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NAFIR Noureddine

Thème

Systèmes de Commande par Réseau Stabilité, Robustesse et Application.

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Department of Electrical Engineering
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Thesis

For the Degree of

Doctor of Philosophy

Domain: Science and Technology
Section: Automatic
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Presented by

NAFIR NOURREDDINE

Theme

Networked Control Systems, Stability, Robustness and Application.

Presented publicly on**: …/...**/2022

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Abstract

Feedback control systems wherein, actuators, sensors, controllers and other components are distributed around a digital communication network called Networked Control Systems (NCSs), that can be shared or not with other applications. Today's NCSs are widely used in many fields because of its appealing advantages, such as enabling remote data transmission, reducing the cabling complexity, minimizing costs and providing easy maintenance. However, the insertion of a communication canal in the control loops bring some communication constraints such as induced time delays and packets dropout. Those constraints can affect the performance of the system or take it to instability. With the aim to keep good performances of the controlled system face of possible changes introduced by the network, it is interesting to introduce new approaches or ameliorate and improving some existing results. In this thesis we are concerned with stability analysis and stabilization of uncertain networked control systems with network induced time-delay and packets dropout.

This work aims at maximizing the allowable upper bound of network induced delay, together with the number of consecutive packets dropout, from which uncertain and disturbed NCS plants are stable. To achieve this goal a novel augmented Lyapunov-Krasovskii functionals (LKF), dependent on both the lower and upper bounds of the network induced time varying delay is considered to derive the proposed delay dependent LMIs based H_∞ stability conditions. The relaxation of these conditions are obtained by exploiting simple and double integral Wirtinger's inequalities to introduce more degree of freedom from the LKF terms than existing inrelated conditions from the literature. Finally, numerical examples and simulation results are presented to illustrate the effectiveness and the conservatism improvement raised by this proposal compared to previous related results.

Keywords: Networked Control Systems (NCSs), Stability analysis, Robust stabilisation, Linear Matrix Inequalities (LMIs), Delay, Packets dropout, Event triggred.

Resumé

Les systèmes de commnade dans lesquels, les actionneurs, les capteurs, les contrôleurs et autres composants de commande sont répartis autour d'une canale de communication, sont appelés Systems de Commande a travers les Reseaux (SCRs). Aujourd'hui les (SCRs) sont largement utilisés dans de nombreux domaines en raison de leurs avantages attrayants tels que la transmission de données à distance, réduire la complexité du câblage, minimiser les coûts et faciliter la maintenance. Cepandant l'insertion d'une canale de communication dans une boucle de commande entraine certain contraintes de communication tels que les retards et les pertes des paquets d'inforamtion, ces contraintes peuvent affecter les performances du système ou meme l'amener à l'instabilité. Dans le but de conserver de bonnes performances du système des (SCRs) face aux changements possibles introduits par le réseau, il est intéressant d'introduire de nouvelles approches ou d'améliorer certaines resultas. Dans cette thèse, nous nous intéressons à l'analyse de stabilité et à la stabilisation de systèmes de commande a travers les réseaux (SCRs) incertains en presence de retard induit par le réseau et des pertes des paquets d'information.

Ce travail vise à maximiser la borne supérieure admissible du retard induit par le réseau, ainsi que le nombre des pertes consécutifs des paquets, à partir desquels les SCRs incertaines et perturbées sont stables, pour atteindre cet objectif, une nouvelle fonction augmentée de Lyapunov-Krasovskii (LKF), dépendante à la fois des bornes inférieure et supérieure du retard variable dans le temps induit par le réseau est prise en consideration pour dériver les conditions de stabilité basées sur inégalités matricielle lineaires (LMI)dépendant du retard. La relaxation de ces conditions est obtenue en exploitant les inégalités de Wirtinger intégrales simples et doubles pour introduire plus de degrés de liberté à partir des termes LKF, a fin d'avoir des conditions meilleurs que celle existantes dans la littérature. Enfin, des exemples numériques et des résultats de simulation sont présentés pour illustrer l'efficacité et la amélioration du conservatisme soulevée par cette proposition par rapport aux résultats déja existantes dans la litterature.

Mots-clés: System Commandé par Réseaux, analyse de stabilité, stabilisation robuste, inégalités matricielle linéaire (LMIs), retard, perte des paquets .

ملخص

انظمة التحكم التي من خلالها يتم توزيع كلا من الملتقطات المحثات و المراقبات و باقي عناصر التحكم حول قناة اتصال او شبكة مواصلات تسمى بأنظمة التحكم بوسطة الشبكات .هاته الأنظمة تعرف حاليا استخداما واسعا في شتى المجالات لما توفره من مزايا و خصائص على غرار امكانية نقل البيانات عن بعد ، تقليل تعقيد الأسلاك، تقليل التكاليف ، وتسهيل الصيانة بالرغم من كل هاته المزايا ان ادراج شبكة اتصال ضمن حلقة تحكم، يتسبب في قيود اتصال مثل التأخيرات الزمنية وكدا ضياع حزم المعلومات ،هاته الظواهر او القيود يمكن أن تؤثر على أداءات النظام بل يمكن ان تؤدي الى فقدان استقرار نضام التحكم ادن . من أجل الحفاظ على استقرار نضام التحكم في وجود هاته القيود اصبح لزاما ايجاد طرق جديدة من اجل الحفاض على الادءات الجيدة لنضام التحكم بوجود شبكة التحكم او تحسين نتائج بعض الأبحاث .

في هاته الاطروحة نفدم دراسة تحليلية حول استقرار و اعادة ا استقرار نضام تحكم بوسطة الشبكات غير مؤكد ومضطرب بوجود تأخيرات زمنية وضيعات حزم المعلومات يهدف هذا العمل إلى ايجاد الحد الأعلى المقبول للتأخير الناجم عن ادراج الشبكة ضمن حلقة تحكم الدي من اجله يبقى النظام مستقرا من اجل بلوغ هدا الهدف نقترح دالة ليابونوف محسنة تحوي على الحدود الدنيا والعليا للتأخير الزمني المتغير لاشتقاق شروط الاستقرار. اعتماداً على المتباينات المصفوفاتية الخطية. تليين شروط الاستقرار يتم اعتمادًا على استغلال متباينات ورثانجر تحوي تكامل بسيط او مضاعف بغية الوصول ال درجة حرية اكبر لحدود دالة ليلبونوف.من اجل الحصول على شروط احسن من تلك الموجودة حاليا .اخيرا نقدم بعض الا مثلة الرقمية و نتائج محاكاة هاته الامثلة من اجل اثبات فاعلية الطريقة المقدمة و تحسين قيمة الاحتفاظية مقارنة بتلك الموجودة حاليا . .

الكلمات المفتاحية

ضياع الحزم. , المتباينات المصفوفاتية الخطية, اعادة الاستقرار, تحليل لاستقرار, التحكم بوسطة الشبكات

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Nourreddine Nafir

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Abbreviations

BMI	Blinear Matrix Inequality
CSMA/CD	Carrier-Sense Multiple Access/ collusion Detection
ETDTP	Event-Triggered Data Transmission Protocol
EVP	Eigen-Value Problem
FDE	Functional Differential Equations
GEVP	Generalized Eigen-Value Problem
LKFs	Lyapunov Krasovskii Functionals
LMI	Linear Matrix Inequalities
LTI	Linear Time Invariant
MAUB	Maximum Allowable Upper Bound
METC	Mixed Event-Triggered Control
NTP	Number of Transmitted Packets
ODEs	Ordinary Differential Equations
TCP	Transmission Control Protocol
TR	Transmission Rate
UDP	User Data Protocol
ZOH	Zero Order Hold

List of Symbols

$x(t) \in \mathbb{R}^{n \times m}$	The system state vector.
$y(t) \in \mathbb{R}^{p \times n}$	The output measurement vector.
$u(t) \in \mathbb{R}^{n \times m}$	The input vector.
$\mathbb{R}$	The set of real number.
$\mathbb{C}$	The set of continuous functions.
$\tau(t)$	Time varying delay.
τ	Time delay.
τ_k^{sc}	The induced time delay from sensor to controller.
τ_k^{ca}	The induced time delay from controller to actuator.
τ_{min}	The lower network-induce delay.
τ_{max}	The upper network-induce delay.
τ_k	The total network induced time delay.
η_k	The number of consecutive packet dropouts.
ΔA	The system parametric uncertainties.
$\omega(t) \in \mathbb{R}^q$	The external system disturbances.
$\gamma > 0$	The disturbance attenuation level.
h	The sampling period.
$e_k(t)$	The actual error between two states.
$\rho(t)$	The varying event triggered threshold.

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Chapter 1

General introduction and literature survey

1.1 Introduction

Until the past 50 years, most systems were controlled with analogical control signals based on the so called point-to-point architecture. The point-to-point architecture is the traditional communication structure, where both sensors and/or actuators are situated in the same physical area, and are connected to the controllers via wires, this architecture has been successfully implemented in industry for decades. However, due to the expansion of physical setups and functionality, the traditional point-to-point architecture is fueled by the high costs of wiring, the difficulty of maintenance, and the difficult introduction of additional components into the systems when a change is needed. So, the point-to-point architecture becomes no longer able to meet new requirements that are particularly demanding in control of complex systems [63, 17, 122, 107]. Consequently, a novel control strategy that takes all those requirements into account is proposed to replace the traditional communication architecture named Networked Control Systems (NCSs)[46].
In recent years, with the explosive advancements in microelectronics, information, communication technology, and the fast development of the embedded computational devices, a communication network can be combined with a modern control system to form and gives rise to the so-called (NCSs). This new technological concept is considered to be the third control system generation [14, 3, 46, 63].

By definition, NCSs are a type of distributed control systems where sensors, actuators, and controllers are located in different areas and interconnected through a digital communication network as shown in Fig1.1. As a result, in NCSs informations are exchanged through those communication networks to interact with the physical environment. Some typical characteristics of those systems are reflected in their asynchronous operations, diversified functions, and complicated organizational structures. This distributed control system structure is the basis of a great many future applications in information technology [46]. Compared with the traditional point-to-point control, and due to the new requirement that

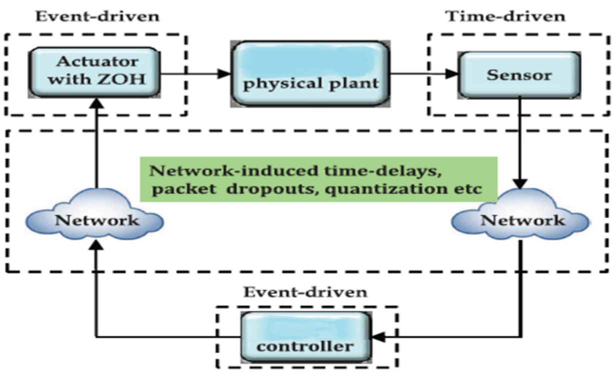

Figure 1.1: Schematic of a Networked Control System NCS

demand the remote control of complex systems [67, 8, 17, 122, 107], NCSs present some appealing features and advantages, such as modularity, integrated diagnostics, quick and easy maintenance, and low cost [14]. Moreover, one of the biggest advantages of a system control over a network is scalability, where many sensors and actuators situated at different locations can be added to the control loops through a single communication network [46]. The introduction of a communication network to connect the different components of the control loop can effectively reduce the complexity of systems, with nominal economical investments. Furthermore, it is easy to fuse the global information to take intelligent decisions over a large physical space. Consequently, because of these attractive benefits, NCSs are rapidly increasing and have been finding application in a broad range of areas, where many industrial companies and institutes have shown great interest in applying various networks to remote control systems and manufacturing automation and many other applications, including telecommunications, remote process control, altitude control of airplanes, automated highway systems and unmanned aerial vehicles, the internet of things, cyber physical systems, smart homes, and so on [67, 8, 122, 179, 73].

Evidently, the field of networks and controlhas emerged as a new discipline in control circles. From this perspective, there are two essentially distinct directions of research.The first one, concerns the *control of network*. In this direction we consider the control of communication networks, which falls into the broader field of information technology that includes problems related to wireless networks and control of data congestion, network protocols. This field concerned mainly to providing a certain level of performance to a network data flow, while achieving efficient and fair utilization of network resources. The second direction is the *control over networks*, dealing with the control of closed loop system through a communication network. The focus in this direction is on stability issue, and the design of feedback control strategies adapted to control systems in which control data is exchanged through unreliable communication links. In this thesis we mainly focuses on "control over network" and more details about research and evolution of networking technologies for NCSs are in the next sections.

Every advancement in technology has its pros and cons, despite the huge advantage and widespread use of networked communication in control systems. It turns out that the insertion of the communication networks in the control loops leads to some communication challenges when transmitting control signals and exchanging data through those networks [120, 76], that makes the stability analysis and control design of NCSs complicated. The main reason for this is that the network itself is a dynamic system with some issues eg., network induced delays, packet losses, quantization error, variable transmission intervals, bandwidth limitation and so on. Note that if these challenges are not taken into account, they may make undesirable behaviour in the closed loop, that can degrades its performance and even lead to its instability, more details of the network-induced challenge are depicted in the next chapter. An other important challenge in NCSs is robust stability analysis of NCSs, especially in the presence of several uncertainties in NCSs, e.g., external disturbances modeling error, system dynamics changes that are unavoidable in the context of practical applications, since delay varies with network topology, network protocol, network load, network bandwidth and package size and their value may be bounded, unbounded, time varying, constant or random [84, 88]. Up to date the problem of robust stabilization for uncertain networked control systems with induced time delay and packet dropout have been dealt with using a number of different approaches. For example, the Riccati equation approaches, Linear Matrix Inequality (LMI) approaches, and the Lyapunov min-max approaches. some research results have been obtained in stability and robustness analysis of systems.

1.2 Literature survey

The first researchers analyzed the stability of NCSs starting in the 1990s [14]. A current survey of the emerging field of NCSs is provided in [46, 42]. Now, there has been considerable research works appeared to address modelling, stability analysis, control and filtering problems for NCSs and summarizing the updated results on NCSs, where most of the works in the area of NCSs is inclined to model them into conventional control systems with some of the communication constraints, where different approaches are adopted.
We distinguish three different approaches towards the modeling, stability analysis and controller synthesis for NCSs.
A categorization of the available literature can be done, firstly, on the basis of the types of network-induced imperfections considered (time varying sampling intervals, time varying delays, packet dropouts, and quantization) as mentioned in the introduction. the past decade. Secondly, on the modeling and analysis approach adopted to study the stability of the NCS under these network induced imperfections.
Before studied the different approach, we notice that it exists two essentially different ways to model networked constraints. The first class of models assumes bounds on the delays, sampling intervals , and the number of subsequent packet dropouts. With some abuse of terminology, we will call this the deterministic approach. A second approach is called the stochastic approach in which information about the stochastic nature of the constraints is taken into account [43, 170, 201].
Based on those two methods many other approaches have been appeared in literature (eg., discrete time approach, continuous time or emulation approach).

1. **The Discrete time approach:** see e.g, [23, 28, 37, 39, 47, 108, 127, 159, 153] have mainly focused on linear NCSs. The first step is to construct discrete time representations of the sampled data NCSs system (which for linear systems can be done exactly), leading to an uncertain discrete time system in which the uncertainties appear in an exponential form (due to discretization). The discrete-time modeling approaches can be further categorized by time driven or event driven models. In time

driven models the continuous time model is integrated from sample transmission time to the next sample transmission time, while in event driven models integration is done from each event time (being control updates times, sample times, etc), see, e.g., [47] for the latter. Here we will mainly focus on time driven linear NCSs models. Typically, this approach is applied to NCSs with linear plants and controllers since in that case exact discrete time models can be derived, although recently new results have been obtained that apply to NCS with nonlinear plants and controllers based on approximate discretization, see [154].

2. **The sampled- data- approach** The sampled- data- approach uses continuous time models that describe the sampled-data NCS dynamics in the continuous-time domain (so without exploiting any form of discretization) and perform stability analysis and controller synthes is based on these sampled data NCS models directly. In [36] the author applied a descriptor system approach to model the sampled data dynamics of systems with varying sampling intervals in terms of infinite dimensional delay differential equations ($DDES$) and study their stability based on the Lyapunov Krasovskii functional method. In [38, 181] the sampled data approach is used for the stability analysis of NCSs with time varying delays and constant sampling intervals, using linear matrix inequality based techniques. The recent results in [38] show how time varying delays and dropouts can be formulated in one framework based on $DDES$, and stability analysis and H_∞ control design methods, based on LMIs, are presented. However, in [97] Mikrin showed that the use of such an approach for digital control systems neglects the piecewise constant nature of the control signal due to the zero order hold mechanism thereby introducing conservatism when exploiting such modeling for stability analysis. More specifically, the conservatism is introduced by the fact that the zero order hold and delay jointly introduce a particular linearly increasing time varying delay within each control update interval , whereas in the modeling approach mentioned above it is replaced by an arbitrary bounded time varying delay.
An alternative approach, proposed in [101, 102, 152], is based on impulsive $DDES$ and does take into account the piece wise constant nature of the control signal due to the zero order hold mechanism. It has been shown in [97] that this approach is less conservative than the descriptor approach. More specifically, the impulsive $DDES$

are based on introducing impulses (discontinuous updates) at the moment new information arrives at the controller or the plant. this manner the true behavior of the underlying *NCS* is captured. As also noted in [60], the usage of infinite dimensional *DDE* models and Lyapunov functionals to analyze the stability of essentially finite dimensional sampled data *NCS* does not seem to offer any advantage. The approach in [102, 152] is able to deal simultaneously with time varying delays and time varying sampling intervals but does not explicitly include packet dropouts in the model (although they might be considered as variations in the sampling intervals or delays). Here, we will focus mainly on the approach towards the modeling and stability analysis using the sampled data approach, where constructive stability conditions have only been obtained for linear NCSs.

3. **Continuous-time approach** known also as emulation approach one can cite [25, 45, 103, 104], where the authors present a technique to stabilize the continuous time plant without any network induced imperfections by designing a continuous time controller. Next, based on a sampled data model of the NCSs (in the form of a hybrid system) the stability analysis will be established and allows to quantify the level of network-induced uncertainty in terms of, maximal allowable delay *MAUD* for which the NCSs conserve the stability properties from the closed loop system without the network. This approach is applicable to a wide class of nonlinear NCSs, since well developed tools for the design of (nonlinear) controllers for nonlinear plants can be employed. A drawback is the fact that the controller is formulated in continuous time, whereas for NCSs one typically designs the controller in discrete time.
   Summarizing, the discrete time approach considers discrete time controllers (or discretized continuous-time controllers) and a discrete time NCSs model, while the sampled data approach also considers discrete time controllers, but has a continuous-time (sampled data) NCS model. Finally, the continuous time (emulation) approach focuses on continuous time controllers using a continuous time (sampled data) NCSs model. Within all these different approaches different techniques towards stability analysis are used. While the discrete time approach uses common quadratic or parameter dependent Lyapunov functions for the discrete time model, the continuous time approach uses continuous time Lyapunov functions constructed

by combining separate Lyapunov functions for the network free closed loop system on the hand and the network protocol on the other hand . The sampled-data approaches exploit extensions of Lyapunov Krasovskii functionals.

1.2.1 The types of network induced phenomena

In recent years, much research effort has been devoted to study of phenomena appeared in NCSs, where several methodologies have been reported in the open literature to handle with the problems mentioned above in networked control systems. Many of those works consider only one of these network induced imperfections. The effect of time varying sampling is studied [23, 47, 100, 127, 153, 196, 94], scheduling [40, 43, 72, 134], transmission delays [15, 34, 45, 193, 11, 12, 126], packet dropouts [29, 90, 95, 77], power limitation [16, 125, 66], signal to noise ratio constraints[54, 55], quantization [85, 105, 152, 176, 30, 115, 183, 174] and data-rate constraints [24]. Most of existing works considering these phenomena separately where a wealth literature can be presented on networked control systems with time delay and networked control systems with packet dropouts.

Time delay Concerning the literature on induced time delay, as far as the network-induced delay is concerned, NCSs are naturally modelled as a special class of time delay systems. The research method covers a vast range of research on NCSs.

An interesting issue here is to determine the Maximum Allowable Delay Bound (*MADB*) of NCSs, which is the upper bound of the transfer interval that ensures the stability or other performance objectives of NCSs. The determination of *MADB* is important in theory and can also play a guiding role for practical applications. One can refer to the survey paper in [89]for more information on this issue much attention has been paid to the study of the effect of time varying transmission intervals and delays on NCSs stability [126, 88, 71, 192, 146, 81, 195, 135]. To the best the authors in [126] present a stochastic network induced delay in given interval with known lower and upper bounds. Therefore, the NCS is modeled as linear system with probabilistic time varying delay distribution. In [141]the random communication delays have been considered as white in nature with known probability distributions. In [60] the authors investigate the Markov chains to model the time delay of NCSs such that the closed loop systems are jump systems. In [136] the Bernoulli binary distributed white sequence taking values of zero or one are used to model the random delays with certain probability. In [196] a stability criteria is presented in the case when NCSs have network delays shorter as well as longer than sampling interval. In addition they have also also proposed a technique that compensates the effect of network delays having time varying nature by using state feedback controller based on conventional estimator. In [88] the author present an online algorithm by using gradient method for handling the random delays, where some novel concepts were introduced and feedback matrices with a switched strategy were designed for studying the stability of networked control systems with variable network induced delays. In [192, 81] the stability of networked control system with time varying delay is examined, where the global time delays interval is divided into a sub interval with *'m'* pieces of sections unevenly and the stability conditions are derived by using Lyapunov Krasovskii Functional with the help of free-weighting matrices approach. Based on an input delay approach, the authors in [146, 71] present a novel delay distribution approach for a continuous-time networked control system with time varying transmission delays by representing the NCSs as an hybrid system with multiple input delay subsystems. The distribution of input delays is modeled as a continuous dependent and non-identically distributed ($d.n.d.$)process. Very recently, in [195] sensor to controller delay and controller to actuator delays are modeled independently with a novel approach with free parameters and input-output method is extended to study the stability problem for the NCSs.

The works of [135]were focused to model linear networked control systems as a sampled-data model with both input delay and input saturation, where the delay induced is approximately modeled as polytopic inclusion, the goal is to give a robust model predictive controller (MPC) such that the uncertain system is asymptotically stable at the origin with a certain level of quadratic performance [98].

Packet loss Concerning the effect of packet loss, it exist many research papers, among these papers, two basic control strategies are applied when the packet dropping happens, if the packet is lost, the actuator input is set to zero, otherwise the previous control input is used again when the control packet drops, Generally, the two methods are applied only in the forward channel concerning whether the control packet is obtained or not. Here, the methods are extended in both feedback and forward channels.

A powerful mathematical tool adopt for these strategies is reported in [175, 76] where packet dropouts and channel delays are modeled as Markov chains with the usual assumption, that is all the transition probabilities are completely accessible. Further research are proposed to studied this imperfection as in [196, 164, 46, 191, 163, 162, 96, 172, 202, 109, 200, 119]. In[196], Zhang and al, consider the deterministic single packet loss model with packet dropouts occurring at an asymptotic rate. While the same authors in 2005 considered the packet loss in correspondence with random time delay model, they have generalized also the assumption that data packets will be lost at the controller side, when the delays are greater than sampling interval. Moreover in [46] a Bernoulli's probability distribution function is used in the goal to derive the mathematical model of single packet loss as well as multiple packet loss in the case when the network delay is greater than sampling interval. In [164] to compensate the effect of packets loss in the discrete time domain, the author's propose a method based on the concept of the state estimation. Recently, in [139] a mathematical model using Markov chain process have proposed to express the packet loss, this model have given very best results both in the case of single packet drop as well as successive packet drops. Reference [156] present three modelling approaches for NCSs with packet dropout, where the focus of the work was on the extension of two existing techniques to describe dropouts, in the first approach dropouts are modelled as prolongation of the delay, the second approach consider that dropouts are modelled as prolongation of the sampling interval, and the third approach is based on explicit dropout modelling using automata. Reference [96] investigates the problem of linear quadratic Gaussian feedback control over unreliable communication channels, they focuses on the loss communication networks, where data loss occurs due to the unreliable nature of the link, transmission errors, and/or congestion or long time delays, they derives stability conditions based on the linear quadratic formalism (LQ). A contribution in [163] have investigated a class of NCSs where the plant has time varying norm bounded parameter uncertainties, both the sensor to controller and controller to actuator channels implement multiple packet transmission scheme and impose and experience random packet dropouts. The work in [162] considered a model of a class of switched NCSs with periodical packet dropouts, and variable sampling intervals with a novel piece wise Lyapunov function, the properties for this model with protocols containing periodical packet dropouts are discussed. Whereas in [172], the problem of controller design for such network based iterative learning control (ILC) Iterative Learning Control processes is treated considering data dropout occurring during transfers from the remote plant to the ILC controller. Study in [200] investigate the adaptive event-triggered dynamic output feedback fuzzy control problem for nonlinear NCSs subject to packet loss. The predictive control of network control system with packet dropouts in the feedback and forward channels is presented in [109], introducing data based networked predictive control method in which a sequence of control increment predictions are calculated in the controller, where the number of consecutive packet dropouts in both channels are assumed to be bounded.

Works concern models incorporating multiple network phenomena.

In the literature listed above, the phenomena of packet dropouts and induced time delay were treated separately, but in practice, networked control systems are subject to the the previous phenomena at the same time, the missing of observations is rarely concerned, which is inclined to encounter by taking into account the presence of multiple phenomena simultaneously. The authors in [23], present a model of NCSs that incorporates all network phenomena, time varying sampling intervals, packet dropouts, and time-varying delays that may be both smaller and larger than the sampling interval. Only a few pieces of work deal simultaneously with two network induced imperfection, in the case of considering time delay and packet loss among them we cite [121, 145, 123, 19, 80, 180, 89], where in [173, 167, 166] a study of the effect of quantization error and packet dropout on the stability of NCSs is addressed. The authors of [123] focuses on the linear minimum mean square estimator for a networked discrete time-varying linear system subject to data quantification and communication constraints. While [80, 32, 145] treat the problem of stability and control design of NCSs subject to network induced delay and packet dropout. Especially the paper in [145] focuses on the mean-square stabilization problem for discrete time networked control systems, where the control signal is sent to plant over a lossy communication channel, where a network induced delay and packet dropout occur simultaneously. The stabilization condition is developed in terms of the unique positive-definite solutions to some coupled algebraic Riccati equations (CAREs), moreover [155] treat simultaneously the effect of quantization, packet dropouts, and time-varying transmission delays, on the stability of NCSs. Also in [171] both random packet dropout and quantization effect are token into consideration for studying the robsut H_∞ control problem of networked linear time-delay systems.

1.2.2 Works on stability of networked control systems

As discussed in the previous section, the varying time-delays induced by the network and packet loss are the major source of instability in NCSs, that's why the stability of a closed loop control system should always be the first concern when designing controllers. According to the related previous results in literature, stability criteria can be classified into two categories according to their dependence on the size of the network imperfections (time delay and packet loss); delay independent stability and delay-dependent ones. Both approaches have their own advantages; delay independent approach don't take into account the size of the imperfection information it can provides less conservative results when time delays are more significantly influencing the system dynamics [124, 182, 48, 150, 50]. In the other hand, delay dependent approaches are generally more useful than delay independent ones because it take into account the maximum information of negative network-induced effects, which leads often to less conservative than imperfections independent ones, specially when the sizes of time-delays are small [164, 204, 105, 6, 194, 201, 79, 126].

Choosing appropriate Lyapunov Krasovskii Functional candidate and estimating the upper bound of its derivative more tightly are crucial to enhance the feasible region of stability criteria [4, 165]. There are several techniques used commonly to study the stability problem of networked control systems. Park's Lemma [111], model transformation [186, 64], free weighting matrices techniques [44], delay partitioning method [78], and reciprocally convex optimization approach [113] are the well known and main techniques in reducing the conservatism of stability criteria. Another approach to reduce the conservatism of stability criteria is to utilize integral terms of states as augmented vectors [83, 117]. Thus, more information of system can be utilized, which can increase the feasible region of stability criteria. Among the recent papers that investigate the choice of new LKFs, an appropriate LKFs with double, triple and quadruple-integral terms were constructed and presented in [68, 65, 26, 192, 195] to provide larger delay bounds, and introduce several augmented functional which include single integral terms $\left(\int_{t-\tau_m}^{t} x(s)ds\right)$ and $\left(\int_{t-\tau_m}^{t-\tau_M} x(s)ds\right)$, double integral terms $\left(\frac{1}{\tau_m}\int_{-\tau_m}^{0}\int_{t+\beta}^{t} x(s)dsd\beta\right)$ and $\left(\frac{1}{\tau_M-\tau_m}\int_{-\tau_M}^{-\tau_m}\int_{t+\beta}^{t-\tau_m} x(s)dsd\beta\right)$ and triple integral term $\left(\frac{2}{\tau_m^2}\int_{-\tau_m}^{0}\int_{\phi}^{0}\int_{t+\beta}^{t} x(s)dsd\beta d\phi\right)$ (where τ_m and τ_M are the lower and upper bound of network induced delay) to involve more cross terms between the elements of the resulting augmented vector. But the use the of LKFs including more than single integral term, increase the computational complexity and lead to the problem of estimation of the LKFs time derivative. Among the papers issued in this regard, the bounding techniques of the cross terms and integral terms in the derivative of the LKFs are extensively investigated, including various integral inequalities. The papers in [74, 53, 192, 83, 195, 189] use of Jensen inequality to estimate simple and double integral terms of Lyapunov time derivative where in [192] and [195] the authors use Jensen inequality to estimate both simple and double integral terms, while in [53]the extended Jensen inequality are used to estimate simple integral terms. In [189] the authors use Jensen inequality to estimate simple, double and triple integral terms. The authors in [144, 87, 114, 32, 132, 193, 18] use Wirtinger based inequalities to estimate the integral resulting of LKF time derivative (single, double and triple integrals forms) in deriving process. where in [193, 59, 82, 117] the authors use an extended Wirtinger's integral inequality which includes the celebrated Wirtinger based integral inequality as a special case.

When we discuss on the free weighting matrix technique the authors in [64, 86, 32, 187] use free weighting matrix to handle the cross terms in the derivative of LKFs, wherein the model transformation and bonding techniques are avoided. In [113, 70, 81, 18], the convex approach is employed to handle the cross terms without enlargement, in the same way, In order to add systems parameters in the stability conditions the use of Finsler's lemma are involved[11, 10].

On the other hand the H_∞ control technique has been used to minimize the effects of the external disturbances. It is the objective of H_∞control to design the controllers such that the closed loop system is internally stable and its H_∞ norm of the transfer function between the controlled output and the disturbances will not exceed a given level γ. Very recently, improvedH_∞ performance analysis and stability for uncertain systems with interval time varying delays were proposed in [51]. However, there are rooms for further improvements in the feasible region of criteria forH_∞ performance and stability. since the theory of H_∞ control was presented by Zames[65], H_∞ performance analysis and control for various systems have been reported in [65, 126, 11].

1.3 Objectives and contributions

The main objective of this thesis is to propose computationally efficient stability and stabilization criteria for linear sampled- data networked control systems. We address systems with constant sampling rates, data packet losses, and varying time delays. In literature, most researches works dealt with time delay and packets dropout separately, in this context our first contribution is to take into account those two constraints simultaneously, by considering packet dropout as a time delay.

The main contribution of this thesis is to provide less conservative (w.r.t., previous related studies) LMIs based conditions for the design of NCSs controllers for uncertain and disturbed systems, which can be view as the main contribution. More specifically, our work follows the way borrowed in [83], where a specific packets dropout modeling with a Lyapunov- Krasovskii Functional (LKF) decomposition techniques are proposed to reduce the conservatism of the obtained LMIs conditions. In this context, the contributions bring by our proposal is to improve the results in [83] with the following extensions:

1) We propose a new LKF (see eq. (4.25) and (4.57)) where $V_1^1(t)$ and $V_2^1(t)$ are extended with new extended state vectors $\theta_1(t)$ and $\theta_2(t)$ so that their decision matrix fully compensate each blocks of the further obtained LMIs. Also, the LKF terms (4.26) and (4.58) are augmented with new integrals.

2) The LMIs conditions are obtained by the application of extended Wirtinger inequalities (Lemma 3 from [169]), which provide less constraining bounds of triple integral terms than the standard Wirtinger inequalities considered in [83].

3) New free weighting matrices (slack decision variables that relax the degree of freedom to the obtained LMIs) are introduced from the application of the Finsler Lemma [138, 137], similarly to some of our previous studies in some other control contexts. (Such approach is not considered in [83].

4) Finally, the considered class of NCSs in this thesis is extended to the case of uncertain and disturbed linear systems, while [83] only consider nominal ones. As usual, we employed the Peterson Lemma to cope with the uncertainties and we minimize a H_∞ criterion to attenuate the external disturbances. All of the mentioned points provide improvements regarding to the previous related studies, which are illustrated with the provided four numerical benchmarking examples where the results obtained in our thesis always outperform several previous ones, listed as follows: [184, 52, 65, 69, 33, 133, 118, 117]

1.4 *Publications*

1.4.1 Journal paper

Nafir Nourreddine, Ahmida Zahir, Kevin Guelton, Faycal Bourahala and Rouamel Mohamed, Improved robust H_∞ stability and stabilization of uncertain disturbed networked control system with induced time delay and packet dropouts, International Journal of Systems Science vol. 52, no. 16, PP. 3493-3510, May. 2021.

1.4.2 Conference papers

1.N.Nafir, Z.Ahmida, Stabilisation of networked control system under communication constraints with state feed buck controller, International Conference on Technological Advances in Electrical Engineering $ICTEE'$14, October 2014. Skikda, Algeria.

2.N.Nafir, M.Rouamel, F.Bourahala, S.Bouzoualeg. Delay dependent stability improvement for networked control systems a sampled data approach, 3rd International Conference on Advanced Engineering in Petrochecimical Industry (ICAEPI) 2021 November 2021. Skikda, Algeria.

3.Nafir Nourreddine. A linear matrix inequality to robust stabilisation of networked linear system by state feedbuck control, Journees National sur les Mathematiques Appliqués(JNMA'15), Decembre 2015. Skikda, Algerie.

4. F.Bourahala, N.Nafir An LMI approach to robust H_∞ control and stabilization analysis for uncertain T-S fuzzy systems with state and input time delays 3rd International Conference on Control, Engineering and Information Technology ($CEIT$), May 2015. Telemecen, Algeria.

5.M.Rouamel, F.Bourahala, A.Lopez, N.Nafir and K.Guelton, Mixed Actual and Memory Data based Event Triggered H_∞Control Design for Networked Control Systems, 4th IFAC Conference on Embedded Systems, Computational Intelligence and Telematics in Control (CESCIT 2021), Valenciennes, France, July 5-7, 2021.

1.5 Thesis organization

Our thesis is organized as following :

Chapter1 The first chapter is a literature survey. The general features of NCSs are given as well as the main research directions in the domain. A side of the advantages of the network in the closed loop control systems, this chapter include also he state of arts of NCSs and the existing problems especially network induced delays and packet dropouts are analyzed, which are necessary to the comprehension of our work. Then, some recent research results are recalled in the order of the different modeling method of networked control systems .

Chapter2 The main objective of this chapter is to present the main ideas and the methodology that will be used throughout the thesis where some generalities on networked control system in detail and the different components of an NCSs, also we present in this chapter the philosophie of modelization of networked control systems in presence of both induced time delay and packets dropouts. To this end, we address stability analysis and controller synthesis of linear single rate sampled-data networked control systems which is the simplest network structure studied in this thesis, some lemma are presented to be exploited in the next chapter the use of those lemma help us to develop the stability conditions

Chapter3 In this chapter we present an overview of the Lyapunov - Krazovski function as a powerful mathematical tool to analysis the stability of a networked control system. Following the methodology presented in this chapter on the stability analysis using Lyapunov - Krasosvki functionals, the derivation of stability conditions is obtained by following four steps, the first step is the selection of the functional, the second step is the differentiation of the functional along the trajectories of the system, the third step is the application of some integral inequality to estimate the integral of the derivative function with the help of the integral of the function, such that the Jensen's integral, and the last step is to drive the stability conditions in the form of linear matrix inequality, to end with this chapter some useful lemma that can be used to bound the time derivative of the Lyapunov- krazovski functional are presented.

Chapter4 The main contribution of this chapter is the derivation of new sufficient stability and stabilization conditions for linear networked control system in term of LMIs by taking into account all of the factors mentioned before by taking into account the induced time delay and packet dropouts, the class of uncertain and disturbed NCSs is considered (as usual, the Peterson Lemma will be employed to cope with structural uncertainties while the minimization of a H_∞ criterion will be considered to attenuate the external disturbances). The stability conditions are based on a modified LKF, where the global delay interval is divided into two sub interval. The stability results are also applied to the case where limited information on the delay bounds is available. Furthermore, this chapter also formulates the problem of finding a lower bound on the maximum network induced delay that preserves asymptotic stability as a convex optimization program in terms of LMIs. This problem can be solved efficiently from a theoretical point of view. Finally, as a comparison, we show that the stability conditions proposed in this chapter compare favorably with the ones available in the open literature for different benchmark problems.

Chapter5 In this chapter, a new mixed event-triggered control scheme, based on the actual and memory sampled data is proposed to reduce the network usage in Networked Control Systems (NCSs) subject to network induced delay. In this context, LMIs based conditions are provided from the selection of a convenient Lyapunov Krasovskii functional, for the design of both the event generator for the proposed Mixed Event Triggered Control(METC) strategy and the associated memory based sampled-data controller. The goal is to relax the number of transmitted packets with closed loop NCSs stability guarantee, so that the network work load is mitigated.

A numerical example is proposed to show the effectiveness and improvement of the proposed METC design procedure, regarding to some previous related results from the literature.

Chapter 6 As all scientific work we conclude our thesis by presenting some conclusions, and proposing some perspectives, and future directions, for the extension of our results, and guiding authors to ameliorate the proposed methods .

Chapter 2

Networked control systems structures, modeling and control approaches

2.1 Introduction

As mentioned in the previous chapter, the control signals in a networked system are exchanged between the components of the system through a common limited communication network. To deal with this new architecture, this chapter summarizes the background necessary for NCSs that include the basic description and preliminaries of NCSs components and its roles in the networked control loop, the functionality modes and the architectures of NCSs, the different techniques and approaches for the mathematical modeling by taking into account the modeling of the challenges brought by this new architecture and finally, we present the different techniques that are used for stability analysis and control design of NCSs subject to one or more imperfections.

2.2 NCSs description and preliminaries

In NCSs structure, the communication networks are used for transmitting the measurements of the physical plant output to the distributed controllers. The physical plants are in real continuous-time, and to share their output via the network the output signal has to be sampled then quantified to be able to send to the controller. the networked controllers are generally in discrete-time and are driven by time or event. In the following, we discuss the the NCSs preliminaries :

2.2.1 NCSs Components

As illustrated in figure 2.1, NCSs is compose of the following components:

2.2.1.1 Sensors In addition to the important role of sensors in the conventional control loop that converts the physical quantities into standard electrical or pneumatic ones, the sensor in NCSs has another role which it samples these measured signals and converted them into a numerical formula by the quantifier. this important role of the sensor allows us to adapt the data required to be transmitted through a digital communication network to the control node.

2.2.1.2 Communication network The good implementation of an NCSs is based on the performance parameters and the quality of the communication network used where all the measured and control signals in the industrial area are transmitted in one communication channel in the networked control system. This new architecture leads to a limited data bandwidth of control system nodes, congestion of the packet shared in this communication channel, and systems node competition. the presence of these challenges in a communication network leads to many challenges in the modeling of NCSs such as transmission rate, delays, packet losses, this may degrade the performance of systems over a network.

2.2.1.3 Controller The controller manipulates the output of a process subject to the inputs needed to drive the process variable toward the desired set-point. The networked controller is generally implemented by a computer or an industrial CPU and is modeled as a discrete-time model driven by the event (at each received of a new packet). the networked controller must be taken the challenge bring by the communication network, external and internal disturbances to generate a control signal to derive the process variable toward the desired set-point. And then, the control node transmits the discrete control signals via the same communication network to the actuator node.

2.2.1.4 Actuator In a traditional control loop, the actuator is responsible for moving the system subject to its limits safety states by converting the signal energy into mechanical motion (limited movements or position). In NCSs, the actuator is composed of a conventional actuator in addition to Zero Order Hold (ZOH). The role of ZOH is converting the discrete-time control signal received from the control node to a continuous-time signal to manipulate the continuous physical system.

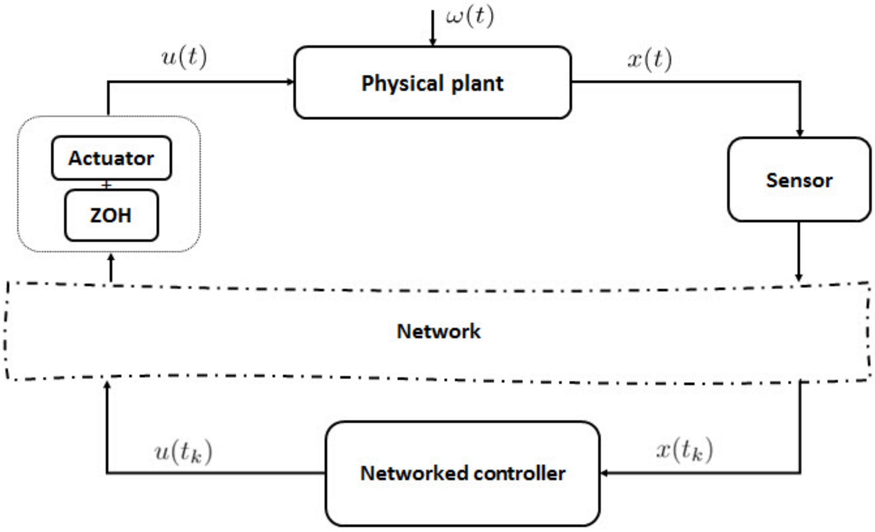

Figure 2.1: Networked control system architecture

2.3 Structure of network control system

Under control over network, there are two major types of control systems that use communication networks, *Shared-network control systems* and *Remote control systems* as depicted in Fig 2.2 and Fig 2.3. Each of the NCS structures has many challenges to maintain the quality of service (QoS) and the quality of control (QoC).

2.3.1 Shared-network control system

As indicate in Fig 2.2 it can be noticed that using shared-network control system the transfer of information from sensors to controllers and control signals from controllers to actuators can greatly reduce the complexity of connections and provides more flexibility in installation, ease of maintenance and troubleshooting. Moreover, this structure is widely adapted for the smart/intelligent actuators and sensors networks. Some autonomy is given to the sensors to make them capable of processing and communicating [22, 42, 197, 192].

2.3.2 Remote control system

In this case, the place where the central controller is installed is called a local site and the place where the plant is installed is called a remote site. The data transfer between local site and remote site is done through a communication network. There are two general approaches to designing an NCS using remote control

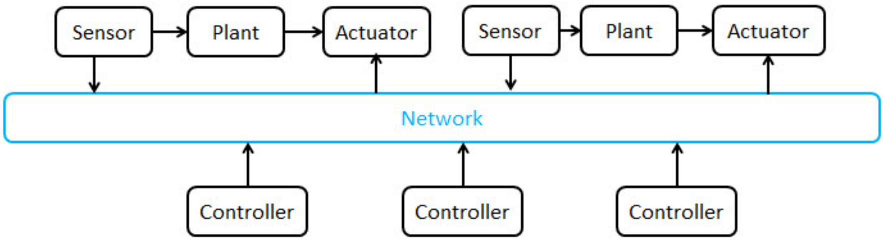

Figure 2.2: Shared NCSs structure.

system, hierarchical structure and direct structure.

2.3.2.1 Hierarchical structure In hierarchical structure there are several subsystems that are connected to central controller through communication network, each subsystem contains sensor, actuator, and controller by itself as depicted in Fig 2.3, in this case a subsystem controller receives a set-point from the central controller. The subsystem then tries to satisfy this set-point by itself. The sensor data or status signal is transmitted back via network to the central controller.

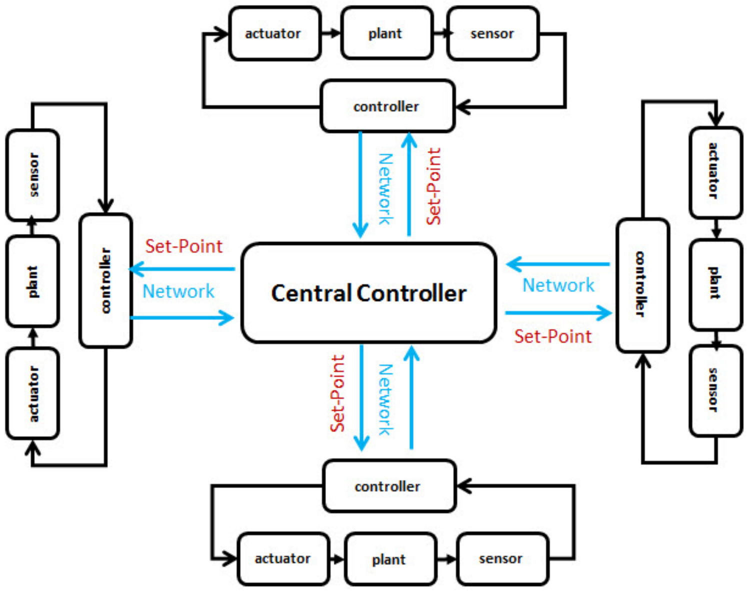

Figure 2.3: Hierarchical NCSs structure

2.3.2.2 Direct structure In this case, the sensor and actuator are attached to a plant, while the controller is separated from the plant by a network connection as shown in Fig2.4. The sensor transmits the signal to the controller through the network and controller sends back the processed control signal to the plant via actuator through the network. This type of configuration is used mainly in the process industries and haptic surgery. Many complex network control system use the combination of both the structures defined as hybrid structure. Both structures have their advantages, the direct structure has better

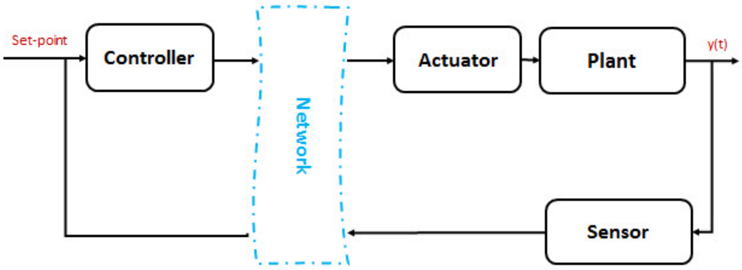

Figure 2.4: Direct NCSs structure

interaction between system component and faster reaction of the network and s, while the hierarchical one is more modular and has better robustness.

2.4 Modeling of NCSs

In this section, the modeling procedure of networked control systems (NCSs) is presented, it is to an extensive literature is available in the modeling of NCSs, including the different NCSs imperfections such as, time-varying delays, time-varying sampling intervals, and packet dropouts, modeling NCSs as discrete-time and continuous-time NCSs models, linear and non-linear NCSs model, uncertain and disturbed NCSs model.

2.4.1 The network induces delays:

The transmission time for the data packets introduces network-induced delays to NCSs, which are well known to degrade the performance of the control systems.As depicted in Figure 2.1. There are two types of network-induced delays according to where they occur.

τ_k^{sc}: is the sensor-to-controller network induced delay, that is backward channel delay.

τ_k^{ca}: the controller-to-actuator network induced delay, that is, forward channel delay.

The two types of network-induced delays may have different characteristicswe can also add an other kind of delay which is computational delay in the controller, the network delays can be combined for analysis purposes:

$$\tau_k = \tau_k^{sc} + \tau_k^{ca} + \tau_k^{c}$$

where τ_k^c is the controller computational delay which can be included into τ_k^{sc} or τ_k^{ca}. The time-variant delays induced by the network, τ_k^{sc} and τ_k^{ca} can be lumped together into one delay τ_k given by :

$$\tau_k = \tau_k^{sc} + \tau_k^{ca}$$

This is the delay of each sample and the subscript k refers to sampling instants. our analysis in this chapter only concentrate on NCSs with constant sampling period. Depending on the communications protocol used, these delays can be constant, time-varying or random.

Remark 1 *The induced delay $\tau(t)$ could be extended to include the packet dropouts by representing it as a special case of time delay; thus*

$$\tau(t)_d = \tau(t_k) + \eta h \tag{2.1}$$

where $\tau(t)_d$ is the total delay including the dropouts delay, η is the number of packet dropouts, and h is the sampling period.

2.4.2 General assumptions on the time delay functions

It is very important to specify the type of delay that will allow an exact modeling of the systems, to study the effect of networked induced time delay, it is important to go through the delay modeling phase. For this purpose, the delay can appear in one of the following form [129, 5, 9] under constant form ,variable in time form, in discrete or distributed form. In this section we will present the main types and models of delay.

2.4.2.1 Constant Delay

The delay is qualified to be constant when it is defined by a positive real number,$\tau(t) = \tau$,$\tau \in \mathbb{R}^+$. If the delay parameter $\tau(t)$ is unknown but constant, then the energy of $X(t - \tau(t)$ and $X(t)$ have the same energy, this type of model can be a good model despite the fact that the delay in the network is often variable or random, for example, if the process time constant controlled by the network is much larger with respect to the induced delay.

2.4.2.2 Time varying Delay: In this category of delay, the hypothesis that the delay is constant is not always verified, hence the need to take into account the potential delay variations, in this case the delays are represented by a continuous functions $\tau(t)$ defined in bounded time intervals. According to these intervals, we can distinguish two types of delays namely:

• **Bounded time-varying delays:** In networked control systems applications, because of congestion in the network and the packet loss phenomenon which may lead to non negligible variations on the delay functions, the assumptions on constant delay is too conservative, it is seldom verified and not justified to oversize resources, so bounded time-varying delay consists of allowing the delay to vary from sample to sample but within an upper maximum value not to be exceeded, so in this case there is a real number $\bar{\tau} \geq 0$ such that:

$$0 \leq \tau(t) \leq \bar{\tau}, \forall \bar{\tau} \geq 0 \tag{2.2}$$

where $\bar{\tau}$ represents the maximum admissible upper bound of the delay $\tau(t)$.

• **Interval time-varying delays or non-small delays:** In Ethernet a collision detection protocol (CSMA / Collision Detection) is the main source of random delays; indeed, when there is a collision, all the affected nodes stop the transmission, each station waits a random time and retransmits the message again.(2.3)

Hence, the assumption saying that the delay functions belong to an interval of the form $[0, h2]$ becomes too restrictive and the associated stability analysis may lead to inherent conservatism ways to tackle with this type delay consist of decomposing the delay into two parts: a nominal part and a residual part, which is considered as a perturbation with respect to the nominal delay, it is interesting to take into consideration a lower limit of the delay $\tau(t)$ for which there are two values τ_1 and τ_2 such that:

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2 \tag{2.3}$$

with $0 \leq \tau_1 \leq \tau_2$. It should be noted that the two types of delay mentioned above are often dealt with in the literature, involve delay-independent stability criteria, see for example [130], [4].

2.5 Networked control system with time delay

From a mathematical perspective, time delay systems, also called systems with after-effects or death time, hereditary systems, equations with deviating arguments or differential difference equations, belong to the class of Functional Differential Equations (FDE). In a general way, a time-delay system is a class of dynamical system represented by differential equations, in some unknown functions and some of its derivatives, evaluated at arguments which are distributed over some intervals in the past. A time delay system is described by the following functional differential equation [129, 9]:

$$\begin{cases} \dot{x}(t) = f(t, x_t, U_t), t \geq t_0 \\ y(t) = g(t, x_t, U_t), \end{cases} \tag{2.4}$$

where

$$f : \mathbb{R}_+ \times \mathbb{C}^{\mathbb{O}}([-\bar{\tau}, 0] \to \mathbb{R}^n) \to \mathbb{R}^n, \phi \in \mathbb{C}^{\mathbb{O}}([-\bar{\tau}, 0] \to \mathbb{R}^n). \tag{2.5}$$

We notice by $\mathbb{C} = \mathbb{C}^{\mathbb{O}}([-\bar{\tau}, 0] \to \mathbb{R}^n)$ the set of continuous functions of $[-\bar{\tau}, 0]$ in $\mathbb{R}^n$ where the initial conditions need to be specified as:

$$\begin{cases} x_{t_0} = \phi(\theta)), \forall t \in [t_0 - \tau, t_0] \\ u_{t_0} = \psi(\theta), \forall t \in [t_0 - \tau, t_0], \end{cases} \tag{2.6}$$

where $x(t) \in \mathbb{R}^n$ is the state vector , $u_t \in \mathbb{R}^n$ is the input vector, $y(t) \in \mathbb{R}^{\shortparallel}$ is the output vector, f and g are vectors functions ,$\tau \geq 0$ is the greatest delay of the system , $(\phi, \psi)^2 \in [t_0 - \tau, t_0]^2$ are continuous function represent the initials conditions, x_t and U_t are functions that represent respectively the state and the input of the system at the instant time t and are defined as :

$$x_t : \begin{cases} [-\tau, 0] \to \mathbb{R}^n, \\ \theta \to x_t(\theta) = x(t + \theta), \end{cases} \tag{2.7}$$

$$u_t : \begin{cases} [-\tau, 0] \to \mathbb{R}^n, \\ \theta \to u_t(\theta) = u(t + \theta). \end{cases} \tag{2.8}$$

Here, $\tau \geq 0$ is termed **delay** factor.

2.5.1 Classification of NCSs subject to time delay

In this part, we will present the four different types of delay systems. Most commonly encountered in the literature, namely: delayed type systems, neutral systems, discrete delay LTI systems and distributed delay systems.

2.5.1.1 Modeling of NCSs with constant time delay Hence, the NCSs can be treated as a deterministic system thatcan apply many deterministic system that can apply many deterministic control methods on this NCSs. The closed-loop on NCSs with constant delay is represented as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t-\tau), \\ y(t) = Cx(t) + DKx(t-\tau), \\ x(t) = \phi(t), \forall t \in [0, \tau]. \end{cases} \tag{2.9}$$

where $\tau(t) = \tau_{sc} + \tau_{ca} + \tau_c$ with τ is the total network-induced delay, τ_{sc}, τ_{ca} and τ_c are the sensor-to-controller, controller-to-actuator and the processing delays, respectively. Therefore, it would be convenient to get rid of the delay in the characteristic equation, so that conventional analysis and design techniques could be applied.

2.5.1.2 Mutually stochastic delay model In NCSs, the network-induced delay is affected by many stochastic factors such as network load, nodes competition, and network congestion hence, delay tends normally to be stochastic. so, the constant delay and the corresponding deterministic control methodologies could hardly satisfy the performance requirement of the system. The stochastic network-induced delay is divided into two delays and its interval into to sub-interval:

$$\tau(t) = \tau_1(t) + \tau_2(t). \tag{2.10}$$

where:
$$\begin{cases} \tau_1(t) = \sigma(t)\tau(t), \\ \tau_2(t) = (1-\sigma(t))\tau(t). \end{cases}$$
with $\tau(t) \in [\tau_{min}, \tau_{max}]$ and $(\tau_{max} \geq \tau_{min} > 0)$, and where τ_{max} and τ_{min} is a given the upper and the lower of the network-induce delay, respectively, and $\sigma(t)$ is Bernoulli process describe the delay distribution describe as:

$$\sigma(t) = \begin{cases} 1 \text{ if } t \in \Omega_1 = \{t : \tau(t) \in [\tau_{min}, \tau_{med})\}, \\ 0 \text{ if } t \in \Omega_2 = \{t : \tau(t) \in [\tau_{med}, \tau_{max}]\}. \end{cases} \tag{2.11}$$

with $\tau_{med} \in [\tau_{min}, \tau_{max}]$ a parameter to be chosen in order to take benefit of distribution of the delay. From (2.11), we can notice that the network-induced delay $\tau(t)$ changes randomly and the probability of $\tau(t) \in \Omega_1$ and $\tau(t) \in \Omega_2$ can be known. Moreover, it is straightforward that $\Omega_1 \cup \Omega_2 = [\tau_{min}, \tau_{max}]$ and $\Omega_1 \cap \Omega_2 = 0$. Therefore, the closed-loop dynamics expressed as:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)BKx\,(t-\tau_1(t)) + (1-\sigma(t))\,BKx\,(t-\tau_2(t))\,. \\ y(t) = Cx(t) + \sigma(t)DKx\,(t-\tau_1(t)) + (1-\sigma(t))\,DKx\,(t-\tau_2(t))\,. \\ x(t) = \phi(t), \forall t \in [-\tau_{max}, -\tau_{min}]. \end{cases} \tag{2.12}$$

2.5.1.2.1 Markov chain model

Sometimes, stochastic delays are not always mutually independent such, there are some probabilistic dependency relationships among the delays, such as Markov chain. According to the relation between the current time delay and previous delays in the real network environment, reasonably modeling the stochastic network-induced delays as Markov chains and the delay could be modeled in two ways, one chain to represent the sum of two delays (full round trip delay); two chains to represent the two delays separately, one for the sensor-to-controller delay and one for controller-to- actuator delay.

Markov chain model for stochastic time delays provides another advantage that the packet dropouts can be included naturally. Markov chain is described as follows: There is a set of $S = \{s_1, s_2, ..., s_r\}$, the process starts in one of these states and moves successively from one state to another, each move is named step. For clarification If the chain is currently in state s_i, then it moves to state s_j at the next step with a probability denoted by p_{ij}, and this probability does not depend upon which states the chain was in before the current state. An initial probability distribution (defined on S) specifies the starting state. Usually, this is done by specifying a particular state as the starting state.

2.5.1.3 Neutral time delay systems In this type of systems, the evolution of the system depends not only on the values of the past state but also its derivative over a time interval. They are described by differential equations of the following form:

$$\begin{cases} \dot{x}(t) = f(t, x_t, \dot{x}_t, u_t), \\ x_{t_0} = \phi(\theta)), \forall t \in [t_0 - \tau, t_0] \\ u_{t_0} = \psi(\theta), \forall t \in [t_0 - \tau, t_0], \end{cases} \tag{2.13}$$

where $\dot{x}(t-\tau)$ is the derivative of the delayed state vector at the instant t.

2.5.1.4 Systems with discrete delay

In some delayed physical systems one delay can not be sufficient to describe the behaviour of those systems where the representation of those systems need more than one delay , in the following we give a formulation of a system with discrete delay which are usually used :

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{q} D_i \dot{x}(t-\chi_i) + \sum_{J=0}^{r} A_j x(t-\tau_j) + B_j u(t-\tau_j) \\ y(t) = \sum_{j=1}^{r} C_j x(t-\tau_j) \end{cases} \tag{2.14}$$

where;

$x(t) \in \mathbb{R}^{\ltimes}$ is the instantaneous state vector , $u_t \in \mathbb{R}^{\ltimes}$ is the inputs vector, $y(t) \in \mathbb{R}^{''}$ is the outputs vector.and D_i, A_j,B_j ,C_j: are constants matrices with appropriate dimensions $\chi_i$and τ_j are positives discrete delays that can be bounded under the form:

$0 < \tau_j < \tau_{max}$ and $0 < \chi_i < \chi_{max}$.

2.5.1.5 system with distributed delay

This delay appear in many real process like traffic control in this class of systems , the delays have a particular form which acts in a distributed manner over all the interval of time, the mathematical model ofof this delay is given by :

$$\dot{x}(t) = Ax(t) + \int_{t-\tau}^{t} A_d(\theta)x(t+\theta)d\theta \tag{2.15}$$

with : $x(t) \in \mathbb{R}^{\ltimes}$ is the state vector ,is the outputs vector.and A_i,is a constant matrix with appropriate dimensions,τ is a scalar which define the distribution time interval of the delay, θ and $A_d(\theta)$ represents the delay matrix which is called the distributed delay kernel.

2.5.2 Packet based data transmission

Due to the use of the network to connect the components of NCSs, It is required before sending the signals of the systems to be sampled and grouped. Each group is called a “packet”, the size of the packet depends on the network used. A typical data packet is shown in Fig 2.9 The transmission of packets could be either single or multiple, the sending of signals in a single packet transmission is when all data from sensors or controllers are sampled and grouped together. In a second way, the data are transmitted in several packets which causing non-simultaneous arriving of data to the actuators or controllers due to limited size of the network and the distribution of sensors and actuators practically

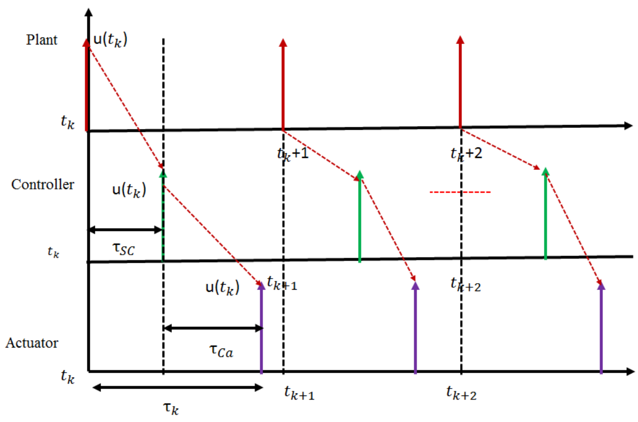

Figure 2.5: Timing diagram of Network induced delay

over a large area which make it difficult to lump data into one network packet.

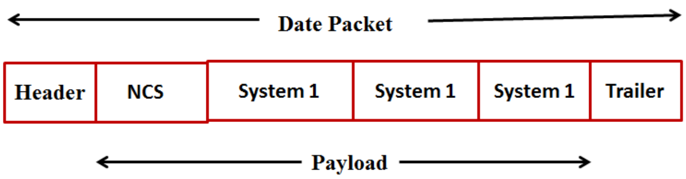

Figure 2.6: The typical data packet structure.

2.5.2.1 Packet dropouts

Data transmission error in communication networks is inevitable, when there are packets

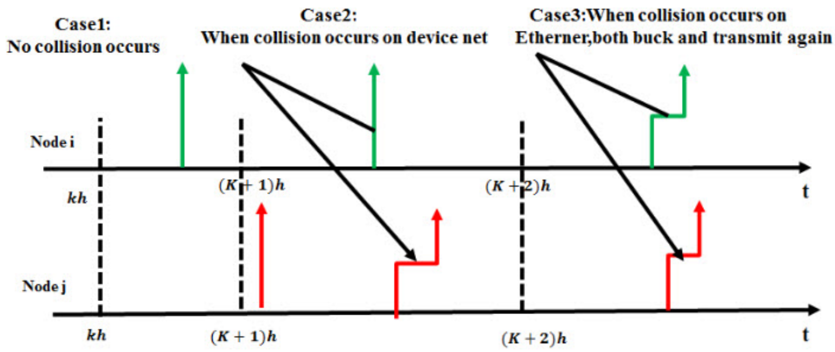

Figure 2.7: Timing diagram for two nodes

collisions, network congestion or buffer overflows, then produce a situation called "data packet dropout" fig 2.10. and creates another critical problem for NCSs, leads this dropout in packets a degradation in system performance or even makes unstable. when a data packet is lost, two different strategies are applied, that is, either to send the packet again or simply discard it.these two strategies are called Transmission Control Protocol (TCP) and User Datagram Protocol (UDP) respectively[142]. it is to note that with TCP protocol, all the data packets will be successfully received , although it may take a considerably long time for some data packet, while with UDP, some data packets will be lost forever. due to the real time requirement and the robustness of control systems, UDP is used in most applications, as far as NCSs is concerned.

2.5.2.2 Data packet disorder

When transmitting signals throughout a communication network, data packets suffer from delays. As a result, a data packet sent earlier may arrive at the destination later, or vice versa, see Fig. 2.8. This phenomenon is known as packet disorder. The existence of data packet disorder can mean that a newly arrived control signal in $NCSs$ may not be the latest. Consequently, [160, 158]; the control performance will be inevitably degraded if

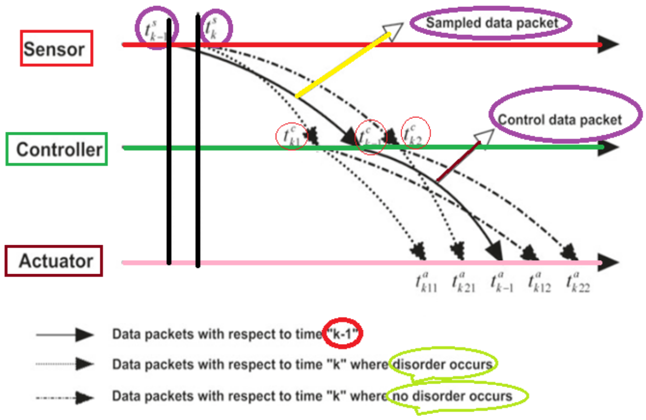

Figure 2.8: Packet disorder

the control algorithm has not taken explicit consideration of the disordered data.
To this end, researchers take into account the problem of packet loss and packet disorder [158]in the design of the networked controller for NCSs, by designing a controller to withstand the upper bound of dropouts in the system where they consider the packet dropout as a random process and model it as a Bernoulli distribution or a Markov process or they considered as kind of time delay because if packet dropout occur the receiver (actuator or controller) wait time for receiving another packet (the lost packet has expired) this waited time is modeled as .

$$\tau_p(t) = (\eta(t) + 1) * h. \tag{2.16}$$

where $N \in \mathbb{N}$ is the number of packets loss and h is the sampling time, and the number of packet loss can be denoted by:

$$\eta(t) = \frac{t_{k+1} - t_k}{h} - 1. \tag{2.17}$$

where $[t_1, t_2, \ldots, t_k, t_{k+1}]$ are the sampling instants of data transmitted from the sensor to the ZOH.According to the previous model of packet dropout, it can lumped the network-induced delay and the packet dropout as one input delay system as

$$\tau(t) = \tau_d(t) + \tau_p(t). \tag{2.18}$$

And the closed-loop system formulated as an input delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ y(t) = Cx(t) + DKx(t - \tau(t)), \\ x(t) = \phi(t), \forall t \in [\underline{\tau}, \bar{\tau}]. \end{cases} \tag{2.19}$$

with $\underline{\tau} = \inf_{k \in \mathbb{N}}(\tau_k) = \tau_{min}$
and $\bar{\tau} = \sup_{k \in \mathbb{N}}(\tau_{k+1}) = \tau_{max} + (\bar{\eta} + 1)h$
and $\bar{\eta}$ is the maximum number of consecutive packet dropout.

2.5.3 Quantization error

In information theory, quantization is a phenomenon occurring in all data networks, and is considered as information encoders and thus as an integral part of the whole system where a real-valued signal is transformed into a piece-wise constant signal. By definition, a quantizer is a function that maps a real-valued function into a piece-wise constant function taking on a finite set of values. In the literature, there are two common types of quantization and they are the uniform quantizer and logarithmic ones, for the first kind maps the real-valued function to a finite number of quantization regions with the rectilinear shape []or arbitrary shape. The study of a system affected by a uniform quantizer is always based on a "zoom" strategy, which is usually composed of two stages, i.e. "zooming-out" and "zooming-in". In the first stage, the range of quantizer is increased to guarantee the states of a system can be adequately measured. In the second stage, the quantization error is decreased to drive the states to the origin. When a system is affected by logarithmic quantizer, in which the quantization levels are linear on a logarithmic scale, the simple classical approach to analysis and mitigation of quantization effects is to treat the quantization error as uncertainty or non-linearity and bound it using a sector bound. The logarithmic quantizer is expressed as follows:

$$q_L(v) = \begin{cases} sgn(v)\mathcal{V}_i & \text{if } \frac{\mathcal{V}_i}{1+\Delta} < |v| < \frac{\mathcal{V}_i}{1-\Delta}, \\ 0 & \text{if } v = 0. \end{cases} \tag{2.20}$$

where $\mathcal{V}_i = \rho^i \mathcal{V}_0$, $i = \pm 1, \pm 2, \pm 3, \ldots$, $0 < \rho < 1$, $\mathcal{V}_0$ is a positive scaling constant, $\Delta = \frac{1-\rho}{1+\rho}$ and $sgn(.)$ is a sign function satisfying

$$sgn(v) = \begin{cases} 1 & \text{if } v > 0, \\ -1 & \text{if } v < 0, \\ 0 & \text{if } v = 0. \end{cases} \tag{2.21}$$

The corresponding quantization error satisfies:$q_L(v) = \delta v$. where $\delta \in \begin{bmatrix} -\Delta & \Delta \end{bmatrix}$.
Logarithmic quantizer provide a sector bound method to deal with the quantization errors, and have been widely used in various control and filtering problems.
The other typical quantizer is the uniform quantizer, which is described as follows

$$q_U(v) = \begin{cases} sgn(v)\frac{\lfloor 2^{N-1}v \rfloor + 0.5}{2^{N-1}} & \text{if } |v| < \bar{v}, \\ sgn(v)(1 - \frac{0.5}{2^{N-1}})\bar{v} & \text{if } |v| = \bar{v}. \end{cases} \tag{2.22}$$

where $\bar{v} > 0$ is the given constant quantizer limitation, N is the number of given quantization bits and $\lfloor x \rfloor = \{max\, z \in \mathbf{N}, z \leq x\}$.
The corresponding quantization error is obtained as $|q_U(v) - v| \leq \frac{\bar{v}}{2^{N-1}}$.
The main problem with quantization is to find a quantified feedback control law to stabilize the given system which can be stabilized by linear time-invariant feedback.

2.5.4 Variable sampling transmission intervals

Before grouped the signals of NCSs and transmitted them through the network it needs to be sampled, which in the conventional systems the sampling period are usually fixed due to their simplicity in design and analysis and called "periodic sampling", but in the recent NCSs, the sampling period is varying since there is awaiting in a queue before the transmission process which will be based on the availability of the network and the protocol used and called “Aperiodic sampling”. It was proved recently that aperiodic sampling may have better performance than sampling at fixed intervals.

2.5.5 Bandwidth limitation

Any communication network has limited bandwidth, which is the number of bits that can be transmitted per second, this limitation of the bandwidth depends not only on the physical bandwidth but also on the efficiency of encoding the data into packets, how efficiently the network operates in terms of inter-frame times (long or short), and whether network time is wasted due to message collisions. Which imposes significant constraints on the operation of NCSs. For this purpose, the researchers made tremendous efforts to determining the minimum bit rate that is needed to achieve the control objectives, Among these research the involving of **Minimum bit rate** necessary to stabilize an LTI system has been derived, **Average bit rate** where is a measure of how infrequent feedback information is needed (in digital networks) to guarantee that the system remains stable, **Intermittent feedback** in which the open-loop is closed for certain fixed or time-varying periods, leading to opportunistic situations where the sensor sends bursts of information when the network is available, this helps in taxing the network less. **Quantified feedback** which is provided in the digital system implementation of NCSs, and if the open-loop system is unstable, only then can we determine the minimum average bit rate to process feedback information. Further research on communication constrained feedback channels is establishing a connection between stabilizability and an inequality relating feedback channel data to open-loop eigenvalues.

2.6 Control approaches of networked control system

As we all know, that there are advantages and challenges to a system controlled over a network such as scalability (adding sensors connected through the network at different locations), low maintenance costs, and so on. However, it is difficult to find the NCSs controller makes the closed-loop stable and guarantee the system performance. For this fact, researchers have given us precise and optimum control strategies emerging from classical control theory, starting from PID control, optimal control, adaptive control, robust control, intelligent control and many other advanced forms of these control algorithms. Applying all these control strategies over a network, however, becomes a challenging task where manifested in ensuring stability and good performance of systems. To deal with the stability problem of NCSs the research community develops several methods when considering one or more of the imperfections discussed in the previous section, where the main issue is to guarantee the stability or other performance objectives of NCSs subject to these imperfections. In this section, a discussion on each of these approaches which include the input delay approach, switched approach, Markovian approach, impulsive approach, stochastic approach ...etc.

2.6.1 Input delay system approach

In the input delay approach, the NCSs are modeled as a system with time-varying delay where the time-varying delay includes the delay from the sensor-to-controller, the delay from controller-to-actuator, the computation delay, and the packets dropout represented as a delay. This approach was developed also to solve the synchronization problem of the complex network by considering the signal sampling. The main issue in this approach is determining the maximum allowable upper bound (MAUB) of transmission delay which guarantees the stability and the good performance of NCSs. Also, the determination of the maximum allowable upper bound of transmission delay is an important factors for practical applications. Therefore, the NCSs closed-loop was described by the following time-varying system with input delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ x(t) = \phi(t), \qquad \tau(t) \in [-\tau_M, -\tau_m) \,. \end{cases} \tag{2.23}$$

with

$$\begin{cases} 0 \leq \tau_m \leq \tau(t) \leq \tau_M \text{ or } (\tau_M + (\eta + 1)h \text{ if packet dropout}), \\ \tau_m > 0, \tau_M > 0, \eta \geq 0. \end{cases} \tag{2.24}$$

where τ_m, τ_M, η are the upper value and lower value and number of packet loss, respectively.A sufficient condition of stability of this NCSs is derived based on a proper Lyapunov function and presented using the LMI method in chapter 5.

2.6.2 Stochastic system approach

In general, the constraints of communication in NCSs such time delay and packet dropout are stochastic in nature, so the application of conventional stochastic control approaches to NCSs is possible by using Bernoulli distribution or Markov chain. This system model simulates the NCSs model in realities and the closed-loop of NCSs was described by the stochastic time distribution delay system with random delay using a Bernoulli distribution sequence or Markov chain. Firstly, the stochastic system using a Bernoulli distribution sequence where the closed-loop of NCSs was described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)\bar{B}Kx\,(t-\tau_1(t)) + (1-\sigma(t))\,\bar{B}Kx\,(t-\tau_2(t))\,, \\ x(t) = \phi(t), \qquad \tau(t) \in [-\tau_{max}, -\tau_{min})\,, \end{cases} \tag{2.25}$$

where $\sigma(t)$, is describe in (2.11) and

$$0 < \tau_{min} \leq \tau_{med} \leq \tau_{max}, \tag{2.26}$$

with τ_{med} a parameter to be chosen in order to take benefit of distribution of the delay and the network-induced delay $\tau(t)$ changes randomly along the interval $[\tau_{min}, \tau_{max}]$ and its probability is be known. A sufficient condition of stability and controller design of this model of NCSs is derived based on a Layapunov Krasovskii functional and formulated as LMI conditions in chapter 4.

2.6.3 Markovian system approach

The NCSs modeled as Markovian jump systems generally consist of a finite number of subsystems and a jumping law governing the active or deactivate mode switches among these subsystems. These subsystems are typically modeled as differential equations, and the jumping law is a Markov chain. Markovian jump systems are a powerful modeling tool in Networked control systems. The NCSs can be modeled as having different dynamics at the abrupt changes, and the changes are typically memoryless, thus resulting in a Markovian jump system. The NCSs as Markovian jump linear systems are expressed as follow:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t), \\ y(t) = C(r(t))x(t) + D(r(t))u(t). \end{cases} \tag{2.27}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^p$ are the state, input, and output, respectively. The parameter $r(t)$ denotes a continuous-time Markov process on a probability space, which takes values in a finite set $S := \{1, 2, \ldots, N\}$ with transition probabilities given by

$$Pr((t+\delta) = j | r(t) = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & if\ i \neq j \\ 1 + \pi_{ii}\sigma + o(\delta). & if\ i = j \end{cases} \tag{2.28}$$

where $\delta > 0$, and π_{ij} denotes the transition probability rate from mode i to mode j when $i \neq j$. Furthermore, for all $i \in S$, π_{ij} satisfies $\pi_{ij} > 0, (i \neq j)$ and $\pi_{ij} = -\sum_{j\in S, i\neq} \pi_{ij}$.
The Markov process $\{r(t), t > 0\}$ is assumed to have an initial process $r(0) = (\mu_1, \mu_2, \ldots, \mu_n)$. The matrices $A(r(t))$, $B(r(t))$, $C(r(t))$ and $D(r(t))$ are contained in $\{A_1, A_2, \ldots, A_N\}$, $\{B_1, B_2, \ldots, B_N\}$, $\{C_1, C_2, \ldots, C_N\}$ and $\{D_1, D_2, \ldots, D_N\}$, respectively, and if $r(t) = i \in S$, we have $A(r(t)) = A_i$, $B(r(t)) = B_i$, $C(r(t)) = C_i$, and , $D(r(t)) = D_i$. When studying the NCSs as a markovian jump system, generally the jumping parameters are often assumed to be precisely known, this assumption is usually incorrect in practice, while the system states can often be observed. So, the *Wonham filter* is used to estimate the jumping parameters using the given system matrices. On a parallel line, in analog systems, imprecise measurements are often present and the quantization error sometimes can not be ignored which makes the implementation of control almost impossible. In addition to that and which make it worse, the poor stability margins of systems will be present if not implemented the robust control such as common techniques H_∞, L_2 or μ synthesis, etc.

2.6.4 Switched system approach

A switched linear system is a hybrid system that consists of several linear subsystems and a rule that orchestrates the switching among them. Switched linear systems provide a framework that bridges the linear systems and the complex and/or uncertain systems. On one hand, switching among linear systems may produce complex system behaviors such as chaos and multiple limit cycles. On the other hand, switched linear systems are relatively easy to handle as many powerful tools from the linear and multilinear analysis is available to cope with these systems. Moreover, the study of switched linear systems provides additional insights into some long-standing and sophisticated problems, such as intelligent control, adaptive control, and robust analysis and control. In a switched system approach, the NCSs is represented as a switched system that is typically used by modeling different network conditions in NCSs as different system modes with a finite number of subsystems. This approach is able to easily deal with network constraints such as transmission delay and packet loss. The use of this approach is difficult and the difficulty to use it grounded on how well we understand the properties of the changes in the network conditions. The continuous-time switched linear networked control system

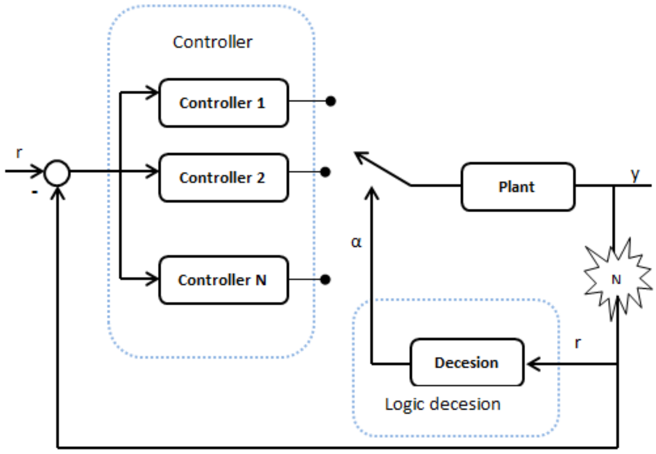

Figure 2.9: Switched System Architecture.

represented as following:

$$\dot{x}(t) = Ax(t) + BK_{\sigma}x(t - \tau_{\sigma}) + E\omega(t). \tag{2.29}$$

initialized at $x(t_0) = x_0$ with input u and switching signal σ.

$\omega(t)$ is a defined function that express disturbances.

Suppose that the switching signal is well-defined and its switching sequence is

$$\{x_0, (t_0, i_0), (t_1, i_1), \ldots, (t_l, i_l)\}.$$

As the i_0 the subsystem is active during $[t_0, t_1)$, we have

$$\dot{x}(t) = Ax(t) + BK_i x(t - \tau_i) + E\omega(t), \quad x(t_0) = x_0 \quad t \in [t_0, t_1). \tag{2.30}$$

Definition[126, 99] The system (2.29) is exponentially stable under the switching signal σ if there exist positive constants γ andα such that the solution of $x(t)$ of the system (2.29) satisfies

$$\|x(t)\| \leq \gamma \|x(t_0)\|_e e^{-\alpha(t-t_0)}, t \geq t_0.$$

where

$$\|x(t_0)\|_e \overset{\Delta}{=} sup_{-h_{M+1} \leq \theta \leq 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}.$$

For a switched linear NCS, the system permits a unique solution for the forward time-space, hence, any discrete-time switched NCS is well-posed.

The state transition matrix is multiple multiplications of matrices. Accordingly, the properties of matrix multiplication play an important role in analyzing the switched NCS. Finally, for both continuous-time and discrete-time switched linear NCS, the state trajectory possesses several nice properties under mild conditions.

2.6.5 Impulsive system approach

Among the most important kinds to represent the NCSs model is linear impulsive systems where the reason is that the linear impulsive systems are a class of hybrid systems where the state propagates according to linear continuous-time dynamics except for a countable set of times at which the state can change instantaneously, as is the case in NCSs where the plant is a linear continuous-time system and the controller is a linear discrete-time system. In impulsive systems, the impulsive effects can be event-driven and/or time-driven.
The linear impulsive systems can be represented as:

$$\begin{cases} \dot{x}(t) = A_C x(t) + B_C u(t) + E_C \omega(t), \quad t \in R\,\mathcal{T}, \quad x(t_0) = x_0, \\ x(t_k) = A_I x(t_k{}^-) + B_I u[k] + E_I \omega[k], \quad t \in Z^+ \\ Z(t) = C_C x(t), \\ y(t_k) = C_I x(t_k{}^-). \end{cases} \tag{2.31}$$

where $x(t)$ is the continuous-time state, $u(t)$ is a continuous-time control input, $\omega(t)$ is a continuous-time exogenous input, $u[k]$ is a discrete-time control input, $w[k]$ is a discrete-time exogenous input, $z(t)$ is a continuous-time output to be controlled, and $y[k]$ is a discrete-time measurement, $\mathcal{T} = \{\tau_1, \tau_2, \ldots\}$ is a set of impulse times assumed to contain a finite number of elements on any finite time interval. The state space for (2.31) is denoted by $\mathcal{X}$. For a function of time $f()$ that is continuous from the right but possibly discontinuous at some t,$f(t^-)$ denotes $\lim_{\varepsilon\to 0^+} f(t-\varepsilon)$, assuming that the limit exists. To distinguish continuous-time signals from discrete-time signals, using the parentheses surrounding a real-time argument to identify the quantities that evolve in continuous time, and the square brackets surrounding an integer index to represent the quantities that are available or computed only at the impulse instants.
In the event driven case, we can represent the impulsive system with the same minor or formulate but occur at a countable number of events when the state vector crosses a given hyper-surface of the state space. In such a case, impulses can still be indexed by time, and the set of impulse times can be expressed as $\{\tau_i(x)|i \in N\}$ [177, 178]. Although,

transmission.

2.7 Conclusion

In this chapter, we presented the major components of NCSs with the main role of each one, then we introduced different real structures and topologies with their benefits and disadvantages. After that, we focused on a very important part of this thesis concerning the modeling of NCSs including continuous and discrete models, and the network induced imperfections (time delays, packet dropouts .etc.) with theirs effects on the stability of NCSs. On the other hand, we have presented several techniques of analyzing stability and designing controllers for NCSs.

Chapter 3

Lyapunov stability analysis and $\mathcal{H}_\infty$ robust control of NCSs

3.1 Introduction

As it is well known, network induced delay and packet dropouts often leads NCSs to instability. Therefore, the stability analysis of NCSs is strongly required, and has formed a sturdy research field during the past years. Lyapunov approach is the most popular approach for the stability analysis of networked control systems [203], in which the main purpose is to construct less conservative stability criteria which can be checked by maximum allowable upper bound ($MAUB$) of the delay, number of packet dropouts and the number of decision variables (NDV).

As well known, less conservative approaches would play a role to improve system performances and relax the operating environment of the systems. The conservativeness of the Lyapunov Krasovskii Functionals method comes from two things: the first one is the choice of functional by constructing suitable Lyapunov Krasovskii Function (LKF) including multiple integral terms, the second way is to develop new integral inequalities to estimate and bounding the time derivative of the LKF.

So choosing an appropriat Lyapunov Krasovskii functional is crucial to the derivation of less conservative criteria. Therefore, to develop less conservative methods are a key issue for many researchers, because it is important in both theoretical and practical points. For this issue, several useful inequalities were developed such as Jensen's inequality [151], Wirtinger based inequality [131], free matrix based integral inequality [187], auxiliary function based integral inequalities [114]. For systems with time varying delay, alternative attention is focused on seeking the solution of matrix inequalities about the time varying delay where several methods are proposed, for example, linear convex combination approach [112], reciprocally convex lemma [113], improved reciprocally convex inequality [190, 192, 199], and quadratic convex function approach [58].

This section gives an overview of the theoretical approaches concerning the stability analysis of time delay systems. The scope of this thesis will include only time domain approach and the second method of Lyapunov.

3.1.1 Stability theory

Out of the need for an accurate and practical means for the modelling, analysis and design of physical systems, the old control theory has emerged as a rigorous framework with which to approach a wide range of mathematical and engineering problems. Starting from a mathematical description of the system of study, control theory techniques are used to characterize a system's autonomous activity, predict its response to external influences and methodically modify a system to shape its behavior.

A structured way in which to view a system is to regard it as a module that interacts with the physical world through a set of quantifiable input and output signals. In addition, an internal description of a system is a key way of understanding how such aforementioned inputs shape its output. A key mathematical tool, among others, with which to model these different signal spaces is the Ordinary Differential Equations ($ODEs$).

The ODE provides a map from the input space to the output space of a system whilst maintaining a description of its internal dynamics, a fundamental property associated with physical systems which are of primary practical significance are those of stability, mathemat- Icely, stability is the quality of a system's internal signals (or states) and output signals. remaining 'small' in some sense

Depending on the context, such stability notions can be defined as boundedness, eventual smallness, integral smallness and convergence [140].

A rich theory of system stability has been developed which encompasses several different, but fundamentally related, treatments of the subject, including Lyapunov theory [57, 56], the dissipativity approach, input, output stability, input state stability [140] and the behavioral approach, a brief exposition of some of these approaches which are of relevance to this thesis is given in the rest of this chapter.

3.1.2 State space tools and Lyapunov theory

a natural way with which to describe a physical system is through the ODE state-space representation [57, 56]

$$\begin{cases} \dot{x}(t) = f(t, x, u), \\ y(t) = h(t, x, u). \\ x_{t_0} = \phi(\theta), \forall t \in [-\tau, 0] \end{cases} \tag{3.1}$$

where: $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^n$ is the system's input and $y(t) \in \mathbb{R}^p$ is the system's output. The maps $f(.)$ and $h(.)$ are such that $f : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ and $h : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^p$.

A solution of the unforced system (3.1) is the map from time t and the initial state $x(0)$ onto the state at time t, formally, this is written as the map $x(.) : [0, \infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ which satisfies (3.1). Such a map exists and is unique if $f(.)$ is Lipschitz continuou function, if the solution exists, then the state $x(t)$ starting at a state $x(0)$ evolves over time t according to the *ODE* (3.1). When f is independent of t the system is said to be time invariant. In the case where (3.1) is both unforced and time invariant, (3.1) is written simply as:

$$\dot{x}(t) = f(x) \tag{3.2}$$

Unlike the map $x(t, x_{t_0})$, the output y is a static function of the signals t, x, u and we assume the existence and uniqueness of solutions and, without loss of generality, the solution $x_t = 0$ is an equilibrium.(If the equiibrium is non zero, an equivalent condition can be achieved by coordinate change).

We call equation (3.3) second Lyapunov method, this method is based on the existence of a function of the state variable x_t, denoted $V(t, x_t)$, such that:

$$\begin{cases} V(t, x_t) \geq 0 \\ \dot{V}(t, x_t) \leq 0, x(t) \neq 0 \end{cases} \tag{3.3}$$

In this thesis, we only consider the Lyapunov Krasovskii method, which is based on a functional of the form $V = V(t, x_t)$, instead of a simple function$V$. We notice that the proposed Lyapunov Krasovskii function is valid only for linear NCSs which is the subject of this thesis.

3.2 Classical stability concepts

An important problem in NCSs is the research of stability criteria. Before discussing this aspect, some fundamental concepts from the stability theory are recalled. Intuitively, stability is a system property that corresponds to returning to its equilibrium position when it is punctually disturbed, before going into the contents of this chapter, it is necessary to give some definitions, these definitions will be used throughout this thesis.

3.2.1 Equilibrium point

The trivial $x(t) = 0$ solution of the system (3.1) is said to be stable $\forall t_0 \in \mathbb{R}$ if $\forall \epsilon > 0$, there exists $\delta = \delta(t_0, \epsilon)$ such that $\|x_{t_0}\|_c < \delta$ implies $\|x_t\| < \epsilon$ for $t \leq t_0$, or simply [41] :

$$\forall \epsilon >, \exists \delta = \delta(t_0, \epsilon), \text{ such that } \|x_{t_0}\|_c < \delta \implies \|x(t)\| < \epsilon, \forall t \leq t_0. \tag{3.4}$$

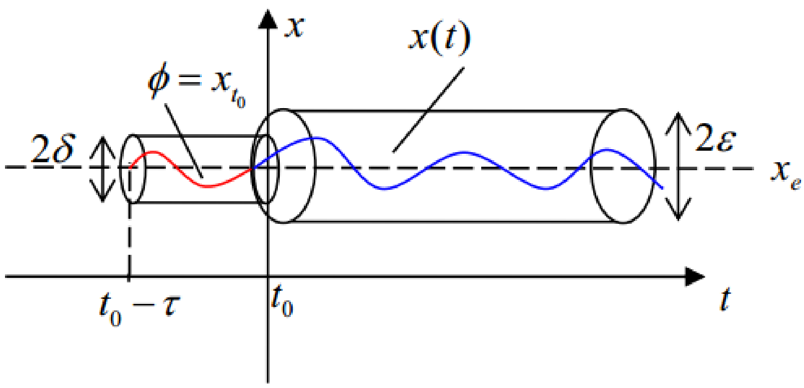

Figure 3.1: Stability in the sense of Lyapunov for an equilibrium point x_e.

3.2.2 Asymptotic and global asymptotic stability

The Lyapunov asymptotic stability can be defined with the help of geometric interpretation as shown in figure 3.2 where we assume that the initial state x_0 is located in a closed ball region with equilibrium state x_e of its center and its arbitrary radius δ when time t tends to infinity, the system state $x(t)$ can be converged to equilibrium state x_e as shown in figure 3.2, then the system is in Lyapunov asymptotic stability.

To more illustration, the mathematical definition of Lyapunov asymptotic stability is presented as follow:

Definition 3.1 *The trivial solution* $x(t) = 0$ *of the system* (3.1) *is asymptotically stable if it is stable and attractive in addition[57, 56]:*

$$\forall \epsilon > 0, \exists \delta_a = \delta(t_0, \epsilon), \text{ such that } \|x_{t_0}\|_c < \delta_a \\ \implies (\|x(t)\| < \epsilon), (\lim_{t\to\infty} \|x(t)\| = 0), \forall t \leq t_0 \tag{3.5}$$

the system described by 3.1 is globally stable if it is stable and globally attractive.

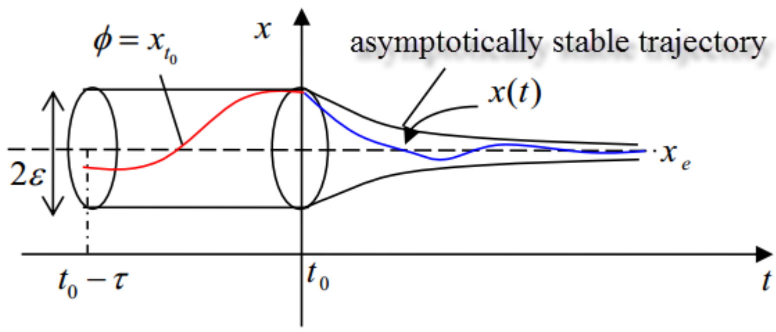

Figure 3.2: Lyapunov asymptotic stability trajectory.

3.2.3 Uniform asymptotic stability

The uniform stability means that the equilibrium point is not getting progressively less stable with time [128] and is expressed mathematically by the following definition:

Definition 3.2 *The trivial solution* $x(t) = 0$ *of the system* (3.1) *is uniformly asymptotically stable if it is uniformly stable and attractive, so if there is* δ_a *as for all* $\eta > 0$ *it exists* $T = T(\delta_a, \eta)$ *such as* $\|x_{t_0}\|_c < \delta_a$ *implies* $\|x_t\| < \eta$ *for all* $t \leq t_0 + T$ *and* $t_0 \in \mathbb{R}$ *or simply:*

$$\exists \delta_a > 0, \text{ such that, } \forall \eta > 0, \exists T = T(\delta_a, \eta), \\ \text{such that, } \|x_{t_0}\|_c < \delta_a \implies \|x(t)\| < \eta, \forall t \leq t_0 + T \tag{3.6}$$

Remark 2 *It is worth mentioning that the definition of the asymptotic stability do not express the speed of convergence of trajectories to the origin, which is exponential in time invariant linear systems either to or from the origin. But, for time varying and nonlinear systems, the rate of convergence can be different according to different system types.*

3.2.4 Exponential and global exponential stability

Lyapunov exponential stability includes both the uniform and the asymptotic stability, it is particularly necessary when studying stability of uncertain systems and is defined as:

Definition 3.3 *The trivial solution $x(t) = 0$ of the system* (3.1) *is exponentially stable if it is stable and if there are three strictly positive constants $\lambda_1 > 0$, $\lambda_2 > 0$, and $\delta > 0$, such as for any initial condition $x_{t_0} = \phi$ the following condition is satisfied:*

$$\forall t \leq t_0 \|x_{t_0}\|_c \leq \delta \implies \|x(t)\| < \left(\lambda_1 \|x_{t_0}\| e^{\lambda_2 (t-t_0)} \|\right), \forall t \geq t_0 \tag{3.7}$$

In this case, the constant λ_2 is called the "decay rate" or the " exponential convergence rate " (see figure 3.3).

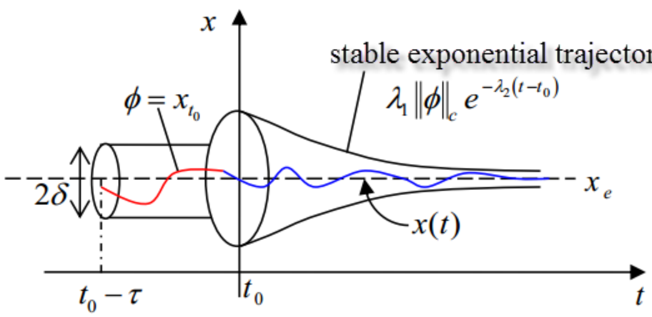

Figure 3.3: Lyapunov exponential stability trajectory.

Remark 3 *We said that the system is globally exponentially stable around an equilibrium state if the bound in equation* (3.7) *holds for all $x(t) = 0 \in \mathbb{R}^n$.*
The uniform and asymptotic stability of a linear system with uncertain coefficients delays or time varying does not imply the exponential stability. To search the decay rate and the constant λ_2 of exponential stability in (3.7)*, the Lyapunov Krasovskii method is the most recommended to give sufficient conditions.*

3.2.5 Robust stability

We say that an uncertain system is robust asymptotically stable if it is asymptotically stable for all allowable uncertainties on the model parameters and we say also that an uncertain system is $\mathcal{H}_\infty$ robustly stable or stable with respect to space $\mathcal{H}_\infty$ if its equilibrium is asymptotically stable for all the admissible uncertainties on the parameters of the model and for any delay function $\tau(t)$ defined in space $\mathcal{H}_\infty$ [7].

Note that Lyapunov second method relies on the existence of a positive definite function $V(t, x_t) > 0$ so that along the trajectories of the system (3.1) we have $\dot{V}(t) < 0$, this method is valid only for a limited class of systems delay, because $V(t, x_t) > 0$ depends on the past values x_t. This is why it is very difficult to apply in the general case of delay systems.

Two techniques based on extensions of Lyapunov second method were developed by *Krasovskii and Razumikhin* respectively in the context of delay systems.

In what follows, we will briefly present these two extensions.

Theorem 3.1 *Suppose that* Ω *is an open set in* $\mathbb{R} \times \mathbb{C}$*,* $f : \Omega \to \mathbb{R}^n$*, is continuous, and* $f(t, \Psi)$ *is Lipschitzian in* Ψ *in each compact set in* Ω*, that is for each given compact set* $\Omega_0 \subset \Omega$ *there exists a scalar* L *, such that [41] :*

$$\|f(t, \Psi_1) - f(t, \Psi_2)\| \leq L\|(\Psi_1 - \Psi_2)\|, \forall (t, \Psi_1) \in \Omega_0, \forall (t, \Psi_2) \in \Omega_0 \tag{3.8}$$

If $(t_0, \Psi) \in \Omega_0$*, then there exists a unique solution of (3.1) through* (t_0, Ψ)*.*

3.2.6 Lyapunov Razumikhin approach

In this approach, the goal is to consider a classical Lyapunov function $V(t, x(t))$ as the one employed for the delay free case (i.e. for ordinary differential equations)[1].

The main idea of the *Lyapunov Krasovskii* theorem is that it is not necessary to ensure the negative definiteness of $V(t, x(t))$ along all the trajectories of the system. Indeed, it is sufficient to ensure its negative definiteness only for the solutions that escape the neighborhood of $V(t, x(t)) \leq c$ of the equilibrium, this idea is formalized in the following theorem:

Theorem 3.2 *Let* u*,* v *and* $W : \mathbb{R} \to \mathbb{R}_+$ *be non decreasing functions such that:* $u(\theta)$ *and* $v(\theta)$ *are strictly positive for all* $\theta \geq 0$*. Assume that the vector field* f *of* 3.1*is bounded for bounded values of its arguments [2, 62], if there exists a continuous and differentiable function* V*:* $\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}_+$ *such that :*

$$\begin{cases} u(\|\phi(0)\|) \leq V(t, \phi) \leq v(\|\phi\|) \\ \dot{V}(t, \phi) \leq -W(\|\phi(0)\|) \end{cases}$$

for all trajectories of (3.1) *satisfying:*

$$V(t+\theta, \phi(t+\theta)) \leq V(t, \phi(t)), \forall\theta \in [-\tau, 0] \quad (3.9)$$

The solution x_t is uniformly stable. Moreover, if $w(\theta) \geq 0$, and it exists a strictly increasing function $p : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $p(\theta) \geq \theta$, $\forall\theta \geq 0$ and

$$\begin{cases} u(\|\phi(0)\|) \leq V(t, \phi) \leq v(\|\phi\|) \\ \dot{V}(t, \phi) \leq -W(\|\phi(0)\|), \forall t \leq t_0 \end{cases}$$

for all trajectories of (3.1) *verifying:*

$$V(t+\theta, x(t+\theta)) \leq pV(t, x(t)), \forall\theta \in [-\tau, 0] \quad (3.10)$$

then such a function $V$is called *Lyapunov Razumikhin* function and solution $x_t = 0$ is uniformly asymptotically stable for system. In practice, the functions p are usually considered as $p = q(\theta)$ where $q \geq 1$. Moreover, Lyapunov functions more commonly employed in the *Razumikhin* approach are of the form:

$$V(t) = x^T P x(t) \quad (3.11)$$

where P is a symmetric positive definite matrix of dimension $n \times n$, equation (3.9) thus becomes:

$$x^T(t+\theta)Px(t+\theta) \leq qx^T(t)Px(t), \forall\theta \in [-\tau, 0], and \quad q \geq 0 \quad (3.12)$$

Although the *Lyapunov Razumikhin* approach generally leads to more conservative results it allows taking into account variable delays without restriction on derivative of the delay function (2.9) and generally leads to delay independent stability conditions, it has also been shown that for constant delays, the existence of a *Lyapunov Razumikhin* function is equivalent to the existence of the so called *Lyapunov Krasovskii* functional.

3.2.7 Lyapunov Krasovskii approach

The *Lyapunov Krasovskii* method is an extension of the second Lyapunov method dedicated to the stability analysis of functional differential equations [2]. It consist of the research of a functional of the form $V(t, x_t)$, which is a decreasing functional along the trajectories of (3.1) similarly, V is called a functional because it depends on the delayed state vector x_t, which is a vector function considered in the interval$[t-\tau, t]$, this approach is summurized in the following theorem [61]:

Theorem 3.3 *Let u, υ and W : $\mathbb{R}_+ \to \mathbb{R}_+$ be continuous and increasing functions, with*

$$\forall \lambda \geq 0, \forall u(\lambda) \geq 0, \forall \upsilon(\lambda \geq 0, u(0) = \upsilon(0) = 0, \exists V : \mathbb{R} \times \mathbb{C} \to \mathbb{R}_+$$

such that

$$\begin{cases} u\left(\|\phi(0)\|\right) \leq V(t,\phi) \leq \upsilon(\|\phi\|) \\ \dot{V}(t,\phi) \leq -W(\|\phi(0)\|), \forall t \leq t_0 \end{cases}$$

Assuming that the vector field f in (3.1) is bounded. Then the solution $x_t = 0$ of (3.1) is uniformly stable (Vis a functional). Moreover, if$\lambda \geq 0$, $w(\lambda) \geq 0$ the solution of (3.1) is uniformly asymptotically stable if V satisfies the following conditions [2, 61] :

$$\begin{cases} u\left(\|\phi(0)\|\right) \leq V(t,\phi) \leq \upsilon(\|\phi\|) \\ \dot{V}(t,\phi) \leq -W(\|\phi(0)\|), \forall t \leq t_0 \end{cases}$$

V is Lipschitz, with respect to its second argument. Then, the solution x_t of (3.1) is exponentially stable and the functional V is called a *Lyapunov Krasovkii* functional (LKF).

Comment : In the previous theorem, the derivative $\dot{V}(t,\phi)$ refers to the derivative in the sense of **Dini**, that is to say :

$$\dot{V}(t,\phi) = lim\ sup_{\Delta t \to 0} + \frac{1}{\Delta t}\left[V(t+\Delta t, x_{\Delta t \to 0}(t,\phi)) - V(t,\phi)\right] \tag{3.13}$$

However, a pain point in this approach is the design of such functional V when it exists. Classical form of such functional V is given by:

$$\begin{aligned} V(t,\phi) =& \phi^T(0)P(t)\phi(0) + \phi^T(0)\int_{-\tau}^{0} Q(t,\sigma)\phi(\sigma)d\sigma \\ &+ \int_{-\tau}^{0}\int_{-\tau}^{0} \phi^T(\sigma)R(t,\sigma,\rho)\phi(\rho)d\sigma d\rho) + \int_{-\tau}^{0} \phi^T(\xi)S\,\phi(\xi)d\xi \end{aligned} \tag{3.14}$$

where, P, Q, R and S are square matrix functions of dimensions $n \times n$. $P(t)$ and $S(\xi)$are symmetric and positive definite matrices, while R satisfies $R(t,\sigma,\rho) = R^T(t,\sigma,\rho)$.

It assumed that each matrix element is bounded and that their derivatives are piece wise continuous functions. In practice, the determination of these functionals set difficult problems due to the time varying aspects of the matrices, it is preferable to restrain to the use of functionals in which the matrix functions P, Q, R, and S are constant, under such restriction, the *Lyapunov Krasovskii* functional becomes:

$$\begin{aligned} V(t,\phi) =& \phi^T(0)P\phi(0) + \int_{-\tau_{slot}}^{0} \phi^T(\sigma)S\phi(\sigma)d\sigma + 2\phi^T(0)\int_{-\tau_{slot}}^{0} Q\phi(\sigma)d\sigma \\ &+ \int_{-\tau_{slot}}^{0}\int_{-\tau_{slot}}^{0} \phi^T(\sigma)R\phi(\sigma)d\sigma d\rho + \int_{-\tau_{slot}}^{0}\int_{-T_\theta}^{0} \dot{\phi}^T(\sigma)R\dot{\phi}(\sigma)d\sigma d\theta) \end{aligned} \tag{3.15}$$

This class of LKF has been widely used in the literature in order to assess stability of the system for the delay τ. The main problem within the application of this theorem is the design functional and then to provide some conditions that guarantee its positive definiteness and the negative definiteness of its derivative.

The derivation of stability conditions using *Lyapunov Krasovskii* functionals usually involves quite elaborate developments.

To give an idea of the procedure involved in this approach and to provide a glimpse of its technical flavor, we present here some basics on the procedure to follow in order to derive asymptotic stability criteria for networked control systems considered as a special case of time delay system expressed in terms of Linear Matrix Inequality (LMI). The basic steps for deriving constructive stability conditions are illustrated as follows [129].

Step 1(*Propose a Lyapunov- Krasovskii functional condidate*): The *Lyapunov Krasovskii* functional that is necessary and sufficient for the stability of LTI systems with delay has a rather complex form, even for the case of constant delays. Let us provide a non exhaustive list of usual terms that are employed in the literature.

• **Complete Lyapunov-Krasovskii functionals**:[130] defined by the following expression:

$$\begin{aligned} V(t,x_t) =& x_t^T(0)P(t)x(t) + 2x_t^T(0)\int_{-\tau}^{0} Q(s)x_t(s)ds \\ &+ \int_{-\tau}^{0}\int_{-\tau}^{0} x_t^T(s)\mathcal{T}(s,\theta)x_t(\theta)dsd\theta) + \int_{-\tau}^{0} x_t^T(s)(S+(h+s)R)x_t(s)ds \end{aligned} \tag{3.16}$$

With:$P = P^T$, $R = R^T$ and $S = S^T$. Q , and $\mathcal{T}$ are real matrices of appropriate dimensions.

• **Delay independent Lyapunov- Krasovskii functionals** This functional is a special case of one described in the last paragraph by setting the parameters Q, $\mathcal{T}$ and R to zero in (3.15) and is given by the following expression :

$$V_0(x_t) = x_t^T(0)P(t)x_t(0) + \int_{-\tau}^{0} Q(s)x_t(s)ds. \tag{3.17}$$

This functional leads to conservative results since a large class of the delay systems may be stable only for some values of the delay, this comes from the fact, when deriving stability conditions from this functional, the resulting conditions does not depend on the value of the delay τ. Hence, if the condition holds, it implies that the system remains stable for any values of the delay.

• **Delay dependent Lyapunov- Krasovskii functionals**: expressed by the following equation:

$$V(x_t, \dot{x}_t) = x_t^T(0)Px_t(0) + \int_{-\tau}^{0} x_t^T(s)S\,x_t(s)ds + \tau \int_{\tau}^{0}\int_{\theta}^{0} \dot{x}_t{}^T(s)R\dot{x}_t(s)dsd\theta \tag{3.18}$$

this expression can be simplified and its equivalent is given by :

$$V(x_t, \dot{x}_t) = x_t^T(0)Px_t(0) + \int_{-\tau}^{0} x_t^T(s)S\,x_t(s)ds + \tau \int_{\tau}^{0} (\tau + s)\dot{x}_t{}^T(s)R\dot{x}_t(s)ds \tag{3.19}$$

where P, S and R are symmetric positive definite matrices.

The newness in the definition of this functional relies on the last terms, which depends on $\dot{x}$, which not meet the requirement of the *Lyapunov Krasovskii* theorem which does not indicate the possibility for a functional to have $\dot{x}_t$ as an argument. Nevertheless, extensions of the *Lyapunov Krasovskii* theorem including this particularity has been investigated and is now admitted (see [35]).

• **Multiple integral functionals:** This class of functionals, provided new possibilities to enrich the functional by many terms [143] and take the following form:

$$V_m(\dot{x}_t(t)) = \tau^2/2 \int_{\tau}^{0}\int_{\theta_1}^{0}\int_{\theta_2}^{0} \dot{x}_t{}^T(s)R_m\dot{x}_t(s)dsd\theta_2 d\theta_1 \tag{3.20}$$

which can be rewritten in its equivalent form as :

$$V_m(\dot{x_t}(t)) = \tau^2/2 \int_{\tau}^{0} (\tau + s)^2\dot{x}_t{}^T(s)R_m\dot{x}_t(s)ds \tag{3.21}$$

From the numerical point of view, an additional quadratic term is required in this functionals, the introduced additional term depends on $\int_{\tau}^{0} x_t(s)ds$ or on $\int_{-\tau}^{0}\int_{\theta}^{0} x_t(s)dsd\theta$ to led to some numerical improvements. Many extensions to more general multiple integral functional have been considered in the literature see [110, 31].

Step2 (*Compute the derivative of the LKFs*): The main idea when computing de derivative of the LKFs is to rewrite this derivative as a quadratic form expressed using all the relevant information on the state function. From equation (3.17)the derivative is given by the following expression :

$$\begin{aligned}\dot{V}(x_t, \dot{x}_t) =& 2x_t^T(0)P(t)x_t(0) + x_t^T(0)S\,x_t(0) - x_t^T(-\tau)S\,x_t(-\tau)\\ &+ \tau^2\dot{x}_t^T(0)R\dot{x}_t(0) - \tau\int_{t-\tau}^{t}\dot{x}^T(s)R\dot{x}(s)ds\end{aligned} \tag{3.22}$$

Corresponding to the “LMIzation” of the expression of $\dot{V}(x_t, \dot{x}_t)$ the relevant information are in this situation composed by $x_t(0)$, $\dot{x}_t(0)$, $x_t(-\tau)$.

Step3 (*Over approximate the integral terms*): It is noted that the analysis of the sign of the derivative $\dot{V}(x_t, \dot{x}_t)$, leads that some terms are common terms in the derivative of *Lyapunov Krasovskii* functionals and they need to be included using over approximation methods, that is to say some common terms must be replaced by other more simple expressions on a quadratic form to be included in the formulation of the derivative of the *Lyapunov Krasovskii* , this procedure is called integral inequalities, using those integral inequalities introduces some conservatism in the analysis and consequently in the resulting stability conditions.

We note that much efforts have been dedicate to the selection and construction of a good *Lyapunov Krasovskii* functional, in most of the case, the main procedure is to follow the paradigm “try and check”.

3.3 Useful lemma and integral inequalities

The objective of this section is to provide a powerful tools that enable the “LMIzation” process, by transforming the expression of the derivative of *Lyapunov Krasovskii* $\dot{V}(x_t, \dot{x}_t)$, in a more appropriate form to obtain the stability conditions under an LMI formulation, this step is crucial and must be studied carefully. In the sequel, we will consider the following integral inequalities which will be used in the proof of our main results in the next chapter.

3.3.1 Jensen's inequality

Finding integral inequalities for quadratic functions plays a key role in the field of stability analysis, in such circumstances, the Jensen inequality has become a powerful mathematical tool for stability analysis of networked control systems. In the following we will consider the problem of providing integral inequalities which deliver a lower bound of an integral quadratic term of the form: $\int_{\tau}^{0} x_t^T(u)Rx_t(u)du$ or $\int_{\tau}^{0} \dot{x}_t^T(u)R\dot{x}_t(u)du$ where τ is a positive scalar.

This approach requires fewer decision variables than other existing approaches having a fine performance behavior. Naturally, Jensen's inequality is likely to entail some inherent conservatism this inequality is expressed bey the following lemma.

Lemma 1 *(Jensen's lemma) For a given $n \times n$ matrices $R \geq 0$ and for any piece wise continuous function*
$x \in [-\tau, 0] \rightarrow \mathbb{R}^n$, the following inequality holds [188, 131]:

$$\int_{-\tau}^{0} x^T(u)Rx(u)du \geq (1/\tau)\Omega_0^T(x)R\Omega_0(x), \quad \Omega_0(x) = \int_{-\tau}^{0} x(u)du. \tag{3.23}$$

Many other inequality may be derived from the *Jensen's integral*. Let us focus on the following inequality adapted to our purpose.

Lemma 2 *knonw as Wirtinger based integral inequalty For any positive matrices Z_1 and Z_2, real scalars a and b satisfying $a < b$, the following inequalities hold for all continuously differentiable function $x : [a, b] \rightarrow \mathbb{R}^n$ [114]*

$$-(b-a)\int_{a}^{b} \dot{x}^T(s)Z_1\dot{x}(s)ds \leq \varpi^T(t)\Xi_1\varpi(t) \tag{3.24}$$

$$-\int_{a}^{b}\int_{t+a}^{t+\theta} \dot{x}^T(s)Z_2\dot{x}(s)ds \leq \varpi^T(t)\Xi_2\varpi(t) \tag{3.25}$$

where:

$$\begin{aligned}\Xi_1 = &-(\tilde{e}_1 - \tilde{e}_2)Z_1(\tilde{e}_1 - \tilde{e}_2)^T - 3(\tilde{e}_1 + \tilde{e}_2 - 2\tilde{e}_3)Z_1(\tilde{e}_1 + \tilde{e}_2 - 2\tilde{e}_3)^T \\ &- 5(\tilde{e}_1 - \tilde{e}_2 + 6\tilde{e}_3 - 12\tilde{e}_4)Z_1(\tilde{e}_1 - \tilde{e}_2 + 6\tilde{e}_3 - 12\tilde{e}_4)^T,\end{aligned}$$

$$\Xi_2 = -2(\tilde{e}_1 - \tilde{e}_2)Z_2(\tilde{e}_1 - \tilde{e}_2)^T - 4(\tilde{e}_1 - 4\tilde{e}_2 + 6\tilde{e}_3)Z_2(\tilde{e}_1 - 4\tilde{e}_2 + 6\tilde{e}_3)^T$$
$$- 6(\tilde{e}_1 - 9\tilde{e}_2 + 36\tilde{e}_3 - 60\tilde{e}_4)Z_2(\tilde{e}_1 - 9\tilde{e}_2 + 36\tilde{e}_3 - 60\tilde{e}_4)^T$$

and

$$\varpi(t) = \begin{bmatrix} x^T(b) & x^T(a) & \frac{1}{(b-a)}\int_a^b x^T(s)ds & \frac{1}{(b-a)^2}\int_a^b\int_\lambda^b x^T(s)dsd\lambda \end{bmatrix}^T$$

We denote that the block entry matrices $e_j = \begin{bmatrix} 0_{n\times(j-1)n} & I_{n\times n} & 0_{n\times(12n-j)n} \end{bmatrix}^T \in R^{12n\times n}$ *for example* $e_4 = \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ *and,* $\forall k \in \mathcal{I}_4$, $\tilde{e}_k = \begin{bmatrix} 0_{n\times(k-1)n} & I_{n\times n} & 0_{n\times(4n-k)n} \end{bmatrix}^T \in R^{4n\times n}$, *for example* $\tilde{e}_2 = \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix}^T$.

This lemma is used to estimate the integral of the derivative function with the help of the integral of the function, it is shown that this resulting inequality encompasses the Jensen one and also leads to tractable LMI conditions, the resulting inequality depends not only on the state $x(t)$ and the delayed or sampled state but also on the integral of the state over the delay or sampling interval, it will be used to provide much tighter bounding for cross terms and improve the conservatism .

Lemma 3 *For a given matrix* $R = R^T > 0$ *and any differentiable function* $x \in [a, b] \to \mathbb{R}^n$, *there exist matrices* $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3 \in \mathbb{R}^{4n\times n}$ *such that the following inequality holds [169]:*

$$-\int_a^b \int_\lambda^b \dot{x}^T(s)R\dot{x}(s)dsd\lambda \le \varpi^T(t)\Xi\varpi(t)$$

where:

$$\Xi = \frac{(b-a)^2}{2}\mathcal{M}_1R^{-1}\mathcal{M}_1^T + \frac{(b-a)^2}{4}\mathcal{M}_2R^{-1}\mathcal{M}_2^T + \frac{(b-a)^2}{6}\mathcal{M}_3R^{-1}\mathcal{M}_3^T + (b-a)\mathcal{H}_e(\mathcal{M}_1\Pi_1 + \mathcal{M}_2\Pi_2 + \mathcal{M}_3\Pi_3),$$

$$\Pi_1 = \tilde{e}_1 - \tilde{e}_2, \quad \Pi_2 = \tilde{e}_1 + 2\tilde{e}_2 - 3\tilde{e}_3, \quad \Pi_3 = \tilde{e}_1 - 3\tilde{e}_2 + 24\tilde{e}_3 - 60\tilde{e}_4,$$

and:

$$\varpi(t) = \begin{bmatrix} x^T(b) & \frac{1}{(b-a)}\int_a^b x^T(s)ds & \frac{1}{(b-a)^2}\int_a^b\int_\lambda^b x^T(s)dsd\lambda & \frac{1}{(b-a)^3}\int_a^b\int_u^b\int_\lambda^b x^T(s)dsdud\lambda \end{bmatrix}^T$$

Also, $\forall k \in \mathcal{I}_4$, $\tilde{e}_k = \begin{bmatrix} 0_{n\times(k-1)n} & I_{n\times n} & 0_{n\times(4n-k)n} \end{bmatrix}^T \in R^{4n\times n}$

This lemma is used as relaxation scheme to reduce the conservatism duo to the double integrals structure of the obatained **LMI**, note that among the relaxation lemmas, this lemma constitute a good compromise between complexity and computational burden.

Lemma 4 *Known as Finsler's lemma. Let* $\xi \in \mathbb{R}^n$, $\mathcal{G} \in R^{m\times n}$ *and* $Q = Q^T \in \mathbb{R}^{n\times n}$ *such that rank*$(\mathcal{G}) < n$ *and,* $\forall \xi \neq 0$, $\mathcal{G}\xi = 0$. *The following statements are equivalent [138] .*

$$\xi^T Q\xi < 0, \tag{3.26}$$

$$\exists \mathcal{R} \in \mathbb{R}^{n\times m}, Q + \mathcal{H}_e(\mathcal{R}\mathcal{G}) < 0. \tag{3.27}$$

This lemma will be used to relax the proposed **LMI** conditions by adding slack decision variables and decoupling the system's matrices from the Lyapuonv ones. Then, to cope with bounded uncertainties, the following usual lemma will be employed .

Lemma 5 *(Peterson lemma) Let* H, E *and* $\delta(t)$ *satisfying* $\delta^T(t)\delta(t) \leq I$ *be real matrices of appropriate dimensions. The following inequality holds* $\forall \lambda > 0$ *[168]*

$$H\delta(t)E^T + E\delta^T(t)H^T \leq \lambda EE^T + \lambda^{-1}HH^T \tag{3.28}$$

Lemma 6 *(lemma of separation) Let* Φ_1, Φ_2 *and* Ω *be matrices of appropriate dimensions and* $0 < \underline{\tau} \leq \tau(t) \leq \bar{\tau}$, *then the following inequality [149]:*

$$(\tau(t) - \underline{\tau})\Phi_1 + (\bar{\tau} - \tau(t))\Phi_2 + \Omega \leq 0 \tag{3.29}$$

holds if and only if :

$$\begin{cases} (\bar{\tau} - \underline{\tau})\Phi_1 + \Omega \leq 0 \text{ and} \\ (\bar{\tau} - \underline{\tau})\Phi_2 + \Omega \leq 0 \end{cases}$$

Lemma 7 *Newton Leibniz lemma For any constant matrix* $Q \in \mathbb{R}^{n\times n}$, $Q > 0$, *scalars* $\tau_1 \leq \tau(t) \leq \tau_3$ *and vector function* $\dot{x} : [-\tau_3, -\tau_1] \to \mathbb{R}^n$ *such that the following integration is well defined, then it holds that [116]:*

$$\begin{aligned} &-(\tau_3 - \tau_1)\int_{t-\tau_3}^{t-\tau_1} \dot{x}(s)^T Q\dot{x}(s)ds \leq \\ &-[x(t-\tau_1) - x(t-\tau(t))]^T Q[x(t-\tau_1) - x(t-\tau(t))] \\ &-[x(t-\tau(t)) - x(t-\tau_3)]^T Q[x(t-\tau(t)) - x(t-\tau_3)]\,. \end{aligned} \tag{3.30}$$

3.3.2 $\mathcal{H}_\infty$ robust control

Fig.3.4 shows a block diagram of a standard $\mathcal{H}_\infty$ control problem. G is the generalized plant, which is given in the problem statement, and K is the controller which needs to be designed. Here, we assume that the system and controller are finite dimensional, linear, and time invariant. The external input w, the control input u, the controlled output z, and the measured output y are all vector signals.

Assume $G(s)$ and $K(s)$ in Fig.3.4 are both proper real rational transform function matrices that describe a linear time invariant system. From Fig.3.4 we have

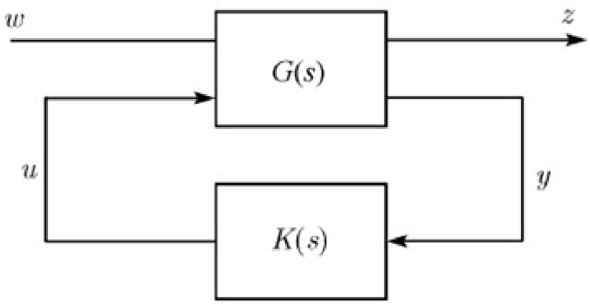

Figure 3.4: Block diagram of standard H_∞ control problem.

$$\begin{pmatrix} z \\ y \end{pmatrix} = G \begin{pmatrix} w \\ u \end{pmatrix} \tag{3.31}$$

The state space realization of $G(s)$ is given by :

$$\begin{cases} \dot{x} = Ax + B_1 W + B_2 u, \\ z = C_1 x + D_{11} W + D_{12} u \\ y = C_2 x + D_{21} W + D_{22} u \end{cases} \tag{3.32}$$

so

$$G(s) = \begin{bmatrix} A_1 & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{3.33}$$

where: $x \in \mathbb{R}^n$is the state vector. also, assume $W \in \mathbb{R}^{m_1}$,$u \in \mathbb{R}^{m_2}$,$z \in \mathbb{R}^{p_1}$ and $y \in \mathbb{R}^{p_2}$ decomposing $G(s)$ into

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{(}s) & G_{22}(s) \end{bmatrix} \tag{3.34}$$

Comparing 3.10 and 3.9 we obtain:

$$G_{ij}(s) = C_i(sI - A)^{-1}B_j + D_{ij}, \qquad i, j = 1, 2. \tag{3.35}$$

The closed loop transform function matrix from w to z is

$$T_{zw}(s) = G_{11}(s) + G_{12}(s)K(s)(I - G_{22}(s))K(s)^{-1}G_{21}(s) = F_l(G, K), \tag{3.36}$$

with $F_l(G, K)$ is called the lower linear fractional transformation (LFT) of $G(s)$ and $K(s)$. The H_∞optimal control problem for the closed-loop control system in Fig. 2.6 involves:
1-finding a proper real rational controller $K(s)$ that stabilizes the system internally
2-minimizing the $\mathcal{H}_\infty$ norm of $Tzw(s)$, that is finding:

$$min_{Kstab}\|F_l(G, K)\|_\infty. \tag{3.37}$$

The $\mathcal{H}_\infty$ sub optimal control problem for the closed loop control system in Fig.2.6 involves:
1)- Finding all proper real-rational controllers, $K(s)$, that stabilize the closed loop system internally
2)- Making the $\mathcal{H}_\infty$ norm of $T_{zw}(s)$ less than a given constant $\gamma > 0$ that is to say:

$$F_l\|(G, K)\|_\infty \leq \gamma \tag{3.38}$$

3.4 Conclusion

This chapter presente a mathematical backgound, we focused on the systems stability in the sense of Lyapunov, where we presented different related definitions . Next, we presented two approaches frequently used for the stability analysis of NCSs, namely the imperfections-independent using *Lyapunov Razumikhin* method and the imperfections dependent approach using*Lyapunov Krasovskii* method, and as it is well known that when using Lyapunov approach for stability analysis a crucial step, is to find the time derivative of the Lyapunov functional, to drive the stability conditions in term of LMIs, by bounding the time derivative including integral terms, to achieve this goal a set of integral inequality is proposed in detail, such as Jenes's integral, Wirtinger lemma, free waiting matrices, and Finsler's lemma, those lemma will be investigated in the next chapter. Finally, we recalled the standard problem of $\mathcal{H}_\infty$ to deal with different disturbances and uncertainties attaining NCSs, and to get robust stability and control.

Chapter 4

Robust stability and stabilization of networked control system with induced time delay and packets dropouts

4.1 Introduction

Hence, in this chapter, the goal is to propose new LMI based conditions for robust H_∞ stability analysis and controller design for a class of uncertain and disturbed NCS under network induced delays and packet dropouts effects. To provide further relaxed LMIs stability conditions, combining several ideas inspired from recent literature, the main contributions of the present chapter are summarized as follows:

- The input time-delay interval is divided into two sub-interval, that includes both network-induced delay and packet dropout, where packet dropout is considered as one induced sample period delay (inspired from [81]).
- The application of recent bounding techniques improving previous Wirtinger's based results (inspired from [169]).
- Introducing free weighing variable by applying the Finsler's lemma [138], see e.g. ([10, 12]).
- The extension to the robust H_∞ stability analysis to cope with uncertainties and external disturbances.

4.2 System description and preliminaries

Let us consider the class of uncertain and disturbed systems described by the following continuous-time uncertain and disturbed state space models

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + B_w\omega(t) \\ y(t) = Cx(t) + Du(t) + D_w\omega(t) \end{cases} \tag{4.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^q$ is the output vector, $\omega(t) \in \mathbb{R}^p$ is a vector of external disturbances belonging to $L_2[0, \infty)$, A, B, C, D, B_w and D_w are real constant matrices with appropriate dimensions, $\Delta A(t)$ and $\Delta B(t)$ are Lebesgue measurable uncertain matrices such that [168]:

$$\Delta A(t) = H\delta(t)E_a \text{ and } \Delta B(t) = H\delta(t)E_b \tag{4.2}$$

where H, E_a and E_b are known constant real matrices with appropriate dimensions and $\delta(t)$ is an unknown real time-varying matrix satisfying $\delta^T(t)\delta(t) \leq I$.

Remark 4 *Model (1) is a standard state space representation of uncertain and disturbed linear dynamical system, where parametric uncertainties ($\Delta A(t)$ and $\Delta B(t)$), as well as unstructured ones ($\omega(t)$), are considered. With the growing complexities of systems to be controlled, taking into account parametric uncertainties for the design of a robust controller is often necessary for more practical applicability, especially to cope with modelling approximations, simplifications or imprecise parameters' identification. Moreover, in most of real applications, some uncontrolled exogenous inputs may affect the systems dynamics as external disturbances (unstructured uncertainties). Once again, it is important to take them into account in the controller synthesis, to make robust the further designed closed-loop dynamics regarding to such external disturbances.*

In this thesis, the goal is to investigate the control of uncertain and disturbed systems (4.1), according to the networked control scheme presented in Figure 4.1. In this context, the following assumptions are considered.

Assumption 4.1 *The sensors are clock driven, the controller and actuators are event driven.*

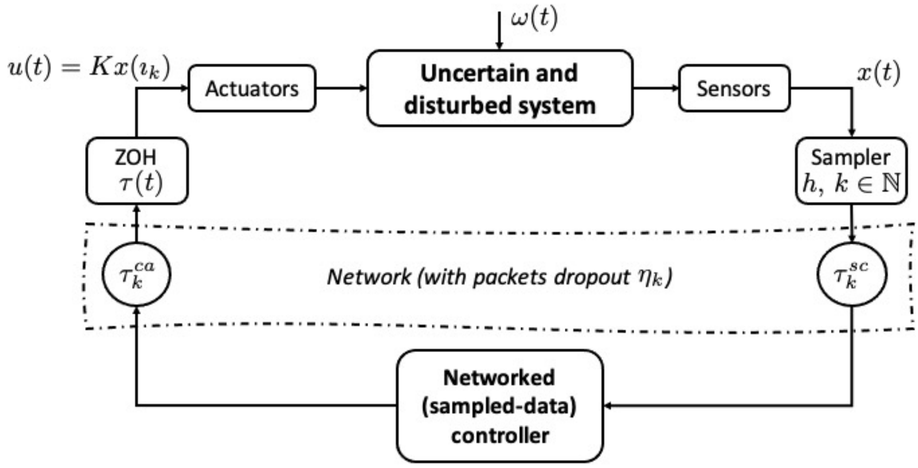

Figure 4.1: Network induced delay

Assumption 4.2 *All the state variables are available from measurements and transmitted to the controller part by single-packets.*

In the sequel, the sampling period $h > 0$ is assumed to be constant. The sampling instants of data transmitted from the sensors to the ZOH are denoted by the set $[\imath_1, \imath_2, \ldots, \imath_k]$, where k are positive integers. Some data may be lost because of the presence of consecutive packet dropouts during network transmissions.
Hence, we denote the number of consecutive packet dropouts by:

$$\eta_k = \frac{\imath_{(k+1)} - \imath_k}{h} - 1$$

In NCS control plants, two kinds of delays can be considered: the sensors-to-controller delay τ_k^{sc} and controller-to-actuators delay τ_k^{ca}. These two delays can be lumped together as $\tau_k = \tau_k^{sc} + \tau_k^{ca}$. Therefore, the updating instants of the ZOH can be denoted by:
$[\imath_1 + \tau_1, \imath_2 + \tau_2, \ldots, \imath_k + \tau_k]$, i.e, every control signal is held by the ZOH over the intervals $[\imath_k + \tau_k, \imath_{k+1} + \tau_{k+1})$.

Remark 5 *Without loss of generality, the system* (4.1) *is used to give the mathematical model of the NCS for a single packet transmission. Nevertheless, in the case of a multi-packet transmission, system* (4.1) *is always exploitable but with buffers before both the controller and the actuators nodes.*

Assumption 4.3 *The number of consecutive packet dropouts ($\eta_k \in \mathbb{N}$) and the network-induced delay (τ_k) are bounded and satisfy respectively $\eta_k \in [0, \bar{\eta}]$, where $\bar{\eta} = \sup_{\{k\in\mathbb{N}\}}(\eta_k)$ and $\tau_k \in [\tau_{min}, \tau_{max}]$, where $\tau_{min} = \inf_{\{k\in\mathbb{N}\}}(\tau_k)$ and $\tau_{max} = \sup_{\{k\in\mathbb{N}\}}(\tau_k)$.*

From assumption 4.3, maximizing $\bar{\eta}$ provides the maximal number of packet dropouts and $MAUB(\tau_{max})$, which can be obtained by the maximization of τ_{max}, with the LMI-based NCS stability analysis and design procedures proposed in the sequel.
To stabilize (4.1) over the network, let us consider the following sampled state feedback control law:

$$u(t) = Kx(\iota_k),\ t \in [\iota_k + \tau_k, \iota_{k+1} + \tau_{k+1}) \tag{4.3}$$

where K is the controller's gain matrix; ι_k are sequence numbers that represent the most recent data available from the sensors to the controller, which are maintained by the ZOH until new data get to actuators.
Let us define:

$$\tau(t) = t - \iota_k,\ \forall t \in [\iota_k + \tau_k, \iota_{k+1} + \tau_{k+1}), \tag{4.4}$$

the control law (4.3) can be rewritten as:

$$u(t) = Kx(t - \tau(t)),\ \forall t \in [\iota_k + \tau_k, \iota_{k+1} + \tau_{k+1}) \tag{4.5}$$

where $\tau(t)$ is such that:

$$\begin{cases} 0 < \underline{\tau} \leq \tau(t) \leq \bar{\tau},\ \forall t \in [\iota_k + \tau_k, \iota_{k+1} + \tau_{k+1}) \\ \dot{\tau}(t) = 1,\ \forall t \neq \iota_k + \tau_k \end{cases} \tag{4.6}$$

with (From assumption(4.2) and assumption (4.3))

$$\begin{cases} \underline{\tau} = \inf_{k\in\mathbb{N}}(\tau_k) = \tau_{min} \\ \bar{\tau} = \sup_{k\in\mathbb{N}}(\tau_{k+1}) = \tau_{max} + (\bar{\eta} + 1)h. \end{cases} \tag{4.7}$$

Substituting (4.5) into (4.1), we can express the closed-loop dynamics as the following uncertain and disturbed system with input time-varying delay:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{B}Kx(t - \tau(t)) + B_w\omega(t), \\ y(t) = Cx(t) + DKx(t - \tau(t)) + D_w\omega(t), \\ x(t) = \phi(t), \qquad t \in [-\bar{\tau}, -\underline{\tau}]. \end{cases} \tag{4.8}$$

where $\bar{A} = A + \Delta A(t)$ and $\bar{B} = B + \Delta B(t)$.

The purpose of this work is first to propose relaxed LMI-based stability conditions for the closed-loop dynamics (4.8), i.e., when considering that the gain K is known. Then, a convexification procedure will be proposed for the design of the controller gain K, i.e. when it is not priory known. Additionally, these conditions will be provided such that, for any non-zero $\omega(t) \in L_2[0, \infty)$, the closed-loop NCS (4.8) guarantee the H_∞ criterion $\| y(t) \|_2 < \gamma \| \omega(t) \|_2$, that is to say:

$$\int_0^\infty \left(y^T(s) y(s) - \gamma^2 \omega^T(s) \omega(s) \right) ds < 0 \tag{4.9}$$

where $\gamma > 0$ is the disturbance attenuation level to be minimized.

To conclude this preliminaries, let us recall the following lemmas, which are useful in the proofs of the theorems proposed in the next sections.

4.3 Main results

In this section, we will first focus on the robust H_∞ stability analysis of the closed-loop NCS (4.8) (assuming that the controller gain K is known). Then, the stability conditions will be convexified to allow the design of the controller gain K. In the sequel, to lighten mathematical expressions, we define $\tau_m = \tau_{max} + h$, $\hat{\tau} = (\tau_m - \underline{\tau})$ and $(\bar{\tau} - \tau_m) = \bar{\eta} h$.

4.3.1 Robust H_∞ stability analysis

The proposed LMI-based robust H_∞ stability conditions of the NCS (4.8), assuming that the controller gain K is known, are summarized by the following theorem.

Theorem 4.1 *For given positive scalars $\underline{\tau}$, $\bar{\tau}$, $\bar{\eta}$, γ and λ such that* $0 < \underline{\tau} \le \tau(t) \le \bar{\tau}$, $\dot{\tau}(t) = 1$ *and known feedback controller gain K, the considered closed-loop NCS* (4.8) *subject to network induced delays, packets dropouts, uncertainties and external disturbances, is robustly stable if there exist positive definite symmetric matrices $P \in R^{4n \times 4n}$, S_1, S_2, W_1, W_2, Q_1, Q_2, Q_3, Z_1 and $Z_2 \in R^{n \times n}$, and slack decision matrices $\mathcal{M}_1$, $\mathcal{M}_2$,, $\mathcal{M}_3$, $\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{N}_3$ $\in R^{4n \times n}$, $\mathcal{L}_1$, $\mathcal{L}_2 \in R^{12n \times n}$ and $\mathcal{T} \in R^{12n \times n}$, such that the following LMIs hold for both $q = 1$*

and q = 2:

$$\Upsilon^q = \begin{bmatrix} \sum_{i=1}^{7}\Phi_i^q + \mathcal{H}_e(\mathcal{T}\mathcal{G}) + \lambda\mathcal{E}^T\mathcal{E} + \mathcal{D}^T\mathcal{D} & \mathcal{T}H & \Gamma_q^2 \\ * & -\lambda I & 0 \\ * & * & \Gamma_q^3 \end{bmatrix} < 0 \tag{4.10}$$

with:

$$\Gamma_1^2 = \begin{bmatrix} \frac{\underline{\tau}}{\sqrt{2}}M_1 & \frac{\underline{\tau}}{2}M_2 & \frac{\underline{\tau}}{\sqrt{6}}M_3 & \frac{\hat{\tau}}{\sqrt{2}}N_1 & \frac{\hat{\tau}}{2}N_2 & \frac{\hat{\tau}}{\sqrt{6}}N_3 & \hat{\tau}\mathcal{L}^q \end{bmatrix},$$

$$\Gamma_2^2 = \begin{bmatrix} \frac{\tau_m}{\sqrt{2}}M_1 & \frac{\tau_m}{2}M_2 & \frac{\tau_m}{\sqrt{6}}M_3 & \frac{\bar{\eta}h}{\sqrt{2}}N_1 & \frac{\bar{\eta}h}{2}N_2 & \frac{\bar{\eta}h}{\sqrt{6}}N_3 & \bar{\eta}h\mathcal{L}^q \end{bmatrix},$$

$$\Gamma_1^3 = diag\{-S_1, -S_1, -S_1, -W_1, -W_1, -W_1, -\hat{\tau}Q_3\},$$

$$\Gamma_2^3 = diag\{-S_1, -S_1, -S_1, -W_1, -W_1, -W_1, -\bar{\eta}hQ_3\},$$

$$\mathcal{G} = \begin{bmatrix} A & 0 & BK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & B_w \end{bmatrix}, \tag{4.11}$$

$$\mathcal{E} = \begin{bmatrix} E_a & 0 & E_bK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{4.12}$$

$$\mathcal{D} = \begin{bmatrix} C & 0 & DK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_w \end{bmatrix}, \tag{4.13}$$

$$\Phi_1^q = \mathcal{H}_e(\Pi_1^{qT}P\Pi_2), \tag{4.14}$$

$$\Phi_2^1 = \Phi_2^2 = e_1Z_1e_1^T - e_2Z_1e_2^T - e_4Z_2e_4^T, \tag{4.15}$$

$$\Phi_3^1 = \Pi_3 + e_{11}(\underline{\tau}^2Q_1 + \hat{\tau}^2Q_2 + \hat{\tau}Q_3)e_{11}^T, \tag{4.16}$$

$$\Phi_3^2 = \Pi_3 + e_{11}(\tau_m^2Q_1 + (\bar{\eta}h)^2Q_2 + \bar{\eta}hQ_3)e_{11}^T, \tag{4.17}$$

$$\begin{aligned}\Phi_4^1 =& \Pi_4 + e_{11}\frac{\underline{\tau}^2}{2}(S_1 + S_2)e_{11}^T + \underline{\tau}\mathcal{H}_e\Big(\mathcal{M}_1\begin{bmatrix} e_1 & -e_5 & 0 & 0 \end{bmatrix}\Big) \\ &+ \underline{\tau}\mathcal{H}_e\Big(\mathcal{M}_2\begin{bmatrix} e_1 & 2e_5 & -3e_7 & 0 \end{bmatrix}\Big) \\ &+ \underline{\tau}\mathcal{H}_e\Big(\mathcal{M}_3\begin{bmatrix} e_1 & -3e_5 & 24e_7 & -60e_9 \end{bmatrix}\Big),\end{aligned} \tag{4.18}$$

$$\begin{aligned}\Phi_4^2 =& \Pi_4 + e_{11}\frac{\tau_m^2}{2}(S_1 + S_2)e_{11}^T + \tau_m\mathcal{H}_e\Big(\mathcal{M}_1\begin{bmatrix} e_1 & -e_5 & 0 & 0 \end{bmatrix}\Big) \\ &+ \tau_m\mathcal{H}_e\Big(\mathcal{M}_2\begin{bmatrix} e_1 & 2e_5 & -3e_7 & 0 \end{bmatrix}\Big) \\ &+ \tau_m\mathcal{H}_e\Big(\mathcal{M}_3\begin{bmatrix} e_1 & -3e_5 & 24e_7 & -60e_9 \end{bmatrix}\Big),\end{aligned} \tag{4.19}$$

$$\begin{aligned}\Phi_5^1 =& \Pi_5 + e_{11}\frac{\hat{\tau}^2}{2}(W_1+W_2)e_{11}^T + \hat{\tau}\mathcal{H}_e\Big(\mathcal{N}_1\begin{bmatrix} e_2 & -e_6 & 0 & 0\end{bmatrix}\Big)\\ &+ \hat{\tau}\mathcal{H}_e\Big(\mathcal{N}_2\begin{bmatrix} e_2 & 2e_6 & -3e_8 & 0\end{bmatrix}\Big)\\ &+ \hat{\tau}\mathcal{H}_e\Big(\mathcal{N}_3\begin{bmatrix} e_2 & -3e_6 & 24e_8 & -60e_{10}\end{bmatrix}\Big),\end{aligned} \tag{4.20}$$

$$\begin{aligned}\Phi_5^2 =& \Pi_5 + e_{11}\frac{(\bar{\eta}h)^2}{2}(W_1+W_2)e_{11}^T + (\bar{\eta}h)\mathcal{H}_e\Big(\mathcal{N}_1\begin{bmatrix} e_2 & -e_6 & 0 & 0\end{bmatrix}\Big)\\ &+ (\bar{\eta}h)\mathcal{H}_e\Big(\mathcal{N}_2\begin{bmatrix} e_2 & 2e_6 & -3e_8 & 0\end{bmatrix}\Big)\\ &+ (\bar{\eta}h)\mathcal{H}_e\Big(\mathcal{N}_3\begin{bmatrix} e_2 & -3e_6 & 24e_8 & -60e_{10}\end{bmatrix}\Big),\end{aligned} \tag{4.21}$$

$$\Phi_6^1 = \Phi_6^2 = \mathcal{H}_e\Big(\mathcal{L}_1(e_3^T - e_4^T) + \mathcal{L}_2(e_2^T - e_3^T)\Big), \tag{4.22}$$

$$\Phi_7^1 = \Phi_7^2 = -\gamma^2 e_{12}e_{12}^T, \tag{4.23}$$

$$\Pi_1^1 = \begin{bmatrix} e_1 & \underline{\tau}e_5 & \underline{\tau}e_7 & 2\underline{\tau}e_9\end{bmatrix}^T,\ \Pi_2^1 = \begin{bmatrix} e_1 & \tau_m e_5 & \tau_m e_7 & 2\tau_m e_9\end{bmatrix}^T$$

$$\Pi_2 = \begin{bmatrix} e_{11} & e_1 - e_2 & e_1 - e_5 & e_1 - 2e_7\end{bmatrix}^T,$$

$$\begin{aligned}\Pi_3 =& -(e_1-e_2)Q_1(e_1-e_2)^T - (e_2-e_4)Q_2(e_2-e_4)^T\\ &- 3(e_1+e_2-2e_5)Q_1(e_1+e_2-2e_5)^T - 3(e_2+e_4-2e_6)Q_2(e_2+e_4-2e_6)^T\\ &- 5(e_1-e_2+6e_5-6e_7)Q_1(e_1-e_2+6e_5-6e_7)^T\\ &- 5(e_2-e_4+6e_6-6e_8)Q_1(e_2-e_4+6e_6-6e_8)^T\end{aligned},$$

$$\begin{aligned}\Pi_4 =& -2(e_2-e_5)S_2(e_2-e_5)^T - 4(e_2-4e_5+6e_7)S_2(e_2-4e_5+6e_7)^T\\ &- 6(e_2-9e_5+36e_7-60e_9)S_2(e_2-9e_5+36e_7-60e_9)^T\end{aligned},$$

$$\begin{aligned}\Pi_5 =& -2(e_4-e_6)W_2(e_4-e_6)^T - 4(e_4-4e_6+6e_8)W_2(e_4-4e_6+6e_8)^T\\ &-6(e_4-9e_6+36e_8-60e_{10})W_2(e_4-9e_6+36e_8-60e_{10})^T.\end{aligned}$$

Proof 1 *Inspired from the delay partitioning strategy proposed in [81], let us divide the delay interval into two sub-intervals $\underline{\tau} \leq \tau(t) \leq \tau_m$ and $\tau_m < \tau(t) \leq \bar{\tau}$. For the first sub-interval, i.e., $\forall\tau(t) \in [\underline{\tau}, \tau_m]$, we propose the following LKF candidate:*

$$V_1(t) = V_1^1(t) + V_1^2(t) + V_1^3(t) + V_1^4(t) + V_1^5(t) \tag{4.24}$$

where:

$$V_1^1(t) = \theta_1^T(t)P\theta_1(t) \tag{4.25}$$

$$V_1^2(t) = \int_{t-\underline{\tau}}^{t} x^T(s)Z_1x(s)ds + \int_{t-\tau_m}^{t-\tau(t)} x^T(s)Z_2x(s)ds \tag{4.26}$$

$$V_1^3(t) = \underline{\tau}\int_{-\underline{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_1\dot{x}(s)dsd\theta + \int_{-\tau_m}^{-\underline{\tau}}\int_{t+\theta}^{t} \dot{x}^T(s)(Q_3+\hat{\tau}Q_2)\dot{x}(s)dsd\theta \tag{4.27}$$

$$V_1^4(t) = \int_{-\underline{\tau}}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_1\dot{x}(s)dsd\theta d\lambda + \int_{-\underline{\tau}}^{0}\int_{-\underline{\tau}}^{\lambda}\int_{t+\theta}^{t} \dot{x}^T(s)S_2\dot{x}(s)dsd\theta d\lambda \tag{4.28}$$

$$V_1^5(t) = \int_{-\tau_m}^{-\underline{\tau}}\int_{\lambda}^{-\underline{\tau}}\int_{t+\theta}^{t} \dot{x}^T(s)W_1\dot{x}(s)dsd\theta d\lambda + \int_{-\tau_m}^{-\underline{\tau}}\int_{-\tau_m}^{\lambda}\int_{t+\theta}^{t} \dot{x}^T(s)W_2\dot{x}(s)dsd\theta d\lambda \tag{4.29}$$

with:

$$\theta_1(t) = \begin{bmatrix} x^T(t) & \int_{t-\underline{\tau}}^{t} x^T(s)ds & \frac{1}{\underline{\tau}}\int_{-\underline{\tau}}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta & \frac{2}{\underline{\tau}^2}\int_{-\underline{\tau}}^{0}\int_{\lambda}^{t}\int_{t+\beta}^{t} x^T(s)dsd\beta d\lambda \end{bmatrix}^T.$$

The LKF candidate (4.24) *is positive if* P, Q_1, Q_2, Q_3, W_1, W_2, S_1, S_2, Z_1 *and* Z_2 *are all positive definite matrices. Moreover, the NCS model* (4.8) *with network-induced time-varying delay* (4.4) *is asymptotically stable if:*

$$\dot{V}_1(t) = \dot{V}_1^1(t) + \dot{V}_1^2(t) + \dot{V}_1^3(t) + \dot{V}_1^4(t)) + \dot{V}_1^5(t) < 0 \tag{4.30}$$

Let us first focus on $\dot{V}_1^1(t)$*, from (4.25) we have:*

$$\dot{V}_1^2(t) = \mathcal{H}_e(\theta^T(t)P\dot{\theta}(t)) = \zeta_1^T(t)\Phi_1^1\zeta_1(t) \tag{4.31}$$

with Φ_1^1 *given in* (4.14) *and:*

$$\zeta_1(t) = \begin{bmatrix} \eta_1^{1T}(t) & \eta_1^{2T}(t) & \eta_1^{3T}(t) & \eta_1^{4T}(t) & \dot{x}^T(t) & \omega^T(t) \end{bmatrix}^T,$$

$$\eta_1^1(t) = \begin{bmatrix} x^T(t) & x^T(t-\underline{\tau}) & x^T(t-\tau(t)) & x^T(t-\tau_m) \end{bmatrix}^T,$$

$$\eta_1^2(t) = \begin{bmatrix} \frac{1}{\underline{\tau}}\int_{t-\underline{\tau}}^{t} x^T(s)ds & \frac{1}{\hat{\tau}}\int_{t-\tau_m}^{t-\underline{\tau}} x^T(s)ds \end{bmatrix}^T,$$

$$\eta_1^3(t) = \begin{bmatrix} \frac{1}{\underline{\tau}^2}\int_{-\underline{\tau}}^{0}\int_{t+\theta}^{t} x^T(s)dsd\theta & \frac{1}{\hat{\tau}^2}\int_{-\tau_m}^{-\underline{\tau}}\int_{t+\theta}^{t} x^T(s)dsd\theta \end{bmatrix}^T,$$

$$\eta_1^4(t) = \begin{bmatrix} \frac{1}{\underline{\tau}^3}\int_{-\underline{\tau}}^{0}\int_{-\lambda}^{0}\int_{t+\beta}^{t} x^T(s)dsd\theta d\lambda & \frac{1}{\hat{\tau}^3}\int_{-\tau_m}^{-\underline{\tau}}\int_{-\lambda}^{-\underline{\tau}}\int_{t+\theta}^{t} x^T(s)dsd\theta d\lambda \end{bmatrix}^T.$$

Now, let us focus on $\dot{V}_1^2(t)$*, from* (4.26) *we have:*

$$\begin{aligned}\dot{V}_1^2(t) =& x^T(t)Z_1x(t) - x^T\left(t-\underline{\tau}\right)Z_1x\left(t-\underline{\tau}\right)\\&+(1-\dot{\tau}(t))x^T\left(t-\tau(t)\right)Z_2x\left(t-\tau(t)\right) - x^T\left(t-\tau_m\right)Z_2x\left(t-\tau_m\right)\\=&\zeta_1^T(t)\Phi_2^1\zeta_1(t)\end{aligned} \tag{4.32}$$

with Φ_2^1 *given in* (4.15).

Then, let us focus on $\dot{V}_1^3(t)$*, we have:*

$$\begin{aligned}\dot{V}_1^3(t) =& e_{11}^T\left(\underline{\tau}^2Q_1 + (\tau_m-\underline{\tau})^2Q_2 + (\tau_m-\underline{\tau})Q_3\right)e_{11}\\&-\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds - \int_{t-\tau_m}^{t-\underline{\tau}}\dot{x}^T(s)(Q_3+\hat{\tau}Q_2)\dot{x}(s)ds\end{aligned} \tag{4.33}$$

Applying lemma 2, we have:

$$\begin{aligned}&-\underline{\tau}\int_{t-\underline{\tau}}^{t}\dot{x}^T(s)Q_1\dot{x}(s)ds\\&\leq -\left(e_1-e_2\right)Q_1\left(e_1-e_2\right)^T - 3\left(e_1+e_2-2e_5\right)Q_1\left(e_1+e_2-2e_5\right)^T\\&\quad -5\left(e_1-e_2+6e_5-6e_7\right)Q_1\left(e_1-e_2+6e_5-6e_7\right)^T\end{aligned} \tag{4.34}$$

and:

$$\begin{aligned}&-\hat{\tau}\int_{t-\tau_m}^{t-\underline{\tau}}\dot{x}^T(s)Q_2\dot{x}(s)ds\\&\leq -\left(e_2-e_4\right)Q_2\left(e_2-e_4\right)^T - 3\left(e_2+e_4-2e_6\right)Q_2\left(e_2+e_4-2e_6\right)^T\\&\quad -5\left(e_2-e_4+6e_6-6e_8\right)Q_2\left(e_2-e_4+6e_6-6e_8\right)^T\end{aligned} \tag{4.35}$$

Therefore, (4.33) *can be majored as:*

$$\dot{V}_1^3(t) \leq \zeta_1^T(t)\Phi_3^1\zeta_1(t) - \int_{t-\tau_m}^{t-\underline{\tau}}\dot{x}^T(s)Q_3\dot{x}(s)ds \tag{4.36}$$

with Φ_3^1 *given in* (4.16).

Let us now focus on $\dot{V}_1^4(t)$, we have:

$$\begin{aligned}\dot{V}_1^4(t) =& \frac{\underline{\tau}^2}{2} e_{11}\,(S_1 + S_2)\, e_{11}^T - \int_{-\underline{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) S_1 \dot{x}(s) ds d\theta \\ & - \int_{-\underline{\tau}}^{0}\int_{t-\underline{\tau}}^{t+\theta} \dot{x}^T(s) S_2 \dot{x}(s) ds d\theta\end{aligned} \tag{4.37}$$

Applying lemmas, 2 and 3, it yields:

$$\begin{aligned}& - \int_{-\underline{\tau}}^{0}\int_{t-\underline{\tau}}^{t+\theta} \dot{x}^T(s) S_2 \dot{x}(s) ds d\theta \\ & \leq -2(e_2 - e_5) S_2 (e_2 - e_5)^T - 4(e_2 - 4e_5 + 6e_7) S_2 (e_2 - 4e_5 + 6e_7)^T \\ & \quad -6(e_2 - 9e_5 + 36e_7 - 60e_9) S_2 (e_2 - 9e_5 + 36e_7 - 60e_9)^T\end{aligned} \tag{4.38}$$

and:

$$- \int_{-\underline{\tau}}^{0}\int_{t+\theta}^{t} \dot{x}^T(s) S_1 \dot{x}(s) ds d\theta \leq \zeta_1^T(t) \Xi_1 \zeta_1(t) \tag{4.39}$$

with:

$$\begin{aligned}\Xi_1 =& \frac{\underline{\tau}^2}{2} \mathcal{M}_1 S_1^{-1} \mathcal{M}_1^T + \frac{\underline{\tau}^2}{4} \mathcal{M}_2 S_1^{-1} \mathcal{M}_2^T + \frac{\underline{\tau}^2}{6} \mathcal{M}_3 S_1^{-1} \mathcal{M}_3^T \\ & + \underline{\tau}\mathcal{H}_e\Big(\mathcal{M}_1 \begin{bmatrix} e_1 & -e_5 & 0 & 0 \end{bmatrix} + \mathcal{M}_2 \begin{bmatrix} e_1 & 2e_5 & -3e_7 & 0 \end{bmatrix}\Big) \\ & + \underline{\tau}\mathcal{H}_e\Big(\mathcal{M}_3 \begin{bmatrix} e_1 & -3e_5 & 24e_7 & -60e_9 \end{bmatrix}\Big)\end{aligned}$$

Hence, from (4.37)-(4.38), we can write:

$$\dot{V}_1^4(t) \leq \zeta_1^T(t)(\Phi_4^1 + \tilde{\mathcal{M}})\zeta_1(t) \tag{4.40}$$

with Φ_4^1 given in (4.18) and:

$$\tilde{\mathcal{M}} = \frac{\underline{\tau}^2}{2} \mathcal{M}_1 S_1^{-1} \mathcal{M}_1^T + \frac{\underline{\tau}^2}{4} \mathcal{M}_2 S_1^{-1} \mathcal{M}_2^T + \frac{\underline{\tau}^2}{6} \mathcal{M}_3 S_1^{-1} \mathcal{M}_3^T$$

Let us now focus on $\dot{V}_1^5(t)$, we have:

$$\begin{aligned}\dot{V}_1^5(t) =& e_{11}\left(\frac{\hat{\tau}^2}{2} W_1 + \frac{\hat{\tau}^2}{2} W_2\right) e_{11}^T - \int_{-\tau_m}^{-\underline{\tau}}\int_{t+\theta}^{t-\underline{\tau}} \dot{x}^T(s) W_1 \dot{x}(s) ds d\theta \\ & - \int_{-\tau_m}^{-\underline{\tau}}\int_{t-\underline{\tau}}^{t+\theta} \dot{x}^T(s) W_2 \dot{x}(s) ds d\theta\end{aligned} \tag{4.41}$$

Applying lemma 2 and 3, it yields:

$$-\int_{-\tau_m}^{-\underline{\tau}}\int_{t+\theta}^{t-\underline{\tau}}\dot{x}^T(s)W_1\dot{x}(s)dsd\theta \le \zeta_1^T(t)\Xi_2\zeta_1(t) \tag{4.42}$$

and:

$$-\int_{-\tau_m}^{-\underline{\tau}}\int_{t-\tau_m}^{t+\beta}\dot{x}^T(s)W_2\dot{x}(s)dsd\theta \le \zeta_1^T(t)\Xi_3\zeta_1(t) \tag{4.43}$$

with:

$$\begin{aligned}\Xi_2 =& \frac{\hat{\tau}^2}{2}\mathcal{N}_1W_1^{-1}\mathcal{N}_1^T + \frac{\hat{\tau}^2}{4}\mathcal{N}_2W_1^{-1}\mathcal{N}_2^T + \frac{\hat{\tau}^2}{6}\mathcal{N}_3W_1^{-1}\mathcal{N}_3^T\\ &+ \hat{\tau}\mathcal{H}_e\Big(\mathcal{N}_1\begin{bmatrix} e_2 & -e_6 & 0 & 0\end{bmatrix} + \mathcal{N}_2\begin{bmatrix} e_2 & 2e_6 & -3e_8 & 0\end{bmatrix}\Big)\\ &+ \hat{\tau}\mathcal{H}_e\Big(\mathcal{N}_3\begin{bmatrix} e_2 & -3e_6 & 24e_8 & -60e_{10}\end{bmatrix}\Big)\end{aligned}$$

and:

$$\begin{aligned}\Xi_3 =& -2(e_4-e_6)W_2(e_4-e_6)^T - 4(e_4-4e_6+6e_8)W_2(e_4-4e_6+6e_8)^T\\ &-6(e_4-9e_6+36e_8-60e_{10})W_2(e_4-9e_6+36e_8-60e_{10})^T\end{aligned}$$

Thus, we have:

$$\dot{V}_1^5(t) \le \zeta_1^T(t)(\Phi_5^1 + \tilde{\mathcal{N}})\zeta_1(t) \tag{4.44}$$

with Φ_5^1 *given in* (4.20) *and:*

$$\tilde{\mathcal{N}} = \frac{\hat{\tau}^2}{2}\mathcal{N}_1W_1^{-1}\mathcal{N}_1^T + \frac{\hat{\tau}^2}{4}\mathcal{N}_2W_1^{-1}\mathcal{N}_2^T + \frac{\hat{\tau}^2}{6}\mathcal{N}_3W_1^{-1}\mathcal{N}_3^T$$

Now, let us come back to the whole derivative of the LKF (4.24), from (4.31), (4.32), (4.36), (4.40) *and* (4.44)*, the inequality (4.30) is satisfied if:*

$$\zeta_1^T(t)\left(\sum_{i=1}^{5}\Phi_i^1 + \tilde{\mathcal{M}} + \tilde{\mathcal{N}}\right)\zeta_1(t) - \int_{t-\tau_m}^{t-\tau(t)}\dot{x}^T(s)Q_3\dot{x}(s)ds - \int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}^T(s)Q_3\dot{x}(s)ds < 0 \tag{4.45}$$

Since:

$$x(t-\tau(t)) - x(t-\tau_m) - \int_{t-\tau_m}^{t-\tau(t)}\dot{x}(s)ds = 0$$

and

$$x(t-\underline{\tau})-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}(s)ds=0.$$

By considering free weighting matrices $\mathcal{L}_1$ and $\mathcal{L}_2$ in $\mathbb{R}^{12n\times n}$, the inequality (4.45) *can be rewritten as:*

$$\begin{aligned}
&\zeta_1^T(t)\left(\sum_{i=1}^{5}\Phi_i^1+\tilde{\mathcal{M}}+\tilde{\mathcal{N}}\right)\zeta_1(t)\\
&+2\zeta_1^T(t)\mathcal{L}_1\left(x(t-\underline{\tau})-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}(s)ds\right)\\
&+2\zeta_1^T(t)\mathcal{L}_2\left(x(t-\tau(t))-x(t-\tau_m)-\int_{t-\tau_m}^{t-\tau(t)}\dot{x}(s)ds\right)\\
&-\int_{t-\tau_m}^{t-\tau(t)}\dot{x}^T(s)Q_3\dot{x}(s)ds-\int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}^T(s)Q_3\dot{x}(s)ds<0
\end{aligned} \tag{4.46}$$

Moreover, for any matrix $Q_3=Q_3^T>0$, the following inequalities hold [10]:

$$\begin{aligned}
&-2\zeta_1^T(t)\mathcal{L}_1\int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}^T(s)ds\\
&\leq\int_{t-\tau(t)}^{t-\underline{\tau}}\dot{x}^T(s)Q_3\dot{x}(s)ds+(\tau(t)-\underline{\tau})\zeta_1^T(t)\mathcal{L}_1Q_3^{-1}\mathcal{L}_1^T\zeta_1(t),
\end{aligned} \tag{4.47}$$

$$\begin{aligned}
&-2\zeta_1^T(t)\mathcal{L}_2\int_{t-\tau_m}^{t-\tau(t)}\dot{x}^T(s)ds\\
&\leq\int_{t-\tau_m}^{t-\tau(t)}\dot{x}^T(s)Q_3\dot{x}(s)ds+(\tau_m-\tau(t))\zeta_1^T(t)\mathcal{L}_2Q_3^{-1}\mathcal{L}_2^T\zeta_1(t).
\end{aligned} \tag{4.48}$$

Therefore, according to (4.46), (4.47) *and* (4.48), *the inequality* (4.45) *is satisfied if:*

$$\begin{aligned}
&\zeta_1^T(t)\Big(\sum_{i=1}^{5}\Phi_i^1+\tilde{\mathcal{M}}+\tilde{\mathcal{N}}+\mathcal{H}_e\left(\mathcal{L}_1(e_2-e_3)+\mathcal{L}_2(e_3-e_4)\right)\\
&+(\tau(t)-\underline{\tau})\mathcal{L}_1Q_3^{-1}\mathcal{L}_1^T+(\tau_m-\tau(t))\mathcal{L}_2Q_3^{-1}\mathcal{L}_2^T\Big)\zeta_1(t)<0
\end{aligned} \tag{4.49}$$

Let us now rewrite the uncertain and disturbed NCS (4.8) *as:*

$$\bar{\mathcal{G}}\zeta_1(t)=0 \text{ with } \bar{\mathcal{G}}=\begin{bmatrix}\bar{A} & 0 & \bar{B}K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & B_w\end{bmatrix}.$$

From (4.49), *applying lemma 4 and 6,* (4.8) *is asymptotically stable if $\exists\mathcal{T}\in\mathbb{R}^{12n\times n}$ such that, $\forall q\in\mathcal{I}_2$:*

$$\zeta_1^T(t)\left(\sum_{i=1}^{6}\Phi_i^1+\tilde{\mathcal{M}}+\tilde{\mathcal{N}}+\mathcal{H}_e\left(\mathcal{T}\bar{\mathcal{G}}\right)+\hat{\tau}\mathcal{L}_qQ_3^{-1}\mathcal{L}_q^T\right)\zeta_1(t)<0 \tag{4.50}$$

with Φ_6^1 *given in* (4.22)*; i.e., from* (4.8) *and* (4.2)*, if:*

$$\zeta_1^T(t)\begin{pmatrix}\sum_{i=1}^{6}\Phi_i^1+\tilde{\mathcal{M}}+\tilde{\mathcal{N}}+\mathcal{H}_e\left(\mathcal{T}\mathcal{G}\right)\\+\hat{\tau}\mathcal{L}_q Q_3^{-1}\mathcal{L}_q^T+\mathcal{H}\delta(t)\mathcal{E}+\mathcal{E}^T\delta^T(t)\mathcal{H}^T\end{pmatrix}\zeta_1(t)<0 \tag{4.51}$$

Now, applying lemma 5 on (4.51)*, it is satisfied if:*

$$\zeta_1^T(t)\begin{pmatrix}\sum_{i=1}^{6}\Phi_i^1+\tilde{\mathcal{M}}+\tilde{\mathcal{N}}+\mathcal{H}_e\left(\mathcal{T}\mathcal{G}\right)\\+\hat{\tau}\mathcal{L}_q Q_3^{-1}\mathcal{L}_q^T+\lambda^{-1}\mathcal{H}\mathcal{H}^T+\lambda\mathcal{E}^T\mathcal{E}\end{pmatrix}\zeta_1(t)<0 \tag{4.52}$$

where $\lambda > 0$ *is an arbitrary scalar,* $\mathcal{H} = \mathcal{T}H$ *and* $\mathcal{E}$ *are defined in* (4.12)*.*

Up to now, $\forall\tau(t) \in [\underline{\tau}, \tau_m]$*, the proof only deals on the asymptotical stability conditions of the NCS without external disturbances (*$\omega(t) = 0$*). In order to take into account the external disturbances (*$\omega(t) \neq 0$*), the* H_∞ *criterion* (4.9) *can be considered to minimize the transfer between the disturbances* $\omega(t)$ *and the outputs* $y(t)$*, which is satisfied by the following inequality:*

$$\dot{V}(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) < 0 \tag{4.53}$$

where $\gamma > 0$ *is the disturbances attenuation level to be minimized.*

Note that, from (4.8)*, we can write:*

$$\begin{aligned} y^T(t)y(t) &= [Cx(t)+DKx(t-\tau(t))+D_w\omega(t)]^T[Cx(t)+DKx(t-\tau(t))+D_w\omega(t)] \\ &= \zeta_1^T(t)\mathcal{D}^T\mathcal{D}\zeta_1(t) \end{aligned} \tag{4.54}$$

with $\mathcal{D}$ *given in* (4.13)*. From* (4.52) *and* (4.54)*, the inequality* (4.53) *is satisfied* $\forall\zeta_1(t) \neq 0$ *if:*

$$\sum_{i=1}^{7}\Phi_i^1 + \tilde{\mathcal{M}} + \tilde{\mathcal{N}} + \mathcal{H}_e\left(\mathcal{T}\mathcal{G}\right) + \hat{\tau}\mathcal{L}_q Q_3^{-1}\mathcal{L}_q^T + \lambda^{-1}\mathcal{H}\mathcal{H}^T + \lambda\mathcal{E}^T\mathcal{E} + \mathcal{D}^T\mathcal{D} < 0 \tag{4.55}$$

where Φ_7^1 *given in* (4.23)*.*

To deal with the BMI terms $\tilde{\mathcal{M}}$ *and* $\tilde{\mathcal{N}}$ *respectively defined in* (4.40) *and* (4.44)*, we apply the Schur complement, so* (4.55) *leads to the condition* (4.10) *with* $q = 1$*,* $\forall\tau(t) \in [\underline{\tau}, \tau_m]$*.*

Now, let us consider the second sub-interval $\tau(t) \in (\tau_m, \bar{\tau}]$*. In this case, let us consider* *the following LKF candidate:*

$$V_2(t) = V_2^1(t) + V_2^2(t) + V_2^3(t) + V_2^4(t) + V_2^5(t) \tag{4.56}$$

where:

$$V_2^1(t) = \theta_2^T(t)P\theta_2(t) \tag{4.57}$$

$$V_2^2(t) = \int_{t-\tau_m}^{t} x^T(s)Z_1x(s)ds + \int_{t-\bar{\tau}}^{t-\tau(t)} x^T(s)Z_2x(s)ds \tag{4.58}$$

$$\begin{aligned} V_2^3(t) =& \tau_m \int_{-\tau_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_1\dot{x}(s)dsd\theta \\ &+ \bar{\eta}h \int_{-\bar{\tau}}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)Q_2\dot{x}(s)dsd\theta + \int_{-\bar{\tau}}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)Q_3\dot{x}(s)dsd\theta \end{aligned} \tag{4.59}$$

$$V_2^4(t) = \int_{-\tau_m}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_1\dot{x}(s)dsd\theta d\lambda + \int_{-\tau_m}^{0}\int_{-\tau_m}^{\lambda}\int_{t+\theta}^{t} \dot{x}^T(s)S_2\dot{x}(s)dsd\theta d\lambda \tag{4.60}$$

$$V_2^5(t) = \int_{-\bar{\tau}}^{-\tau_m}\int_{\lambda}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^T(s)W_1\dot{x}(s)dsd\theta d\lambda + \int_{-\bar{\tau}}^{-\tau_m}\int_{-\bar{\tau}}^{\lambda}\int_{t+\theta}^{t} \dot{x}^T(s)W_2\dot{x}(s)dsd\theta d\lambda \tag{4.61}$$

with:

$$\theta_2(t) = \begin{bmatrix} x^T(t) & \int_{t-\tau_m}^{t} x^T(s)ds & \frac{1}{\tau_m}\int_{-\tau_m}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta & \frac{2}{\tau_m^2}\int_{-\tau_m}^{0}\int_{\lambda}^{t}\int_{t+\beta}^{t} x^T(s)dsd\beta d\lambda \end{bmatrix}^T$$

Following the same path as for the first sub-interval (from equation (4.30) *to* (4.55)*), the NCS is asymptotically stable (for* $\omega(t) = 0$*) and satisfies the* H_∞ *criterion* (4.9) *(for* $\omega(t) \neq 0$*) if,* $\forall \tau(t) \in (\tau_m, \bar{\tau}]$*:*

$$\dot{V}_2(t) \leq \zeta_2^T(t)\Upsilon^2\zeta_2(t) < 0. \tag{4.62}$$

with:

$$\zeta_2(t) = \begin{bmatrix} \eta_2^{1T}(t) & \eta_2^{2T}(t) & \eta_2^{3T}(t) & \eta_2^{4T}(t) & \dot{x}^T(t) \end{bmatrix}^T,$$

$$\eta_2^1(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_m) & x^T(t-\tau(t)) & x^T(t-\bar{\tau}) \end{bmatrix}^T,$$

$$\eta_2^2(t) = \begin{bmatrix} \frac{1}{\tau_m}\int_{t-\tau_m}^{t} x^T(s)ds & \frac{1}{\hat{\tau}}\int_{t-\bar{\tau}}^{t-\tau_m} x^T(s)ds \end{bmatrix}^T,$$

$$\eta_2^3(t) = \begin{bmatrix} \frac{1}{\tau_m^2}\int_{-\tau_m}^{0}\int_{t+\theta}^{t} x^T(s)dsd\theta & (\bar{\eta}h)^2\int_{-\bar{\tau}}^{-\tau_m}\int_{t+\theta}^{t} x^T(s)dsd\theta \end{bmatrix}^T,$$

$$\eta_2^4(t) = \begin{bmatrix} \frac{1}{\tau_m^3}\int_{-\tau_m}^{0}\int_{-\lambda}^{0}\int_{t+\beta}^{t} x^T(s)dsd\theta d\lambda & \frac{1}{(\bar{\eta}h)^3}\int_{-\bar{\tau}}^{-\tau_m}\int_{-\lambda}^{-\tau_m}\int_{t+\theta}^{t} x^T(s)dsd\theta d\lambda \end{bmatrix}^T.$$

So now, (4.62) *is satisfied if* (4.10) *holds with* $q = 2$.

Finally, the stability conditions being splitted into two sub-intervals of $\tau(t)$*, we still have to prove the overall decreasing behaviour of the LKF. To do so, note that:*
$\forall k \in \mathbb{N}$, $\lim_{k\to\infty}(\imath_k + \tau_k) = \infty$ *and* $[\imath_1 + \tau_1, \infty) = \cup_{k=1}^{\infty}[\imath_k + \tau_k, \imath_{k+1} + \tau_{k+1})$*. Moreover,*

$$\tau((i_k - \tau_k)^-) = \imath_k - \imath_{k-1} + \tau_k = \tau_k + h \geq \tau(\imath_k - \tau_k) = \tau_k$$

Hence, by evaluating the LKF terms around the instants $\imath_k + \tau_k$*, we can assert that:*

$$V_q^2((i_k - \tau_k)^-) \geq V_q^2(i_k - \tau_k) \quad for \quad q \in \mathcal{I}_2$$

. (the other LKF terms don't change at $\imath_k - \tau_k$*).Consequently, if the* (4.10) *holds, it provides that* $V_q^2(t)$ *(*$q \in \mathcal{I}_2$*) do not increase at* $\imath_k + \tau_k$*.*

In the first sub-interval we fully exploit information to find $MAUB\bar{\tau}$ *the LKF* V_1^q*is considered, once this parameter is found we can compute the parameter* $\bar{\eta}$*, however in the second sub interval we fully exploit information by taking the LKF* V_2^q *to find* $\bar{\eta}$ *and once this parameter is found we can easily compute* $\bar{M}AUB\tau$

Remark 6 *Following the strategy proposed in [81], in order to provide less conservative LMI conditions, the global interval* $[\underline{\tau}, \bar{\tau}]$ *has been splitted into two sub-intervals* $[\tau_{min}, \tau_{max} + h] \cup]\tau_{max} + h, \tau_{max} + (1 + \bar{\eta})h]$*. As quote in [81], this choice allows to fully exploit the information on network-induced delays, packet dropouts and sampling period. Moreover, the computation of the upper bound* τ_{max} *of the time-varying delay and the maximum packets dropout* $\bar{\eta}$ *become easier. Indeed, one of these two parameters can be easily computed when the other one is given. To provide further conservatism improvements, the integral terms of the LKF candidates* (4.24) *and* (4.56) *have been selected to obtain closed-loop stability conditions that are parameter-dependent to* $\underline{\tau}$*,* τ_m*,* $\bar{\tau}$ *and the sampling period* h *(and so also dependent to the number of consecutive packet dropouts* $\bar{\eta}$*, see Assumption 4.3 and equation* (4.6)*). Also, double and triple integral terms have been considered in these LKF terms to reduce the conservatism, especially from the application*

Lemma 2.3 and 2.4. Thus, Theorem 4.1 is proposed to provide tighter bounds of these LKF integral terms. Finally, regarding to previous results (e.g., [81]), the quadratic terms $V_1^1(t)$ and $V_2^1(t)$ involves extended state vectors $\theta_1(t)$ and $\theta_2(t)$ to improve cross-compensations in LMIs from an extended matrix P.

4.4 Controller design

Recall that, when the feedback gain K is unknown, the conditions presented in theorem 4.1 are no more LMIs and so require a convexification procedure. In this context, the following theorem summarizes the proposed relaxed LMI-based H_∞ controller design conditions for NCS (4.8).

Theorem 4.2 *For given positive scalars $\underline{\tau}$, $\bar{\tau}$, $\bar{\eta}$, γ and λ such that $\underline{\tau} \leq \tau(t) \leq \tau_m$ and $\dot{\tau}(t) = 1$, the NCS model (4.8) is robustly stabilized by the NCS controller (5.8) and minimizes the H_∞ disturbance attenuation level $\gamma > 0$, if there exist a scalar $\beta > 0$ and symmetric positive definite matrices $\bar{P} \in R^{4n\times 4n}$, X, $\bar{S}_1$, $\bar{S}_2$, $\bar{W}_1$, $\bar{W}_2$, $\bar{Q}_1$, $\bar{Q}_2$, $\bar{Q}_3$, $\bar{Z}_1$, and $\bar{Z}_2$ $\in R^{n\times n}$, slack decision matrices $\bar{\mathcal{M}}_1$, $\bar{\mathcal{M}}_2$, $\bar{\mathcal{M}}_3$, $\bar{\mathcal{N}}_1$, $\bar{\mathcal{N}}_2$, $\bar{\mathcal{N}}_3 \in R^{4n\times n}$, $\bar{\mathcal{L}}_1$, $\bar{\mathcal{L}}_2 \in R^{12n\times n}$, the matrix $F \in R^{m\times n}$ (the NCS controller gain matrix being retrieved by the change of variable $K = FX^{-1}$), and two arbitrary scalars ϵ_1, and ϵ_2, such that the following LMIs hold for $q = 1$, and $q = 2$:*

$$\begin{bmatrix} \sum_{i=1}^{7} \bar{\Phi}_i^q + \mathcal{H}_e(\mathcal{J}) + \beta\bar{\mathcal{H}}^T\bar{\mathcal{H}} & \bar{\mathcal{H}} & \bar{\mathcal{D}}^T & \bar{\Gamma}_q^2 \\ * & -\beta I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & \bar{\Gamma}_q^3 \end{bmatrix} < 0 \tag{4.63}$$

with:

$$\bar{\Gamma}_1^2 = \begin{bmatrix} \frac{\underline{\tau}}{\sqrt{2}}\bar{M}_1 & \frac{\underline{\tau}}{2}\bar{M}_2 & \frac{\underline{\tau}}{\sqrt{6}}\bar{M}_3 & \frac{\hat{\tau}_{12}}{\sqrt{2}}\bar{N}_1 & \frac{\hat{\tau}_{12}}{2}\bar{N}_2 & \frac{\hat{\tau}_{12}}{\sqrt{6}}\bar{N}_3 & \hat{\tau}_{12}\bar{\mathcal{L}}^q \end{bmatrix},$$

$$\bar{\Gamma}_2^2 = \begin{bmatrix} \frac{\tau_m}{\sqrt{2}}\bar{M}_1 & \frac{\tau_m}{2}\bar{M}_2 & \frac{\tau_m}{\sqrt{6}}\bar{M}_3 & \frac{\bar{\eta}h}{\sqrt{2}}\bar{N}_1 & \frac{\bar{\eta}h}{2}\bar{N}_2 & \frac{\bar{\eta}h}{\sqrt{6}}\bar{N}_3 & \bar{\eta}h\bar{\mathcal{L}}^q \end{bmatrix},$$

$$\bar{\Gamma}_1^3 = diag\left\{-\bar{S}_1, -\bar{S}_1, -\bar{S}_1, -\bar{W}_1, -\bar{W}_1, -\bar{W}_1, -\hat{\tau}\bar{Q}_3\right\},$$

$$\bar{\Gamma}_2^3 = diag\left\{-\bar{S}_1, -\bar{S}_1, -\bar{S}_1, -\bar{W}_1, -\bar{W}_1, -\bar{W}_1, -\bar{\eta}h\bar{Q}_3\right\},$$

$$\mathcal{J} = \begin{bmatrix} AX & 0 & BF & 0 & -X & B_w \\ \varepsilon_1 AX & 0 & \varepsilon_1 BF & 0 & -\varepsilon_1 X & \varepsilon_1 B_w \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \varepsilon_2 AX & 0 & \varepsilon_2 BF & 0 & -\varepsilon_2 X & \varepsilon_2 B_w \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\mathcal{E}} = \begin{bmatrix} E_a X & 0 & E_b F & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\mathcal{D}} = \begin{bmatrix} CX & 0 & DF & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_w \end{bmatrix},$$

$$\bar{\mathcal{H}} = \begin{bmatrix} H & 0 & \varepsilon_1 H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_2 H & 0 \end{bmatrix},$$

$$\bar{\Phi}_1^1 = \mathcal{H}_e(\Pi_1^{1T}\bar{P}\Pi_2), \tag{4.64}$$

$$\bar{\Phi}_1^2 = \mathcal{H}_e(\Pi_1^{2T}\bar{P}\Pi_2), \tag{4.65}$$

$$\bar{\Phi}_2^1 = \bar{\Phi}_2^2 = e_1\bar{Z}_1e_1^T - e_2\bar{Z}_1e_2^T - e_4\bar{Z}_2e_4^T, \tag{4.66}$$

$$\bar{\Phi}_3^1 = \bar{\Pi}_3 + e_{11}(\underline{\tau}^2\bar{Q}_1 + \hat{\tau}^2\bar{Q}_2 + \hat{\tau}\bar{Q}_3)e_{11}^T, \tag{4.67}$$

$$\bar{\Phi}_3^2 = \bar{\Pi}_3 + e_{11}(\tau_m^2\bar{Q}_1 + (\bar{\eta}h)^2\bar{Q}_2 + \bar{\eta}h\bar{Q}_3)e_{11}^T, \tag{4.68}$$

$$\begin{aligned}\bar{\Phi}_4^1 =& \bar{\Pi}_4 + e_{11}\frac{\underline{\tau}^2}{2}\left(\bar{S}_1 + \bar{S}_2\right)e_{11}^T + \underline{\tau}\mathcal{H}_e\left(\bar{\mathcal{M}}_1\begin{bmatrix} e_1 & -e_5 & 0 & 0\end{bmatrix}\right)\\ &+ \underline{\tau}\mathcal{H}_e\left(\bar{\mathcal{M}}_2\begin{bmatrix} e_1 & 2e_5 & -3e_7 & 0\end{bmatrix}\right)\\ &+ \underline{\tau}\mathcal{H}_e\left(\bar{\mathcal{M}}_3\begin{bmatrix} e_1 & -3e_5 & 24e_7 & -60e_9\end{bmatrix}\right),\end{aligned} \tag{4.69}$$

$$\begin{aligned}\bar{\Phi}_4^2 =& \bar{\Pi}_4 + e_{11}\frac{\tau_m^2}{2}\left(\bar{S}_1 + \bar{S}_2\right)e_{11}^T + \tau_m\mathcal{H}_e\left(\bar{\mathcal{M}}_1\begin{bmatrix} e_1 & -e_5 & 0 & 0\end{bmatrix}\right)\\ &+ \tau_m\mathcal{H}_e\left(\bar{\mathcal{M}}_2\begin{bmatrix} e_1 & 2e_5 & -3e_7 & 0\end{bmatrix}\right)\\ &+ \tau_m\mathcal{H}_e\left(\bar{\mathcal{M}}_3\begin{bmatrix} e_1 & -3e_5 & 24e_7 & -60e_9\end{bmatrix}\right),\end{aligned} \tag{4.70}$$

$$\begin{aligned}\bar{\Phi}_5^1 =& \bar{\Pi}_5 + e_{11}\frac{\hat{\tau}^2}{2}\left(\bar{W}_1 + \bar{W}_2\right)e_{11}^T + \hat{\tau}\mathcal{H}_e\left(\bar{\mathcal{N}}_1\left[\begin{array}{cccc} e_2 & -e_6 & 0 & 0 \end{array}\right]\right)\\ &+ \hat{\tau}\mathcal{H}_e\left(\bar{\mathcal{N}}_2\left[\begin{array}{cccc} e_2 & 2e_6 & -3e_8 & 0 \end{array}\right]\right)\\ &+ \hat{\tau}\mathcal{H}_e\left(\bar{\mathcal{N}}_3\left[\begin{array}{cccc} e_2 & -3e_6 & 24e_8 & -60e_{10} \end{array}\right]\right),\end{aligned} \tag{4.71}$$

$$\begin{aligned}\bar{\Phi}_5^2 =& \bar{\Pi}_5 + e_{11}\frac{(\bar{\eta}h)^2}{2}\left(\bar{W}_1 + \bar{W}_2\right)e_{11}^T + \bar{\eta}h\mathcal{H}_e\left(\bar{\mathcal{N}}_1\left[\begin{array}{cccc} e_2 & -e_6 & 0 & 0 \end{array}\right]\right)\\ &+ \bar{\eta}h\mathcal{H}_e\left(\bar{\mathcal{N}}_2\left[\begin{array}{cccc} e_2 & 2e_6 & -3e_8 & 0 \end{array}\right]\right)\\ &+ \bar{\eta}h\mathcal{H}_e\left(\bar{\mathcal{N}}_3\left[\begin{array}{cccc} e_2 & -3e_6 & 24e_8 & -60e_{10} \end{array}\right]\right),\end{aligned} \tag{4.72}$$

$$\bar{\Phi}_6^1 = \bar{\Phi}_6^2 = \mathcal{H}_e\left(\bar{\mathcal{L}}_1(e_3^T - e_4^T) + \bar{\mathcal{L}}_2(e_2^T - e_3^T)\right), \tag{4.73}$$

$$\bar{\Phi}_7^1 = \bar{\Phi}_7^2 = -\gamma^2 e_{12}e_{12}^T, \tag{4.74}$$

$$\begin{aligned}\bar{\Pi}_3 =& -(e_1-e_2)\,\bar{Q}_1\,(e_1-e_2)^T-(e_2-e_4)\,\bar{Q}_2\,(e_2-e_4)^T\\ &-3\,(e_1+e_2-2e_5)\,\bar{Q}_1\,(e_1+e_2-2e_5)^T-3\,(e_2+e_4-2e_6)\,\bar{Q}_2\,(e_2+e_4-2e_6)^T\\ &-5\,(e_1-e_2+6e_5-6e_7)\,\bar{Q}_1\,(e_1-e_2+6e_5-6e_7)^T\\ &-5\,(e_2-e_4+6e_6-6e_8)\,\bar{Q}_1\,(e_2-e_4+6e_6-6e_8)^T\,,\end{aligned}$$

$$\begin{aligned}\bar{\Pi}_4 =& -2(e_2-e_5)\bar{S}_2(e_2-e_5)^T-4(e_2-4e_5+6e_7)\bar{S}_2(e_2-4e_5+6e_7)^T\\ &-6(e_2-9e_5+36e_7-60e_9)\bar{S}_2(e_2-9e_5+36e_7-60e_9)^T,\end{aligned}$$

$$\begin{aligned}\bar{\Pi}_5 =& -2(e_4-e_6)\bar{W}_2(e_4-e_6)^T-4(e_4-4e_6+6e_8)\bar{W}_2(e_4-4e_6+6e_8)^T\\ &-6(e_4-9e_6+36e_8-60e_{10})\bar{W}_2(e_4-9e_6+36e_8-60e_{10})^T.\end{aligned}$$

Proof 2 *: Let $\beta = \lambda^{-1}$, by applying the Schur complement on the inequality* (4.55)*, we have:*

$$\begin{bmatrix} \sum_{i=1}^{7}\Phi_i^q+\mathcal{H}_e\,(\mathcal{T}\mathcal{G})+\beta\mathcal{H}^T\mathcal{H} & \mathcal{E}^T & \mathcal{D}^T & \Gamma_q^2\\ * & -\beta I & 0 & 0\\ * & * & -I & 0\\ * & * & * & \Gamma_q^3 \end{bmatrix} < 0 \tag{4.75}$$

Then, let us define:

$$\mathcal{T} = \left[\begin{array}{cccccc} X^{-T} & 0 & \varepsilon_1 X^{-T} & 0 & \varepsilon_2 X^{-T} & 0 \end{array}\right]^T$$

and

$$D_X = diag\left\{\underbrace{X\ldots X}_{11\ times}\quad I\quad I\quad I\quad \underbrace{X\ldots X}_{7\ times}\right\}^T$$

with $X \in \mathbb{R}^{n\times n}$ *(invertible). Pre-and post-multiplying* (4.75) *by* D_X *and its transpose respectively and with the changes of variables:*
$F = KX$, $\bar{P} = diag\left\{ X \quad X \quad X \quad X \right\}^T P\, diag\{ X \quad X \quad X \quad X \}$, $\bar{S}_\rho = X^T S_\rho X$, $\bar{W}_\rho = X^T W_\rho X$, $\bar{Q}_\sigma = X^T Q_\sigma X$, $\bar{\mathcal{M}}_\sigma = X^T \mathcal{M}_\sigma X$, $\bar{\mathcal{N}}_\sigma = X^T \mathcal{N}_\sigma X$, $\bar{\mathcal{L}}_\rho = X^T \mathcal{L}_\rho X$, *with* $\rho \in \mathcal{I}_2$ *and* $\sigma \in \mathcal{I}_3$, *we obtain the conditions expressed in Theorem 4.2.*

Remark 7 *As usual when considering lemma 4, the conditions proposed in Theorem 4.2 involves arbitrary scalars* ε_1 *and* ε_2*. These parameters can be tuned offline by linear programming to reduce the conservatism, see e.g., [106, 12, 94] and references therein. Also, if the computational cost can be considered today as a drawback of such LMI-based approaches, it doesn't constitute a big obstacle since these are solved offline. Moreover, with the growing advances of computational capabilities and convex optimization solvers, we can fairly expect that such a drawback will be alleviated in a near future.*

4.5 Numerical examples

In the sequel, four numerical examples, drawn from previous related literature, are considered to illustrate the effectiveness of our proposal and for comparison purpose.

4.5.1 Example 1: *Stability analysis of an NCS without disturbance and without uncertainties*

Let us consider a system (4.1) ($\Delta A(t) = \Delta B(t) = 0$ and $\omega(t) = 0$) and a known NCS control law (5.8) with:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},\ K = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

For given scalars $\underline{\tau}$, the goal is to find the maximum allowable upper bound $maub(\bar{\tau})$ of the closed-loop NCS overall input delay $\tau(t)$, which includes the network-induced time-delay τ_{max} and the maximum number of consecutive packet dropouts $\bar{\eta}$, such that the proposed LMI conditions presented in theorem 4.1 are feasible (i.e., guaranteeing the asymptotical stability of the NCS). The results, obtained from Theorem 4.1 (with different values of $\bar{\eta}$) and compared with several related previous studies, are presented in Table 1. We can notice that Theorem 4.1 provides the less conservative results since it always outperforms the $maub(\bar{\tau})$ values obtained from the considered related previous studies. This confirm the superiority of the LMI-based conditions proposed in this paper.

Table 4.1: Comparison of $maub(\bar{\tau})$ obtained for different values of $\underline{\tau}$ and $\bar{\eta}$ (Example 1).

Methods / $\underline{\tau}$ =	2	3	4	5
[33]	3.5836	4.2009	4.9268	5.8774
[83]	3.7293	4.3847	5.1125	6.0497
[118]	3.5667	4.5666	5.5666	6.5666
[20]	3.3764	4.5313	5.6636	6.7748
[81]	3.4388	3.8184	4.4728	5.1592
Th.1 ($\bar{\eta}$ = 0)	**3.7082**	**4.7043**	**5.7042**	**6.7041**
Th.1 ($\bar{\eta}$ = 10)	**3.7692**	**4.7620**	**5.7621**	**6.7620**
Th.1 ($\bar{\eta}$ = 20)	**3.8687**	**4.8616**	**5.8622**	**6.8613**
Th.1 ($\bar{\eta}$ = 50)	**4.1690**	**5.1620**	**6.1617**	**7.1613**

4.5.2 Example 2: *Stability analysis of an uncertain and disturbed NCS*

Let us now consider an uncertain and disturbed system (4.1), and a known networked control law (4.5), specified by:

$$A=\begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}, BK=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}, E_a=\begin{bmatrix}1.6 & 0\\ 0 & 0.05\end{bmatrix}, H=I,$$

$$E_bK=\begin{bmatrix}0.3 & 0\\ 0 & 0.5\end{bmatrix},\ E=\begin{bmatrix}0.01\\ 0.05\end{bmatrix},\ C=\begin{bmatrix}0.1\\ 0.2\end{bmatrix}^T,\ D=F=0.$$

In this example, the goal is still to find $maub(\bar{\tau})$ for different values of $\underline{\tau}$ and $\bar{\eta}$. The obtained results are compared with previous ones from the literature in Table 4.2. Once again, we can conclude on the superiority of the conditions proposed by Theorem 4.1 in terms of conservatism reduction. Note that, for comparison purpose and similarly to what has been proposed in the considered previous related studies [69, 117, 65], it is assumed that $\lambda = 1$ and the H_∞ disturbance attenuation level is set to $\gamma = 1$. To highlight the effectiveness of this result, Fig. 5.3 shows a simulation of the considered NCS where $\tau(t) \in [0.2, 1.334]$ is randomly updated at each sampling time to represent communication noises, allowing the maximal admissible packets dropout $\bar{\eta} = 10$, and with a sampling period $h = 0.025$ (s). In this case, Theorem 4.1 provides a solution where all the tested previous results failed. For the simulation, an initial condition $x(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$

has been considered, an uncertain signal $\delta(t)$ satisfying $\delta^2(t) \leq 1$ is applied as well as a disturbance signal , $\forall t \in [3, 5.5]$, $w(t) = \sin(10t)$, $w(t) = 0$ otherwise.This illustrates the robustness of this NCS against communication noises, structural uncertainties and external disturbances, guaranteed from Theorem 4.1 with large delay range $maub(\bar{\tau}) - \underline{\tau}$, when previous results from the literature failed to provide a solution.

Table 4.2: Comparison of $maub(\bar{\tau})$ obtained for different values of $\underline{\tau}$ and $\bar{\eta}$ (Example 2).

Methods / $\underline{\tau}$ =	0	0.2	0.4	0.6	0.8	1
[69]	1.05	1.06	1.06	1.07	1.10	1.15
[117]	1.057	1.076	1.098	1.104	1.140	1.205
[65]	1.073	1.095	1.095	1.098	1.125	1.174
Th. 1 ($\bar{\eta} = 10$)	**1.150**	**1.334**	**1.493**	**1.640**	**1.786**	**1.937**
Th. 1 ($\bar{\eta} = 20$)	**1.242**	**1.428**	**1.588**	**1.738**	**1.886**	**2.037**
Th. 1 ($\bar{\eta} = 30$)	**1.333**	**1.520**	**1.682**	**1.834**	**1.985**	**2.136**

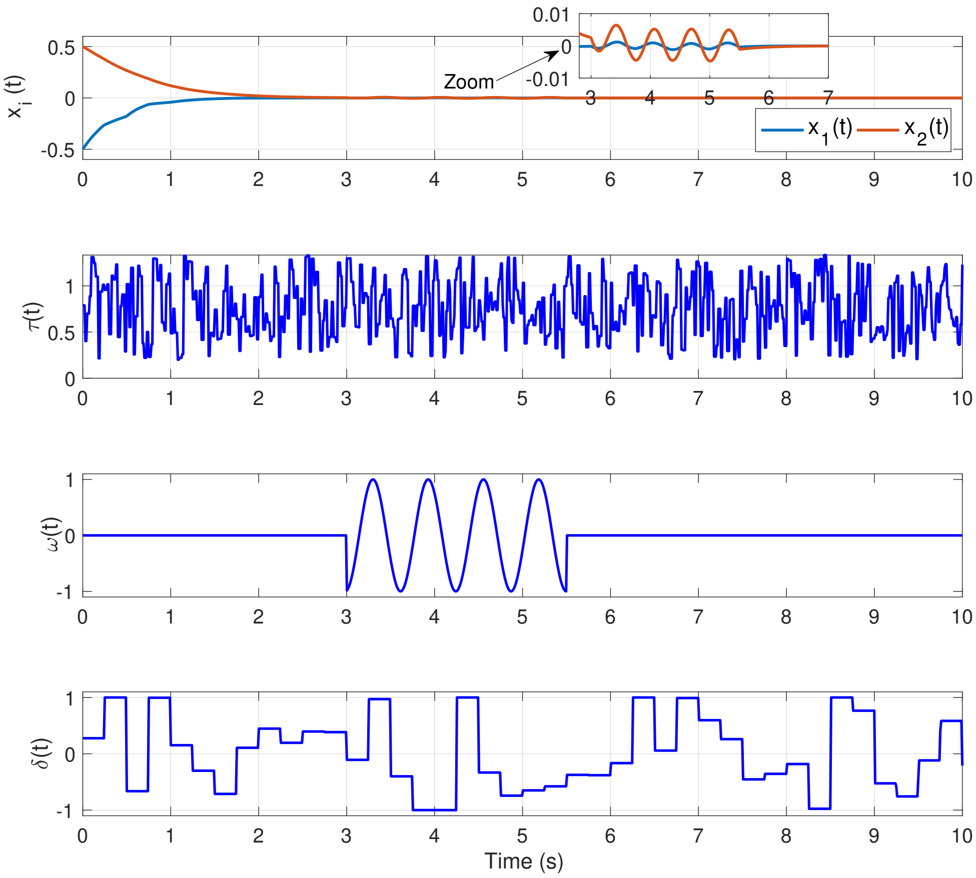

Figure 4.2: Numerical simulation (Example 2).

4.5.3 Example 3: *Packets loss effect on NCS stability analysis*

Let us consider a closed-loop NCS (4.8), without uncertainties and external disturbances, drawn from [81] and describe by the matrices:

$$A = \begin{bmatrix} -0.5 & -2 \\ 1 & -1 \end{bmatrix} \quad B.K = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.6 \end{bmatrix}$$

For this example, without loss of generality, we choose $\underline{\tau} = 0$. Applying Theorem 4.1 for different prescribed values of packet dropouts, the obtained values of $maub(\bar{\tau})$ are listed in Table 4.3. We can observe that, when the maximum number of consecutive packet dropouts increases, $maub(\bar{\tau})$ also increases. Moreover, the results obtained with the present approach are significantly outperforming the ones obtained in [81] for any given packet dropouts. This also shows the conservatism improvement raised by Theorem 4.1.

Table 4.3: $maub(\bar{\tau})$ with given $\bar{\eta}$ and $\underline{\tau} = 0$ (Example 3).

Methods / $\bar{\eta}$ =	10	20	30	40	50	60	70
(Li et al., 2016)	1.1689	1.2585	1.3424	1.4235	1.5001	1.5642	15875
Th.1	**1.4200**	**1.5070**	**1.5951**	**1.6719**	**1.7740**	**1.8531**	**1.9330**

4.5.4 Example 4: *Controller design for NCS subject to uncertainties and external disturbances*

Let us highlight that, in the three previous examples, the controller gains K were assumed to be known for the stability analysis of NCS (4.8) via Theorem 4.1. Therefore, this last example is dedicated to complete this study with the design of a NCS controller (4.3) via the application of Theorem 3.2. To do so, let us consider an open-loop unstable third order uncertain and disturbed system (4.1), drawn from [133] and specified by:

$$A=\begin{bmatrix}-1 & 0 & -0.5\\ 1 & -0.5 & 0\\ 0 & 0 & 0.5\end{bmatrix}, B=\begin{bmatrix}0\\0\\1\end{bmatrix}, C=\begin{bmatrix}1\\0\\1\end{bmatrix}^T, B_w=\begin{bmatrix}1\\1\\1\end{bmatrix}, H=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix},$$
$$E_a=\begin{bmatrix}0.1 & 0.1 & 0\\ 0 & 0 & 0.1\\ 0 & -0.1 & 0\end{bmatrix}, E_b=\begin{bmatrix}0.05\\0\\0.05\end{bmatrix}, D=0.1, D_w=0.$$

Here, the goal is to design of a robust stabilizing NCS controller (4.3), which guarantee the lowest as possible H_∞ disturbance attenuation level γ. In this context and for comparison purpose with previous results from the literature [184, 52, 133], we set $\tau(t) \in [0.1, 0.58]$, $\lambda = 0.1$ and $\bar{\eta} = 0$. Then applying Theorem 3.2 with $\varepsilon_1 = 25$ and $\varepsilon_2 = 13$, we obtain the NCS controller gain matrix $K = \begin{bmatrix}-1.4500 & 0.7734 & -1.0495\end{bmatrix}$, with the minimal disturbance attenuation levels γ_{min} listed in Table 4.4. We can notice that the minimal H_∞ disturbance attenuation level obtained from Theorem 3.2 is smaller than the one obtained from the previous considered results. This confirm again the conservatism improvements raised by the present proposal.

Finally, to illustrate the effectiveness of this result, a numerical simulation of the designed closed-loop uncertain and disturbed NCS is shown in Fig. 4.3 with an initial condition $x(0) = \begin{bmatrix}0.5 & -0.5 & 1.4\end{bmatrix}^T$ and $\tau(t) \in [0.1, 0.58]$, which is randomly updated at each sampling time to represent communication noises. For this simulation, Fig. 4.3 also show the applied uncertain signal $\delta(t)$, which satisfies $\delta^2(t) \leq 1$, and a noisy disturbance signal chosen as, $\forall t \in [25, 35]$, $w(t) = 0.2\sin(10t) + r(t)$, $w(t) = r(t)$ otherwise, where $r(t)$ is a noise disturbance set as a random signal having a normal distribution with mean 0 and standard deviation 0.05. We can notice that the considered uncertain and disturbed systems is properly stabilized by the NCS controller and successfully attenuate the external disturbances with noise. This confirm the robustness of the designed controller with regards to communication noises, structural uncertainties and external/noise disturbances, guaranteed from Theorem 3.2 with larger allowed network-induced delay ranges ($maub(\bar{\tau}) - \underline{\tau}$) and better H_∞ performances than the ones obtained from previous literature.

Table 4.4: H_∞ performance obtained from different methods (Example 4).

Methods	**Theorem 3.2**	[133]	[52]	[184]
γ_{min}	**1.3517**	1.4762	1.6242	1.9

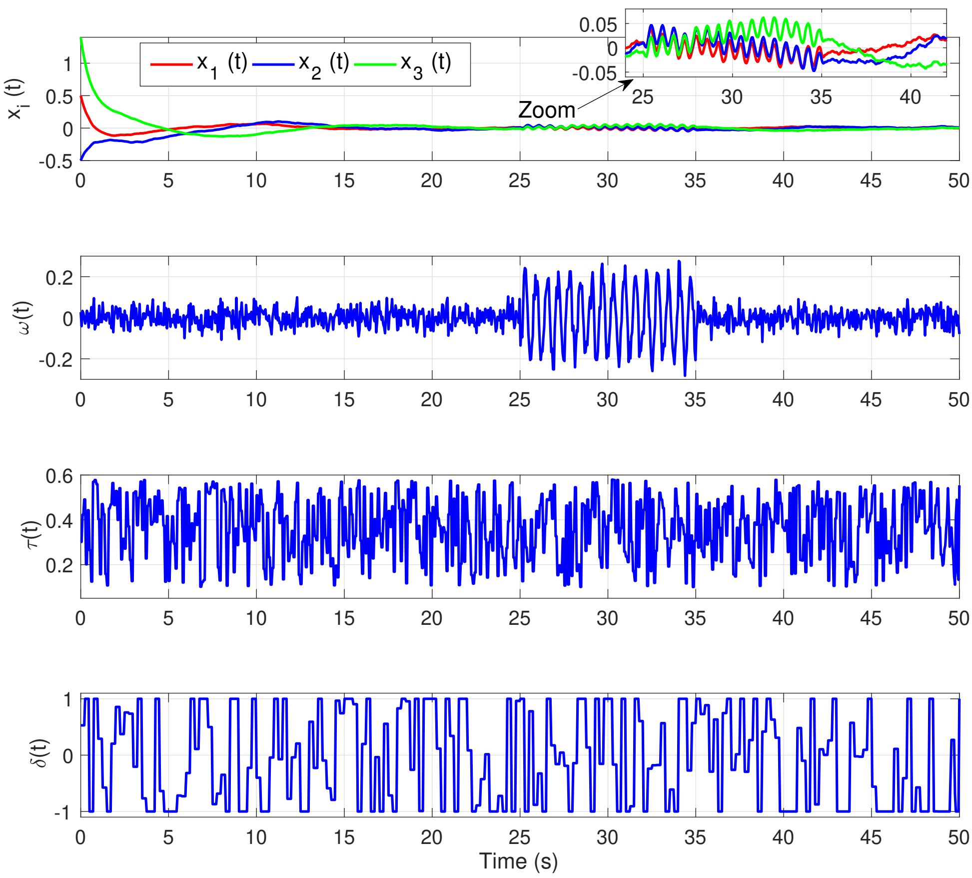

Figure 4.3: Numerical simulation (Example 4).

4.6 Conclusion

In this chapter, the problem of robust H_∞ stability and controller design for a class of uncertain and disturbed networked control system has been addressed. The considered NCS is subject to network-induced delay and packet dropouts. It is modelled as a sampled data control system and, to derive LMI-based delay-dependent conditions, a piecewise LKF is proposed with double and triple integral terms and for the estimation of the time derivative terms of the LKF six lemma have been investigated, to reduce the conservatism of the proposed conditions, free weighting matrices were introduced from null terms and the application of the Finsler's lemma. Two theorems have been proposed: the first one for closed-loop stability analysis, i.e., assuming that the NCS controller is known, and the second one for the design of the NCS control law. Four numerical examples, drawn from the literature for comparison purpose, have been considered. These shown significant improvements in terms of conservatism of the proposed conditions regarding to several recent related results.

Chapter 5

Event Triggered H_∞ control design for networked control systems with time delay and packet dropouts.

5.1 Introduction

Event-triggered control (ETC) is a control strategy in which the control task is executed after the occurrence of an event, generated by some well designed event-triggering condition, it is especially suited for applications where computation and communication resources are scarce [148, 147]. Event-triggered control is currently attracting more and more attention due to its abilities to significantly reduce the communication and computation resources in embedded control systems and distributed systems. Unlike the traditional time-triggered control, in which the controller updates control law periodically, the event-triggered control is aperiodic.

The key point is that the transmission instants in a networked feedback control loop are generated by the triggered events. Generally speaking, an event generator provides the even-triggered conditions, under which the ability, such as convergence and stability, of the systems can still be achieved. Motivated by above observations, the goal of this chapter is

- Propose a new Mixed Event Triggered Control (METC) strategy, which consider both actual and memory clock-driven sensors data. The design of this METC is proposed together with the design of a stabilizing networked sampled-data controller such that the closed loop NCS stability is guaranteed under network induced delays, as well as guaranteeing a prescribed H_∞ performance index to attenuate the effect of external disturbances.

- The proposed controller uses only a single feedback vector for its update. Thus, it further helps in reducing the data transmission through the communication channel.
- The proposed event-triggered controller is free from Zeno behavior, i.e., it avoids the accumulation of triggering instants. This will guarantee the practical feasibility of the proposed scheme.
- The benchmark of a satellite system is proposed to show the effectiveness, compared with previous related studies, of the proposed METC and controller design methodology.

5.2 Preliminaries

In this section, a Mixed Event-Triggered Control (METC) strategy will be proposed for the networked control of linear systems with external disturbances given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_w\omega(t) \\ y(t) = Cx(t) + Du(t) + D_w\omega(t) \end{cases} \tag{5.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^\nu$ is the control input vector, $\omega(t) \in \mathbb{R}^\mu$ is a vector of external disturbances belonging to $L_2[0, +\infty)$, $y(t) \in \mathbb{R}^q$ is the output measurement vector and A, B, C, D, B_w, and D_w are real constant matrices with proper dimensions. The considered networked control scheme with METC is depicted in Fig. 1, where $k \in \mathbb{N}$ denotes the actual sampling number, assuming that the sensors are clock-driven with fixed sampling period h and the controller and actuators are event driven from the METC.
In this thesis, we also assume that all the state variables are available for measurements, and their respective values are broadcast together as single packets at each sampling period, being therefore accessible from the controller device. Moreover, we denote τ_k^{sc} and τ_k^{ca}, respectively the sensors to controller and the controller to actuators network induced delays, and so $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ the overall network induced delay.

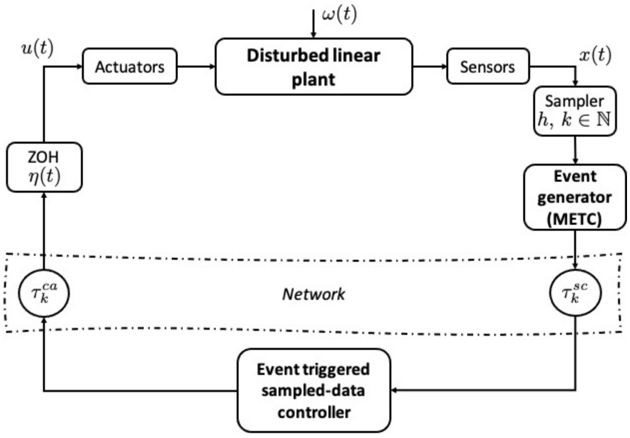

Figure 5.1: Networked control system scheme

5.3 Event triggered data transmission protocol

This subsection presents the problem event-based state feedback control with our event-triggered data transmission strategy.In our design, the sampled data packets are transmitted directly to the event generator. The event generator monitors the event-triggered condition continuously. In order to reduce network traffic and enhance network resource utilization, an Event-Triggered Data Transmission Protocol ($ETDTP$) is designed to determine whether or not the current sampled data packet should be transmitted. It is composed of buffer and an event generator. The buffer stores the latest transmitted data packet $x(t_{kh})$, where $t_k = 1, 2, \ldots, g$ represents the release instant sequence of the $ETDTP$.
The event generator checks the current sampled-data packet $x(t_{kh+jh})$ with regard to the following event triggered condition:

$$[x((t_k + j)h) - x(t_k h)]^T \Omega_1 [x((t_k + j)h - x(t_k h)] \leq \delta x^T(t_k) h \Omega_2 (t_k h) \qquad (5.2)$$

where Ω_1 and Ω_2 are positive matrices, and $\delta \in [0, 1]$.
The event triggered scheme in equation (5.2)is related from the latest released sampled-data $x(t_{kh})$ and current sampled-data $x(t_k h + nh)$, $n \in \{0, 1, ...t_{k+1} - t_k - 1\}$ $k \in \mathbb{N}$, where t_{k+1} is the next latest release time instant the positive weighting matrices Ω_1 and Ω_2 in (equation (3)) present more flexibility in the solution space when the current sampled data is decided whether to be released, which could improve the system's performance.
If the condition expressed by equation (5.2) is satisfied, current sampled data packet will be discarded directly. Otherwise, the current sampled data packet will be the $ETDTP$

released immediately by the $ETDTP$ to the controller via network communications. only if the state error between the current sampling state and the previously transmitted state exceeds a threshold, The sampled data packet will transmitted .the frequency of network transmissions of sampled data will be reduced only when the event triggered scheme is used. Through the NCS communication network the ZOH is reached by a released data packet with a time delay. Let's we consider that data packet $x(t_k h)$ arrives at the ZOH at the time instant $(k = 1, 2, ...)$, the ZOH sends the packet to the controller and holds it until the next data packet comes. if data packets are not dropped so we have $t_1 \leq t_2 \leq ... \leq t_k \leq ....$ If the latest transmitted data packet $x(t_k h)$ is released by the ZOH, the controller will be actuated immediately with$\bar{x}(t)$ given by the following expression :

$$\bar{x}(t) = x(t_k h), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_k+1}) \tag{5.3}$$

5.4 Modelling of NCSs with event-triggered transmission mechanism

In this section, we propose a Mixed Event Triggered Control (METC) of closed loop NCS (5.1) through networks, i.e when the control loop is closed via a communication network, as is depicted in Fig.5.1. Assuming that the sensor is time triggered with fixed sampling period h and its sampling sequence is described by the set $\mathcal{S} = \{0, s_1, s_2, \ldots, s_k\}$, with $s_k = kh, \ k \in \mathbb{N}$. By collecting these sampling data, the sample data $x(s_k)$ which beyond the designed triggering condition should be transmitted.

We denote the event triggered sequence as the set $\mathcal{T} = \{0, t_1, t_2, \ldots, t_k\}$, with $\mathcal{T} \subset \mathcal{S}$. So, the actuators will implement their control actions with these successfully transmitted the released data packets. When a sampled data is released by the event generator to the controller, it is occur a sensor to controller communication delay τ_k^{sc}. Also, data transmissions from the controller to the actuator induce a controller to actuator communication delay τ_k^{ca}. Therefore, the total network induced delay can be lumped together as a time-varying delay $\tau_k = \tau_k^{sc} + \tau_k^{ca}$.

The total network induced delay from sensor to actuator ar bounded and satisfy $\tau_m \leq \tau_k \leq \tau_M$, with τ_M and τ_m is the upper and lower bound of network induced delay, respectively . In order to considerate the limited communication bandwidth in network based systems and to mitigate the burden of the computation in a communication network, we propose an new event triggered control strategy to reduce the redundant release data. In this context, let us define the following state error taking at the last sampling instant and the last release instant define by:

$$e_k(t) = x(t_k) - x(t_k + lh) \tag{5.4}$$

where $l \in \mathbb{N}$ is the number of samples that are not transmitted before the last sampling instant $t_k = kh$. In this chapter, to improve event triggered control strategies available in previous literature (see e.g., [147, 49, 161]), let us also define the following memory based weighted state error:

$$\sum_{i=1}^{m} \varepsilon_i e_{k-i}(t) = \sum_{i=1}^{m} \varepsilon_i (x(t_{k-i}) - x(t_k + lh)) \tag{5.5}$$

where the integer m is the number of memory samples taken into account in the proposed METC strategy and $\varepsilon_i \in [0, 1]$ are given weighting scalars satisfying:

$$\varepsilon_1 \geq \varepsilon_2 \geq ... \geq \varepsilon_m \quad and \sum\nolimits_{i=1}^{m} \varepsilon_i = 1$$

From (5.4) and (5.5), to determine whether the current sampling state measurements should be transmitted or not to the controller over the network, let us propose that the next transmission instant t_{k+1} satisfies the following METC condition:

$$t_{k+1} = t_k + \min_{l \in N} \left\{ lh | \mathcal{E}_k^T(t) \Omega \mathcal{E}_k(t) > \rho(t) \mathcal{X}(t)^T \Omega \mathcal{X}(t) \right\} \tag{5.6}$$

with:

$$\mathcal{E}_k(t) = \begin{bmatrix} e_k(t) \\ \sum\limits_{i=1}^{m} \sqrt{\varepsilon_i} e_{k-i}(t) \end{bmatrix}, \ \mathcal{X}(t) = \begin{bmatrix} x(t_k + lh) \\ \frac{1}{m} \sum\limits_{i=1}^{m} x(t_{k-i}) \end{bmatrix},$$

$$\rho(t) = \rho_1 + \rho_2 e^{-\lambda \|x(t)\|_2},$$

where $\rho(t) \in [\rho_1, \bar{\rho} = \rho_1 + \rho_2] \subseteq [0, 1]$ is a time varying event triggered threshold with a decreasing rate λ, and $\Omega > 0$ is an event triggering weighting matrix to be designed.

With the considered networked control scheme, the ZOH is dedicated to keep the control signal constant during the interval $\mathbb{I}_z = [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, which can be divided, similarly the way borrowed in [185], into several sub intervals such that $\mathbb{I}_z = \bigcup_{l=0}^{\bar{v}} \mathbb{I}_l$ with:

$$\mathbb{I}_z = \begin{cases} \mathbb{I}_{l=0} = [t_k + \tau_k, t_k + h + \tau_k),\ l = 0 \\ \mathbb{I}_l = [t_k + lh + \bar{\tau}, t_k + lh + h + \bar{\tau}),\ l = 1, 2, \dots, \bar{v} - 1 \\ \mathbb{I}_{l=\bar{v}} = [t_k + \bar{v}h + \bar{\tau}, t_{k+1} + \tau_{k+1}),\ l = \bar{v} \end{cases}$$

where $\bar{\tau} = \max_{k \in N} \tau_k$ and $\bar{v} \in \mathbb{N}$ satisfies:

$$t_k + \bar{v}h + \bar{\tau} < t_{k+1} + \tau_{k+1} \leq t_k + \bar{v}h + h + \bar{\tau} \tag{5.7}$$

$\forall t \in \mathbb{I}_l$, let $\eta(t) = t - t_k - lh$, then, $\forall t$, $\eta(t)$ is a piecewise function satisfying $0 \leq \tau_k \leq \eta(t) \leq h + \bar{\tau} = \bar{\eta}$ and $\dot{\eta}(t) = 1$. According to the considered METC strategy, lut us propose a new event triggered sampled data control law which includes actual and memory actions described by.

$$u(t) = Fx(t_k) + \sum_{i=1}^{m} \varepsilon_i K_i x(t_{k-i}) = F(x(t - \eta(t)) + e_k(t)) + \sum_{i=1}^{m} \varepsilon_i K_i (x(t - \eta(t)) + e_{k-i}(t)) \tag{5.8}$$

where $F \in \mathbb{R}^{v \times n}$ and $K_i \in \mathbb{R}^{v \times n}$ are the controller gain matrices to be designed. Replacing (5.8) into (5.1), yields the expression of the closed-loop dynamics:

$$\begin{cases} \dot{x}(t) = Ax(t) + BF\,(x(t - \eta(t)) + e_k(t)) + \sum_{i=1}^{m} \varepsilon_i BK_i (x(t - \eta(t)) + e_{k-i}(t)) + B_w \omega(t) \\ y(t) = Cx(t) + DF\,(x(t - \eta(t)) + e_k(t)) + \sum_{i=1}^{m} \varepsilon_i DK_i (x(t - \eta(t)) + e_{k-i}(t)) + D_w \omega(t) \\ x(t) = \phi(t),\ \forall t \in [-\bar{\eta}, 0) \end{cases} \tag{5.9}$$

where $\phi(t)$ is a vector valued function of initial conditions for $t \in [-\bar{\eta}, 0)$.

To conclude these preliminaries, the purpose of this work is to provide LMI based conditions for the design of the gains matrices F and K_i $(i = 1, ..., m)$ of the event triggered controller (5.8) such that, without external disturbances ($\omega(t) = 0$), the closed loop dynamics (5.9) is asymptotically stable and, when $\omega(t) \neq 0$, the transfer of the external disturbances to the system's outputs is minimized such that the following H_∞ criterion is satisfied:

$$\int_0^\infty \left(y^T(s) y(s) - \gamma^2 \omega^T(s) \omega^T(s) \right) ds < 0 \tag{5.10}$$

where the scalar $\gamma > 0$ denotes the disturbance attenuation level (performance index) to be minimized.

5.5 Main result

The main contribution of this paper is to propose a LMI based procedure for the design mixed event triggered controllers (5.8) for NCS subject to network induced delay, in order to reduce the network traffic by minimizing the number of redundant released packets. The following theorem summarizes the proposed conditions.

Theorem 5.1 *For all $i = 1, \ldots, m$ and for given constant scalars $\bar{\eta} > 0$, $\bar{\rho} > 0$, $\varepsilon_i > 0$, δ and ϵ, the closed-loop NCS* (5.9) *is robustly stable, i.e. the disturbed linear system* (5.1) *is robustly stabilized by the networked controller* (5.8)*, under the release instants defined by the METC condition* (5.6)*, with a H_∞ disturbance attenuation level $\gamma > 0$, if there exist the real matrices $X > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{R} > 0$, $\tilde{\Omega}_1$, $\tilde{\Omega}_2$, $\tilde{\Omega}_3$, $\tilde{W}$, $\tilde{F}$ and $\tilde{K}_i$, such that the following LMIs are satisfied:*

$$\begin{bmatrix} \tilde{\Omega}_1 & \tilde{\Omega}_2 \\ * & \tilde{\Omega}_3 \end{bmatrix} > 0 \begin{bmatrix} \tilde{R} & \tilde{W} \\ * & \tilde{R} \end{bmatrix} > 0 \tag{5.11}$$

$$\begin{bmatrix} \tilde{\Phi} & \tilde{\Pi}_1^T & \bar{\eta}\tilde{\Pi}_2^T \\ * & -I & 0 \\ * & * & -\delta(2X - \delta\tilde{R}) \end{bmatrix} < 0 \tag{5.12}$$

where:

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}^{11} & \tilde{\Phi}^{12} & \tilde{W} & B\tilde{F} & \tilde{\Phi}^{15} & B_w \\ * & \tilde{\Phi}^{22} & \tilde{R} - \tilde{W} & \tilde{\Phi}^{24} & \tilde{\Phi}^{25} & \epsilon B_w \\ * & * & -\tilde{Q}_1 - \tilde{R} & 0 & 0 & 0 \\ * & * & * & (\bar{\rho} - 1)\tilde{\Omega}_1 & \tilde{\Phi}^{45} & 0 \\ * & * & * & * & \tilde{\Phi}^{55} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\Phi^{11} = AX + X^T A^T + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R},$$
$$\Phi^{12} = B\tilde{F} + \sum_{i=1}^{m} \varepsilon_i B\tilde{K}_i + \epsilon X^T A^T + \tilde{R} - \tilde{W},$$

$$\tilde{\Phi}^{15} = \begin{bmatrix}\varepsilon_1 B\tilde{K}_1 & \dots & \varepsilon_m B\tilde{K}_m\end{bmatrix}$$
$$\tilde{\Phi}^{22} = \epsilon\mathcal{H}\Big(B\tilde{F} + \sum_{i=1}^{m}\varepsilon_i B\tilde{K}_i\Big) - 2\tilde{R} + 2\tilde{W} + \bar{\rho}\tilde{\Omega}_1 + \bar{\rho}\tilde{\Omega}_2 + \frac{\bar{\rho}}{m}\tilde{\Omega}_3,$$
$$\tilde{\Phi}^{24} = \epsilon B\tilde{F} + \bar{\rho}\tilde{\Omega}_1,$$
$$\tilde{\Phi}^{25} = \begin{bmatrix}\varepsilon_1\epsilon B\tilde{K}_1 + \frac{\bar{\rho}\tilde{\Omega}_2}{m} + \frac{\bar{\rho}\tilde{\Omega}_3}{m^2} & \dots & \varepsilon_m\epsilon B\tilde{K}_m + \frac{\bar{\rho}\tilde{\Omega}_2}{m} + \frac{\bar{\rho}\tilde{\Omega}_3}{m^2}\end{bmatrix},$$
$$\tilde{\Phi}^{45} = \begin{bmatrix}(\bar{\rho}/m - \sqrt{\varepsilon_1})\tilde{\Omega}_2 & \dots & (\bar{\rho}/m - \sqrt{\varepsilon_m})\tilde{\Omega}_2\end{bmatrix}$$
$$\Phi^{55} = diag\{(\bar{\rho}/m^2 - \varepsilon_1)\tilde{\Omega}_3, \dots, (\bar{\rho}/m^2 - \varepsilon_m)\tilde{\Omega}_3\},$$
$$\tilde{\Pi}_1 = \begin{bmatrix}CX & D\tilde{F} + \sum_{i=1}^{m}\varepsilon_i D\tilde{K}_i & 0 & D\tilde{F} & \varepsilon_1 D\tilde{K}_1 & \dots & \varepsilon_m D\tilde{K}_m & D_w\end{bmatrix},$$
$$\tilde{\Pi}_2 = \begin{bmatrix}AX & B\tilde{F} + \sum_{i=1}^{m}\varepsilon_i B\tilde{K}_i & 0 & B\tilde{F} & \varepsilon_1 B\tilde{K}_1 & \dots & \varepsilon_m B\tilde{K}_m & B_w\end{bmatrix},$$

In that case, the gain matrices of the networked controller (5.8) *are obtained by the bijective change of variable* $F = \tilde{F}X^{-1}$ *and* $K_i = \tilde{K}_i X^{-1}$*. as well as the weighting matrix*

Proof 3 *Let us consider the following delay dependent LKF candidate:*

$$V(t) = V_1(t) + V_2(t). \tag{5.13}$$

where $V_1(t) = \theta^T(t)\mathcal{P}\theta(t)$*, with* $\theta^T(t) = \begin{bmatrix}x^T(t) & x^T(t-\eta(t))\end{bmatrix}$*, and:*

$$V_2(t) = \int_{t-\bar{\eta}}^{t} x^T(s)Q_1x(s)ds + \int_{t-\eta(t)}^{t} x^T(s)Q_2x(s)ds + \bar{\eta}\int_{-\bar{\eta}}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R\dot{x}(s)dsd\theta$$

The LKF candidate (5.13) is positive if $\mathcal{P}$*,* Q_1*,* Q_2 *and* R *are all positive definite matrices. In this case, the closed loop system under METC* (5.9) *is asymptotically stable if:*

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) < 0 \tag{5.14}$$

Assuming that $\mathcal{P} = \begin{bmatrix}P_1 & \phi\\ P_2 & \phi\end{bmatrix}$ *(with arbitrary* ϕ*), taking the derivative of* $V_1(t)$ *yields:*

$$\begin{aligned}\dot{V}_1(t) = {}& 2x^T(t)P_1\Big(Ax(t) + BF(x(t-\eta(t)) + e_k(t)) + \sum_{i=1}^{m}\varepsilon_i BK_i(x(t-\eta(t)) + e_{k-i}(t)) + B_w\omega(t)\Big)\\ &+ 2x^T(t-\eta(t))P_2\Big(Ax(t) + BF(x(t-\eta(t)) + e_k(t)) + \sum_{i=1}^{m}\varepsilon_i BK_i(x(t-\eta(t)) + e_{k-i}(t)) + B_w\omega(t)\Big)\end{aligned} \tag{5.15}$$

Next, taking the derivative of $V_2(t)$*, we obtain:*

$$\dot{V}_2(t) = x^T(t)(Q_1+Q_2)x(t) - x^T(t-\bar{\eta})Q_1x(t-\bar{\eta}) + \bar{\eta}^2\dot{x}^T(t)R\dot{x}(t) - \bar{\eta}\int_{t-\bar{\eta}}^{t}\dot{x}^T(s)R\dot{x}(t)ds \tag{5.16}$$

Applying the reciprocally convex approach (see Theorem 1 in [113]), providing that (3.1) is satisfied, we can write:

$$-\bar{\eta}\int_{t-\bar{\eta}}^{t}\dot{x}^T(s)R\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\eta(t)) \\ x(t-\bar{\eta}) \end{bmatrix}^T \begin{bmatrix} -R & R-W & W \\ * & -2R+2W & R-W \\ * & * & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta(t)) \\ x(t-\bar{\eta}) \end{bmatrix} \tag{5.17}$$

On the other hand, with the definition of $\eta(t)$, the errors (5.4) *and* (5.5)*, from the event-triggering condition* (5.6)*, it can be seen that, for $t \in \mathbb{I}_l$:*

$$\mathcal{E}_k^T(t)\Omega\mathcal{E}_k(t) \leq \rho(t)\mathcal{Z}_k^T(t)\Omega\mathcal{Z}_k(t) \tag{5.18}$$

with $\mathcal{Z}_k(t) = \begin{bmatrix} x(t-\eta(t))+e_k(t) \\ \frac{1}{m}\sum_{i=1}^{m}(x(t-\eta(t))+e_{k-i}(t)) \end{bmatrix}.$

Therefore, from the inequality (5.18) *and from the H_∞ criterion* (5.10)*, the inequality* (5.14) *is sufficiently satisfied if:*

$$\begin{aligned} &\dot{V}(t) + \rho(t)\mathcal{Z}_k^T(t)\Omega\mathcal{Z}_k(t) - \mathcal{E}_k^T(t)\Omega\mathcal{E}_k(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) \\ &= \zeta^T(t)\Big(\Phi + \Pi_1^T\Pi_1 + \bar{\eta}^2\,\Pi_2^T R\Pi_2\Big)\zeta(t) < 0 \end{aligned} \tag{5.19}$$

with

$$\Phi = \begin{bmatrix} \Phi^{11} & \Phi^{12} & W & P_1BF & \Phi^{15} & P_1B_w \\ * & \Phi^{22} & R-W & \Phi^{24} & \Phi^{25} & P_2B_w \\ * & * & -Q_1-R & 0 & 0 & 0 \\ * & * & * & (\bar{\rho}-1)\Omega_1 & \Phi^{45} & 0 \\ * & * & * & * & \Phi^{55} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

and $\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix}$

$$\Phi^{11} = P_1A + A^TP_1^T + Q_1 + Q_2 - R,$$

$$\Phi^{12} = P_1BF + \sum_{i=1}^{m} \varepsilon_i P_1 BK_i + A^T P_2^T + R - W,$$

$$\Phi^{15} = \begin{bmatrix} \varepsilon_1 P_1 BK_1 & \dots & \varepsilon_m P_1 BK_m \end{bmatrix}$$

$$\Phi^{22} = \mathcal{H}\Big(P_2BF + \sum_{i=1}^{m} \varepsilon_i P_2 BK_i\Big)$$
$$-2R + 2W + \bar{\rho}\Omega_1 + \bar{\rho}\Omega_2 + \frac{\bar{\rho}}{m}\Omega_3,$$

$$\Phi^{24} = P_2BF + \bar{\rho}\Omega_1,$$

$$\Phi^{25} = \begin{bmatrix} \varepsilon_1 P_2 BK_1 + \frac{\bar{\rho}\Omega_2}{m} + \frac{\bar{\rho}\Omega_3}{m^2} & \dots & \varepsilon_m P_2 BK_m + \frac{\bar{\rho}\Omega_2}{m} + \frac{\bar{\rho}\Omega_3}{m^2} \end{bmatrix},$$

$$\Phi^{45} = \begin{bmatrix} (\bar{\rho}/m - \sqrt{\varepsilon_1})\Omega_2 & \dots & (\bar{\rho}/m - \sqrt{\varepsilon_m})\Omega_2 \end{bmatrix}$$

$$\Phi^{55} = diag\{(\bar{\rho}/m^2 - \varepsilon_1)\Omega_3, \dots, (\bar{\rho}/m^2 - \varepsilon_m)\Omega_3\},$$

$$\Pi_1 = \begin{bmatrix} C & DF + \sum_{i=1}^{m} \varepsilon_i DK_i & 0 & DF & \varepsilon_1 DK_1 & \dots & \varepsilon_m DK_m & D_w \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} A & BF + \sum_{i=1}^{m} \varepsilon_i BK_i & 0 & BF & \varepsilon_1 BK_1 & \dots & \varepsilon_m BK_m & B_w \end{bmatrix},$$

and

$$\zeta(t) = col\{x(t), x(t-\eta(t)), x(t-\bar{\eta}), e_k(t), e_{k-1}(t), ..., e_{k-m}(t), \omega(t)\}.$$

Let $D_X = diag\{\overbrace{X \dots X}^{m+4 \text{ times}}\ I\ X\ X\}^T$ *with* $X \in \mathbb{R}^{n\times n}$.

Applying the Schur complement on (5.19)*, then taking the congruence by* D_X*, with the changes of variables:*

$P_1 = X^{-1}$, $P_2 = \epsilon X^{-1}$, $\tilde{Q}_1 = X^T Q_1 X$, $\tilde{Q}_2 = X^T Q_2 X$, $\tilde{R} = X^T R X$, $\tilde{\Omega}_1 = X^T \Omega_1 X$,
$\tilde{\Omega}_2 = X^T \Omega_2 X$, $\tilde{\Omega}_3 = X^T \Omega_3 X$, $\tilde{W} = X^T W X$, $\tilde{F} = FX$ *and* $\tilde{K}_i = K_i X$.

Finally, since $-XR^{-1}X < -\delta(2X - \delta\tilde{R})$*, for any constant scalar* δ*, we obtain the LMI condition expressed in theorem 5.1*

Remark 8 *Note that the conditions of Theorem 5.1 are not strictly LMI because of some scalar parameters such like the* ε_i $(i = 1, ..., m)$ *that has to be prescribed. Fortunately, the search for a solution from these non-strict LMI based conditions is done offline, so that these parameters can be tuned by grid search, as usual in some recent related studies dedicated to the conservatism reduction of LMI conditions, e.g. [12, 21, 91]. Moreover, since these are weighting parameters for the memory part of the proposed event triggered controller, their optimization and design will be the subject of one of our future prospects.*

5.6 Illustrative example

In order to illustrate the effectiveness of the proposed METC strategy and networked controller design conditions, let us consider the benchmark of the networked control of a satellite modelled by two solids in rotations (with inertia $J_1 = J_2 = 1$) connected by a spring with torque constant $k = 0.09$ and viscous damping constant $d = 0.0219$ [198, 75]. The linear state space model (4.1) of this benchmark under external disturbance is specified by the following matrices [147]:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_2} & -\frac{d}{J_2} & \frac{k}{J_2} & \frac{d}{J_2} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_1} & \frac{d}{J_1} & -\frac{k}{J_1} & -\frac{d}{J_1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_1} \end{bmatrix}, B_w = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, D = 0, D_w = 0,$$

with : $\quad x(t) = col\{\theta_2(t), \dot{\theta}_2(t), \theta_1(t), \dot{\theta}_1(t)\}$,

where $\quad \theta_1(t)$ and $\theta_2(t)$ denotes the angular position of the solids.

By setting the prescribed H_∞ attenuation level to $\gamma = 0.1$, and with the parameters $\delta = 0.1$, $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.17$, $\varepsilon_3 = 0.13$, $\epsilon = 9.3$ and $\bar{\rho} = 0.4$, the conditions of Theorem 5.1 have been solved via the Matlab LMI Toolbox for $m = 3$ and provide a maximal value of

$\bar{\eta} = 360\,ms$, as well as the following sampled-data controller (5.8) gains matrices:

$$F = \begin{bmatrix} -0.2957 & -1.5094 & -0.2016 & -0.1698 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -11.1298 & -56.8027 & -7.5940 & -6.3817 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -11.2839 & -57.5891 & -7.6991 & -6.4701 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -14.5299 & -74.1646 & -9.9147 & -8.3312 \end{bmatrix}$$

and the METC (5.6) triggering weighting matrices:

$$\Omega_1 = \begin{bmatrix} 0.8963 & 4.6450 & 0.5951 & 0.5401 \\ 4.6450 & 24.4678 & 3.1267 & 2.8542 \\ 0.5951 & 3.1267 & 0.4010 & 0.3649 \\ 0.5401 & 2.8542 & 0.3649 & 0.3347 \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0.6946 & 3.5722 & 0.4672 & 0.4087 \\ 3.5722 & 18.5284 & 2.4193 & 2.1241 \\ 0.4672 & 2.4193 & 0.3166 & 0.2773 \\ 0.4087 & 2.1241 & 0.2773 & 0.2443 \end{bmatrix},$$

$$\Omega_3 = \begin{bmatrix} 45.054 & 230.18 & 30.640 & 25.953 \\ 230.18 & 1178.4 & 156.79 & 132.92 \\ 30.640 & 156.79 & 20.875 & 17.686 \\ 25.953 & 132.92 & 17.686 & 15.007 \end{bmatrix}.$$

Now, we assume that the sampling period of the clock-driven sensors is $h = 0.1\ s$, which allows a maximal network-induced delay $\bar{\tau} = \bar{\eta} - h = 260\,ms$. Under the designed sampled-data controller (5.8) and the METC release condition (5.6), the closed-loop NCS is simulated for this example with the initial condition $x_0 = \begin{bmatrix} 0.2 & -0.3 & 0.3 & 0.2 \end{bmatrix}$ and an external disturbance signal set as $\omega(t) = sgn(sin(t))$ when $t \in [0, 10]$, $\omega(t) = 0$ otherwise. The closed-loop NCS state trajectories are shown in Fig. 5.2. The event-triggering intervals obtained from the proposed METC strategy are plotted at their release instants in Fig. 5.3. Then Fig. 4.3 shows the evolution of the time-varying event-triggering threshold

$\rho(t)$. We can notice that the designed closed-loop NCS is properly stabilized and achieved the origin in less than 15 s, so that, without further changes of the external disturbance, new packet transmissions are not needed. This confirm the effectiveness of the proposed METC strategy and sampled data controller design for NCS.

Recall that the goal of the present study is the reduction of unnecessary transmissions

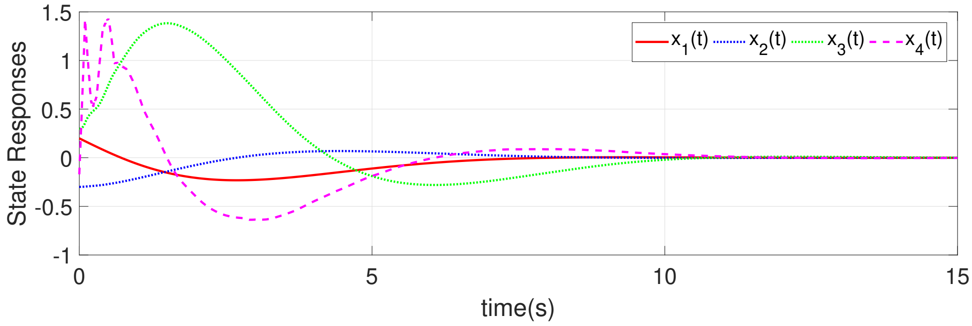

Figure 5.2: States trajectory of satellite system.

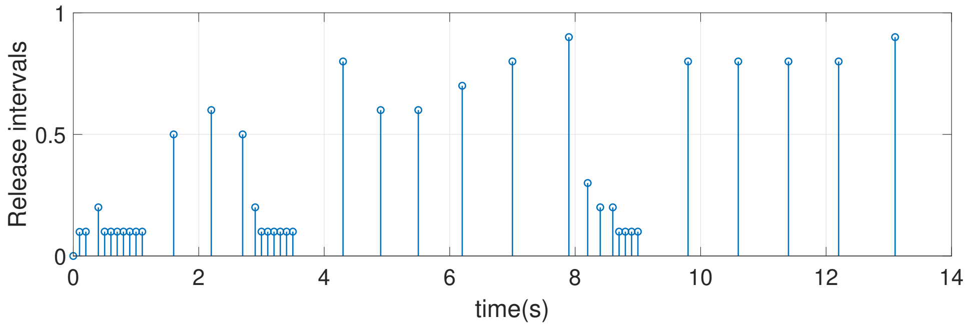

Figure 5.3: Release instants and release intervals.

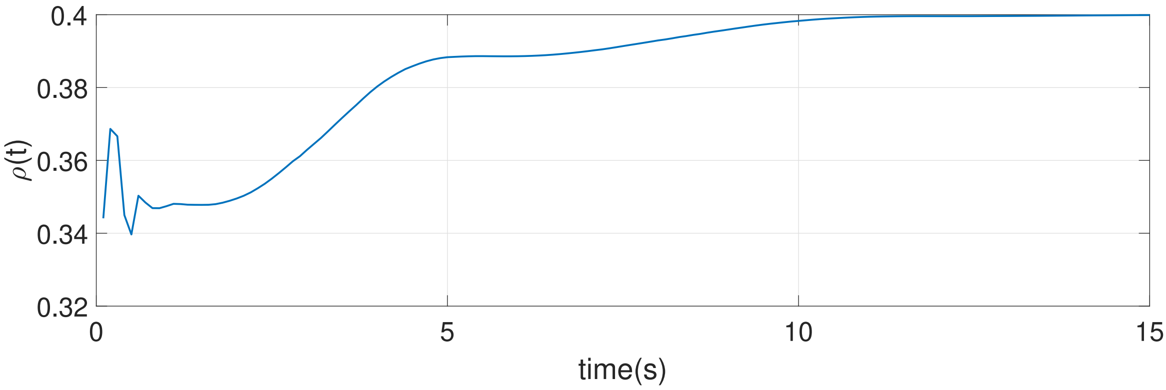

Figure 5.4: The variation of threshold $\rho(t)$

in the designed NCS. To highlight the improvement raised, Table (5.1) lists the Number of Transmitted Packets (NTP) and the Transmission Rate (TR) obtained from the present proposal compared with previous results from related literature.

It is worth to point-out that with the proposed $METC$ event-triggering condition (5.6) and

Table 5.1: Comparison of the AVI and TPN

Methods	**TPN**	**RT**
Our paper $t \in [0, 15s]$	38	25.33%
[147] $t \in [0, 30s]$	81	27%
[27] $t \in [0, 30s]$	432	28.80%
[75] $t \in [0, 80s]$	255	31.9%
[198] $t \in [0, 100s]$	323	32.3%
[157] $t \in [0, 150s]$	711	56.88%

NCS controller design, only 38 data packets (among 150 sampled data packets) are sent to the ZOH through the communication network, which means that the transmission rate is 25.33% during the interval $[0, 15s]$. Thus, 74.67% of the communication resources are saved so that the closed loop system remains stable. Therefore, we conclude that, among all the conditions tested, Theorem 5.1 provides less conservative conditions, which confirm again the improvement brought by this thesis.

5.7 Conclusion

In this chapter, a mixed event triggered scheme and sampled data controller design have been proposed for a class of networked control systems subject to network induced delay and external disturbances. The proposed triggering condition involves the information of the actual sampled-data packet, as well as the memory of some previous packets, in order to reduce the number of unnecessary transmissions, and so to save network bandwidth, with guaranteed closed loop stability performances. Based on the proposed METC and on the choice of a suitable Lyapunov Krasovskii functional, new networked state feedback sampled data controller design conditions have been proposed such that the closed loop dynamics stability is guaranteed with a prescribed H_∞ disturbance attenuation level.

A numerical example have been given to illustrate the effectiveness and improvements of the proposed design conditions regarding to previous related studies from the literature.

Chapter 6

Conclusions and future directions

6.1 Conclusions

In this PhD thesis, we have carefully examined the asymptotic stability, and the robust stabilization of linear networked control system, subject to some communication constraints such as network-induced delay and packet dropouts that are inevitably present in practice, The studied solution is based in some existing results in literature to estimate the present state of the plant.Four main contributions can be noted.

In Chapter 1, we presented the state of the arts in the field of networked control systems, where an introduction including the main challenges in NCSs in a brief History of recent works and recent trends in the modeling and stability analysis of NCSs.

Chapter 2 was dedicated to some fundamentals ans basics on networked control systems, where the different architectures and driving modes for NCSs were presented, then the crucial step in studying NCSs which is the modelling of NCSs,and the different models of delay were presented,we conclude this chapter bu mentioning different control techniques involved the design controllers for NCSs.

In Chapter 3, a mathematical background is presented and dedicated to the concepts of the stability analysis using the Lyapunov theorem where different definitions and notation related to this mathematical tool are presented in detail, then and in the goal of driving stability conditions of NCSs, the Jensen's integral is studied in the large sens moreover some integral inequality lemma such as Wirtinger lemma,Peterson lemma free waiting matrices are in used to estimate the derivative of the lyapunov function.then we moved to the presentation of the robust control techniques where we focused on the $\mathcal{H}_\infty$ technique to avoid perturbations and modeling uncertainties.

In Chapter 4, which is the hearth of this thesis. we presented a first contribution for the stability analysis and controller synthesis of NCSs, where the problem of robust H_∞ stability and controller design for a class of uncertain and disturbed networked control system has been addressed. The considered NCS is subject to network induced delay and packet dropouts. It is modelled as a sampled data control system and, to derive LMI-based delay-dependent conditions, a piecewise LKF is proposed with double and triple integral terms. Also, to reduce the conservatism of the proposed conditions, free weighting matrices were introduced from null terms and the application of the Finsler lemma. Two theorems have been proposed: the first one for closed loop stability analysis, i.e., assuming that the NCS controller is known, and the second one for the design of the NCS control law.

Four numerical examples, drawn from the literature for comparison purpose, have been considered. These shown significant improvements in terms of conservatism of the proposed conditions regarding to several recent related results.

In the last chapter, we presented a second contribution concerning the stability analysis and controller design for NCSs modeled as even triggered system.

A mixed event triggered scheme and sampled data controller design have been proposed for a class of networked control systems subject to network induced delay and external disturbances. The proposed triggering condition involves the information of the actual sampled-data packet, as well as the memory of some previous packets, in order to reduce the number of unnecessary transmissions, and so to save network bandwidth, with guaranteed closed loop stability performances.

Based on the proposed METC and on the choice of a suitable Lyapunov Krasovskii functional, new networked state feedback sampled data controller design conditions have been proposed such that the closed loop dynamics stability is guaranteed with a prescribed H_∞ disturbance attenuation level.

A numerical example have been given to illustrate the effectiveness and improvements of the proposed design conditions regarding to previous related studies. Further works will be done to provide an optimization procedure for the tuning parameters, as well as to extend this result to the case of nonlinear systems.

6.2 Perspectives

The work accomplished in this thesis has explored several NCSs stability analysis and robust controller design, it achieved significant results in this domain where the researches still at these initial stage. We believed that the improvements addressed in this thesis should be more improved in future researches beginning from our obtained results, here below some future research investigations :

- The stability condition derived in chapter 4 and chapter 5 depends on the network induced delay and packet dropout, while the NCSs suffer from others several network induced imperfections and constraints (quantization error, variable sampling, bandwidth limitation,...etc.). It is obvious that the inclusion of all the imperfections will be most suitable for a possible real implementation of the obtained control laws. Therefore, we will focus in our future works on the trade off between all the network induced imperfections to design an NCSs that can be implemented in industries.
- The periodic transmission of closed loop data through the network causes different problems such as packet collision, buffer overflow, and network congestion which lead to packet losses and degrade the network quality of services. this means that it is necessary to design new data transmission strategies to avoid the loss in packets by reducing the amount of data transmitted through a new event triggered control design strategy based on aperiodic transmission and threshold between the latest and current transmission to decide exactly the time required to transmit, or by inserting the buffer overflow challenge into the design of the network control system.
- The main shortcoming of the proposed approach remain in the fact that it is only suitable for the class of linear uncertain and disturbed systems, since most of the real physical systems are inherently nonlinear, for that we will focus in the future on the analysis of the nonlinear NCSs subject to network-induced imperfections by designing sampled-data control for Takagi-Sugeno systems [92, 94, 93] or most generally fuzzy controllers with Takagi–Sugeno (T-S) fuzzy NCSs model subject to network-induced imperfections.

Also, our future works will be focused on network event based triggering strategies, to mitigate unnecessary waste of the networked system resources and reduce redundant and non necessary transmitted packets.

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Appendix A

LMI problems

Linear matrix inequalities are used to solve several automation problems (optimization problems in control theory, system identification and signal processing) which are generally difficult to solve analytically. More specifically, they are used for stability analysis and the synthesis of control laws for NCSs models. The advantage of LMI-based methods is that they can be solved using convex programming.
In this appendix, we will give a brief reminder on convex analysis and linear matrix inequalities as well as the techniques used to solve the LMI problems established during this thesis. For more details on the use of LMI in the field of control, the reader can refer to [13].

A.1 Convex set

A set is said to be convex if:

$$\forall \lambda \in [0, 1], \quad \forall (x_1, x_2) \in C^2 \Leftrightarrow (\lambda x_1 + (1 - \lambda) x_2) \in C. \tag{A.1}$$

- An important property of convex sets is that the intersection of two convex sets is a

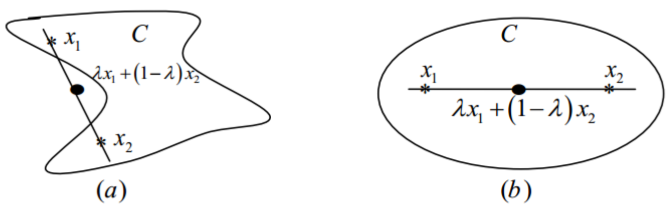

Figure A.1: Convex (a) and non-convex (b) sets.

convex set.

A.2 Convex function

Let be a function $f : C \rightarrow \mathbb{R}$ where the set C is convex, the function f is a convex function if and only if:

$$\forall (x, y) \in C^2 \ \forall \lambda \in [0, 1], \ then \ f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y). \tag{A.2}$$

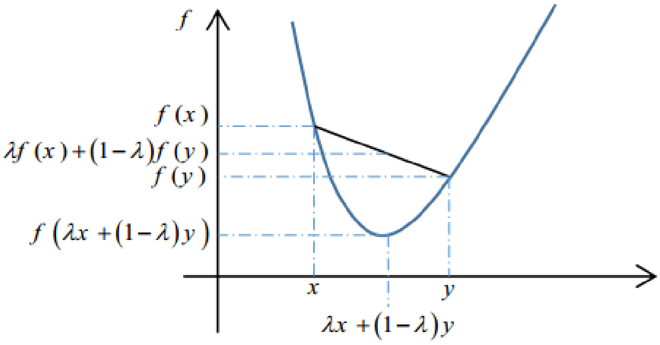

Figure A.2: convex function.

A.3 Linear Matrix Inequalities LMIs

An LMI is a matrix inequality defined in the following form:

$$F(x) = F(0) + \sum_{i=1}^{m} x_i F_i > 0. \tag{A.3}$$

where $x \in \mathbb{R}^{>}$ is the variable to be determined and symmetric matrices $F_i = F_i^T \in \mathbb{R}^{\ltimes \times \ltimes}$, $i = 0, \ldots, m$, are given.

- The inequality ">" in (A.3) means that it is defined positive, that is to say, $u^T F(x) u > 0$ for all $u \in \mathbb{R}$, $u \neq 0$, that is to say all the eigenvalues $\lambda(F(x))$ of $F(x)$ are negative.
- The LMI (A.3) is a convex constraint in, in other words the set $\{x | F(x) > 0\}$ is convex.
- The resolution of the LMI (A.3) amounts to determining the vector $x(t) = [x_1, x_2, \ldots, x_n]$, so $F(x) > 0$ that the convex constraint is verified. This then amounts to minimizing the eigenvalues $\lambda_{max}(F(x)) > 0$. This problem can be solved using the algorithm of the interior point $\lambda(F(x)) > 0$ so that the function is convex.

- It is possible to convert non-linear matrix inequalities (BMI) into LMIs of the form (A.3). The LMI formatting of a convex optimization problem therefore requires writing all the constraints of the problem in the form of affine LMI as a function of the optimization variables.

- We can have a non-strict linear matrix inequality given by: $F(x) \leq 0$.

A.4 Classic LMI problems

There are three types of optimization problems convex encountered in the form of LMI.

A.4.1 Feasibility problem:

It is a question of finding a vector x such that the convex constraint $F(x) > 0$ is satisfied. This problem is generally solved by looking for the vector x minimizing the scalar t such that:

$$-F(x) < t \times I. \tag{A.4}$$

If the minimum value of is negative then the problem is doable.

A.4.2 Eigen-value problem EVP:

It is a question of minimizing the greatest eigenvalue of a symmetric matrix under a constraint of type LMI:

$$\begin{cases} \text{minimize } \lambda, \\ \text{subject to } \begin{cases} \lambda I - A(x) > 0, \\ B(x) > 0. \end{cases} \end{cases} \tag{A.5}$$

where the matrices $A(x)$ and $B(x)$ are symmetrical and linear with respect to the variable x. In the literature, we can find another equivalent formulation of the EVP problem. For example, the problem of minimizing a linear function is posed in the form of an x EVP problem, as a variable as follows:

$$\begin{cases} \text{minimize } c^T x, \\ \text{subject to } F(x) > 0. \end{cases} \tag{A.6}$$

A.4.3 Generalized Eigen-Value Problem GEVP:

It is a question of minimizing the greatest generalized eigenvalue of a pair of matrices, linearly dependent on the

x variable, under LMIs constraints. This problem is expressed as follows:

$$\begin{cases} \text{minimize } \lambda, \\ \text{subject to } \begin{cases} \lambda B(x) - A(x) > 0, \\ B(x) > 0, C(x) > 0. \end{cases} \end{cases} \tag{A.7}$$