	0.01239	0.01239	−0.01239
ΔLa_1	0.01067	−0.01130	0.01229	0.01236	0.01108	−0.01108	0.01228	−0.01175	−0.01259	0.01256	0.01240	−0.01240
ΔLb_1	−0.00972	−0.01146	−0.01213	0.01190	0.01034	−0.01034	0.01250	−0.01234	−0.01189	0.01181	−0.01239	0.01239
$\Delta\alpha_{11}$	0.00385	−0.00968	−0.00258	0.00238	−0.00875	0.00874	0.00929	−0.00931	−0.00267	0.00309	−0.00971	0.00971
$\Delta\alpha_{31}$	0.00845	0.00782	0.00837	0.00708	−0.00779	0.00779	0.00565	−0.00572	−0.00706	0.00678	−0.00828	0.00828
$\Delta\alpha_{51}$	0.00898	0.00782	−0.00680	0.00629	−0.00778	0.00777	0.00695	−0.00698	−0.00694	0.00692	−0.00820	0.00820
$\Delta\alpha_{61}$	0.00845	0.00782	0.00837	0.00708	−0.00779	0.00778	0.00565	−0.00572	−0.00706	0.00678	−0.00828	0.00828
Δq_{11}	−0.00111	0.00789	0.01210	−0.00643	0.00424	−0.00425	−0.00632	0.00659	0.01011	−0.00867	0.00795	−0.00795
Δq_{21}	−0.00352	−0.00502	−0.00803	0.00352	−0.00548	0.00548	0.00267	−0.00269	−0.00616	0.00485	−0.00908	0.00908
Δq_{31}	0.00251	−0.00531	0.00249	−0.00243	−0.00630	0.00631	0.00534	−0.00531	0.00262	−0.00312	−0.00568	0.00568
Δq_{41}	0.00193	−0.00754	0.00671	0.00778	−0.00733	0.00733	0.00714	−0.00715	0.00736	0.00808	−0.00786	0.00786
Δq_{51}	0.00139	−0.00714	0.00608	0.00618	−0.00661	0.00660	0.00542	−0.00552	−0.00790	0.00725	−0.00820	0.00820
Δq_{61}	0.00250	−0.00532	0.00249	−0.00243	−0.00630	0.00629	0.00534	−0.00531	0.00262	−0.00312	−0.00568	0.00568
Δq_{71}	0.00193	−0.00754	0.00671	0.00778	−0.00733	0.00733	0.00714	−0.00715	0.00736	0.00808	−0.00786	0.00786
$\Delta\theta_1$	0.00853	0.00828	0.00578	−0.00550	0.00801	−0.00811	−0.00593	0.00605	0.00841	−0.00764	0.00810	−0.00810
Δd_2	0.01200	−0.01141	0.01253	−0.01235	−0.01088	0.01088	0.01229	−0.01176	0.01244	−0.01248	−0.01237	0.01237
ΔLa_2	0.01181	0.01127	0.01246	−0.01234	0.01087	−0.01087	−0.01235	0.01167	0.01261	−0.01260	0.01248	−0.01248
ΔLb_2	−0.01183	0.01074	0.01244	−0.01202	0.00986	−0.00984	−0.01158	0.01146	0.01213	−0.01211	0.01191	−0.01191
$\Delta\alpha_{12}$	0.00258	−0.00291	0.00173	−0.00206	−0.01035	0.01034	0.00190	−0.00272	0.00200	−0.00281	−0.01061	0.01060
$\Delta\alpha_{32}$	0.00706	−0.00733	−0.00816	0.00797	0.00705	−0.00707	0.00684	0.00834	0.00733	0.00792	0.00736	−0.00736
$\Delta\alpha_{52}$	0.00731	0.00826	0.00577	0.00906	−0.00797	0.00797	0.00630	0.00849	0.00605	0.00888	−0.00827	0.00827
$\Delta\alpha_{62}$	0.00706	−0.00733	−0.00816	0.00797	0.00705	−0.00706	0.00684	0.00834	0.00733	0.00792	0.00736	−0.00736
Δq_{12}	0.01015	−0.00997	−0.00880	0.00809	0.00339	−0.00339	0.00912	−0.00894	−0.00703	0.00699	0.00168	−0.00167
Δq_{22}	−0.00510	0.00625	0.00406	−0.00461	−0.00352	0.00352	−0.00532	0.00596	0.00439	−0.00364	−0.00381	0.00382
Δq_{32}	−0.00254	0.00287	−0.00175	0.00208	−0.00441	0.00441	−0.00190	0.00272	−0.00203	0.00292	−0.00385	0.00385
Δq_{42}	−0.00739	0.00756	0.00864	−0.00829	−0.00675	0.00675	−0.00707	0.00746	0.00814	−0.00770	−0.00632	0.00632
Δq_{52}	−0.00763	0.00786	0.00692	−0.00706	−0.00602	0.00603	−0.00751	0.00762	0.00677	−0.00636	−0.00473	0.00473
Δq_{62}	−0.00254	0.00287	−0.00175	0.00208	−0.00441	0.00441	−0.00190	0.00272	−0.00203	0.00292	−0.00385	0.00385
Δq_{72}	−0.00739	0.0075	0.00864	−0.00829	−0.00675	0.00675	−0.00707	0.00746	0.00814	−0.00770	−0.00632	0.00632
$\Delta\theta_2$	−0.00910	0.00896	0.00515	−0.00587	−0.00630	0.00630	−0.00487	0.00632	0.00985	−0.00876	−0.00496	0.00495
Δd_3	−0.01187	0.01126	0.01281	−0.01252	0.01115	−0.01114	−0.01228	0.01191	0.01252	−0.01250	−0.01233	0.01233
ΔLa_3	−0.01186	0.01125	−0.01255	0.01227	0.01113	−0.01113	−0.01223	0.01154	0.01246	−0.01234	0.01239	−0.01239
ΔLb_3	−0.01097	0.01016	−0.01121	0.01115	−0.01089	0.01088	−0.01090	0.01125	0.01134	−0.01113	0.01147	−0.01147
Δq_{13}	−0.00359	0.00336	0.00143	0.00942	−0.00983	0.00982	−0.00285	0.00217	0.00177	−0.00212	−0.01031	0.01030
Δq_{23}	−0.00729	0.00729	−0.00819	0.00718	−0.00823	0.00822	0.00818	0.00733	0.00593	−0.00573	0.00743	−0.00743
Δq_{53}	−0.00745	0.00734	−0.01011	0.00773	−0.00815	0.00814	0.00847	0.00690	0.00737	0.00743	−0.00806	0.00807
Δq_{63}	−0.00729	0.00729	−0.00819	0.00718	−0.00823	0.00822	0.00818	0.00733	0.00593	−0.00573	0.00742	−0.00743
Δq_{13}	0.01205	−0.01206	0.01266	−0.01119	0.00718	−0.00718	0.01098	−0.01100	0.00793	−0.00869	0.00113	−0.00113
Δq_{23}	−0.00675	0.00659	−0.00403	0.00464	−0.00916	0.00917	−0.00798	0.00869	−0.00146	0.00116	−0.00339	0.00339
Δq_{33}	0.00283	−0.00286	−0.00150	0.00691	−0.00523	0.00524	0.00284	−0.00216	−0.00173	0.00199	−0.00449	0.00449
Δq_{43}	−0.00888	0.00863	−0.00762	0.00795	−0.00774	0.00774	−0.00857	0.00880	−0.00517	0.00494	−0.00549	0.00548
Δq_{53}	−0.00927	0.00882	−0.01048	0.00784	−0.00808	0.00808	−0.00839	0.00853	−0.00383	0.00251	−0.00406	0.00405
Δq_{63}	0.00283	−0.00286	−0.00150	0.00691	−0.00523	0.00521	0.00284	−0.00216	−0.00173	0.00199	−0.00449	0.00448
Δq_{73}	−0.00888	0.00863	−0.00762	0.00795	−0.00774	0.00774	−0.00857	0.00880	−0.00517	0.00494	−0.00549	0.00548
$\Delta\theta_3$	0.01044	−0.01006	0.01055	−0.00859	0.00760	−0.00760	−0.00689	0.00616	−0.00985	0.00872	−0.00510	0.00508

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