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Presented by

ROUAMEL Mohamed

Theme

Robust Control of Networked Systems.

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Abbreviations

ACK	Acknowledgment
AMP	Arbitration on Message Priority
BEB	Binary Exponential Backoff
BMI	Blinear Matrix Inequality
CA	Collision Avoidance
CAN	Controller Area Network
CAREs	Coupled Algebraic Riccati Equations
CD	Collision Detection
CRC	Cyclic redundancy check
CSMA	Carrier Sense Multiple Access
DCSs	Distributed Control Systems
EVP	Eigen-Value Problem
FIFO	First-In-First-Out
GAS	Globally asymptotic stable
GEVP	Generalized Eigen-Value Problem
ILC	Iterative Learning Control
LKFs	Lyapunov Krasovskii Functionals
LMI	Linear Matrix Inequalities

LQ	Linear Quadratic
LRFs	Lyapunov-Razumikhin Functionals
LTI	Linear Time Invariant
MAC	Media Access Control
MAIB	Maximum Allowable Imperfections Bound
MAUB	Maximum allowable Upper Bound
NCSs	Networked Control Systems
OSI	Open Systems Interconnection
QoS	Quality of Service
RTS	Request To Send
RTT	Round Trip Time
TCP	Transmission Control Protocol
TDM	Time-Division Multiplexing
TDMA	Time Division Multiple Access
TRT	Token Rotation Time
ZOH	Zero Order Hold

Abstract

In recent years, the control system is becoming decentralized in location and more extensive in scope, making it difficult to be implemented in the traditional point-to-point configuration. Therefore, the networked control system (NCS) is introduced to replace the traditional control due to its benefits in the wide industrial areas such as reduced installation costs, better maintainability, and greater flexibility, where the control loop is implemented via communication networks, and the data is transmitted between its different components through a limit communication channel.

However, the use of communication channels in the control loop causes many problems and constraints such as network bandwidth limit, network-induced delays, packet loss, packet disordering, scheduling protocols, and variable sampling. These negative factors can degrade the NCSs performance and even lead to its instability. This thesis takes into account these problems in the analysis and the synthesis of NCSs where it analyses the robust stability and design of a new controller for NCSs subject to network-induced delay and packet dropout.

The main contribution of this thesis is based on the reduction of the effect of network-induced imperfections on the stability of the networked system through deriving robust less conservative stability conditions and design a new control low ensuring the stability and good performance of networked systems subject to the maximum possible of these imperfections. And in addition, this work investigates the different modeling of NCSs such as stochastic, deterministic, and hybrid that mimic reality to use it in the analysis and synthesis of networked systems.

Finally, the stability conditions obtained are tested using numerical illustrative examples representing different NCSs with network-induced imperfections, as well as a comparison with other existing methods in terms of conservativeness and robustness.

ملخص

في السنوات الأخيرة، أصبح نظام التحكم لا مركزيًا في الموقع وأكثر اتساعًا في النطاق، مما يجعل من الصعب تنفيذه في التكوين التقليدي من نقطة إلى نقطة. لذلك، تم إدخال نظام التحكم الشبكي (NCS) ليحل محل التحكم التقليدي نظرًا لفوائده في المناطق الصناعية الواسعة مثل انخفاض تكاليف التركيب ، وقابلية صيانة أفضل ومرونة أكبر ، حيث يتم تنفيذ حلقة التحكم عبر شبكات الاتصال ، و تنتقل البيانات بين المكونات المختلفة للنظام من خلال قناة اتصال محدودة. ومع ذلك ، فإن استخدام قنوات الاتصال في حلقة التحكم يسبب مشاكل وقيودًا كثيرة مثل حد النطاق الترددي للشبكة ، والتأخير الناجم عن

الشبكة ، وفقدان الحزمة ، واضطراب الحزمة ، وبروتوكولات الجدولة ، وأخذ العينات المتغير. يمكن لهذه العوامل السلبية أن تؤدي إلى تدهور أداء NCSs وحتى تؤدي إلى عدم استقرار الحلقة المغلقة للنظام. تأخذ هذه الرسالة في الاعتبار هذه المشاكل في التحليل وتوليف NCSs حيث تدرس تحليل الثبات القوي ومشكلات تصميم جهاز التحكم في NCSs الخاضعة للتأخير الناجم عن الشبكة وانقطاع الحزم. تعتمد المساهمة الرئيسية لهذه الأطروحة على تقليل تأثير العيوب الناجمة عن الشبكة على استقرار النظام الشبكي من خلال اشتقاق ظروف استقرار أقل تحفظًا وتصميم تحكم جديد قوي يضمن الاستقرار والأداء الجيد للأنظمة الشبكية الخاضعة لأقصى قدر ممكن من هذه العيوب. بالإضافة إلى ذلك ، يبحث هذا العمل في النمذجة المختلفة لمحطات التحكم بالشبكة مثل العشوائية والحتمية والهجينة التي تحاكي الواقع لاستخدامه في تحليل وتركيب الشبكات المتصلة.
أخيرًا، تم اختبار شروط الاستقرار التي تم الحصول عليها باستخدام أمثلة توضيحية عددية تمثل NCSs مختلفة مع عيوب ناتجة عن الشبكة ومقارنتها مع الطرق الحالية الأخرى من حيث المحافظة والصلابة.

Résumer

Ces dernières années, le système de contrôle est devenu décentralisé et plus étendu, ce qui rend difficile sa mise en œuvre dans la configuration traditionnelle point à point. Par conséquent, le système de contrôle en réseau (NCSs) est créé pour remplacer le contrôle traditionnel en raison de ses avantages dans les zones industrielles étendu telles que la réduction des coûts d'installation, une meilleure maintenabilité et une plus grande flexibilité, où la boucle de contrôle est mise en œuvre via les réseaux de communication et les données est transmis entre ses différents composants via un canal de communication limitée.

Cependant, l'utilisation des canaux de communication dans la boucle de contrôle entraîne plusieurs problèmes et contraintes telles que la limite de bande passante du réseau, les retards induits par le réseau, la perte de paquets, le désordre des paquets, les protocoles de planification et l'échantillonnage variable. Ces facteurs négatifs peuvent dégrader les performances des NCSs et même conduire à l'instabilité du système en boucle fermée. Cette thèse prend en compte ces problèmes dans l'analyse et la synthèse des NCSs où elle étudie l'analyse de stabilité robuste et la conception de contrôleurs du NCSs soumis à des retards induits par le réseau et à la perte de paquets.

La principale contribution de cette thèse est basée sur la réduction de l'effet des imperfections induites par le réseau sur la stabilité du système en réseau par dériver des conditions de stabilité robustes et moins conservatrices et synthétiser un nouveau contrôle robuste assurant la stabilité et les bonnes performances des systèmes en réseau soumis à le maximum possible de ces imperfections. De plus, ce travail étudie les différentes modélisations de NCS telles

que stochastiques, déterministes et hybrides qui imitent la réalité pour l'utiliser dans l'analyse et la synthèse de systèmes en réseau.

Enfin, les conditions de stabilité obtenues sont testées à l'aide des exemples illustratifs numériques représentant différents NCS avec des imperfections induites par le réseau, ainsi qu'une comparaison avec d'autres méthodes existantes en termes de conservativité et de robustesse.

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Chapter 1

General introduction

Classical control systems are designed to bring all sensors information to a central location for processing, decisions about how to act are then sent to the actuators. In this configuration, all components (controllers, sensors, and actuators) are typically situated in the same physical area, and each of them is separately connected by electrical wiring. This classical approach was successfully implemented in the industry for decades.

Nowadays, the classical point-to-point structure barely meet the modern control requirements such as decentralized control, remote control, and so on. In this context, the introduction of communication networks in the control systems was a great revolution, it offered new channels to exchange control data between decentralized system components, thus reducing installation, time and maintenance cost (using considerably fewer wires...etc) and developing the capabilities of the whole control system. A new paradigm called the Networked Control Systems (NCSs) is then appeared from the end of nineties. [19], [160].

The Networked control systems (NCSs) are decentralized or distributed control systems (DCSs) wherein the control loop components (sensors, actuators, and controllers) are closed through a single digital real-time communication channel Fig. 1.1. NCSs is, therefore, a field situated at the intersection of control and communication theories. Thus, the scale of the networked control system is generally much larger than that of conventional control systems. The appearance of this new discipline introduces a new control system challenges concerning the stability analysis and controller design. Among them: the great complexity of the network control system, the negative effect of transmission channel such as communication delay, packet loss, packet disordering and so on. But also the platform heterogeneity, the clock differences between the computers...etc. It is clear that all this represents an extremely heavy burden for the control engineers who must solve these problems when designing the control system loops.[76]

In addition, due to the development of the network access technologies such as: internet,

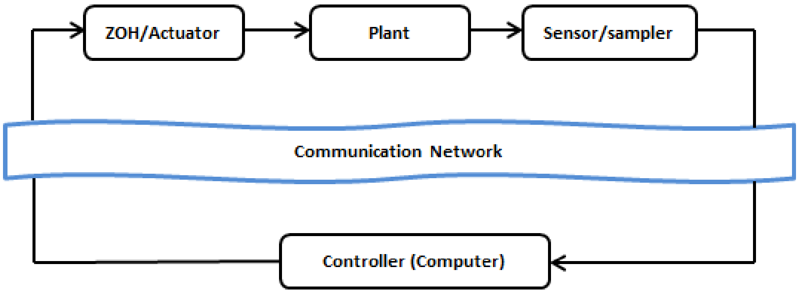

Figure 1.1: Networked Control System Architecture

controller area network (CAN), wireless LAN, Ethernet, etc, NCSs is being employed in many real control systems such urban heating systems, wide area plant automation, intelligent transportation systems, surveillance, remote surgery, distributed power systems, and smart grids [2, 10, 7, 71] and so on. This led to a growing interest in this field from industrial companies, researchers and many control research groups.

1.1 Literature review and recent trends in NCSs

In the late 1990s, researchers began to identify the key distinctive issues of networked control system , driving the main research topics of the next decade to focus on this field and they classify two strategies to address those problems on NCSs. the first is called control of network [28], which focus on the investigations of communication protocols and qualities such that the real-time network-based systems have better network environments. The second is called control over network, which focuses on the control strategies over present networks to reduce the effect of unreliable network-induced imperfections. This thesis addresses the problem of the control over the network in order to minimize the effects of the network-induced imperfections on the system stability and performances.

In recent decades, great effort has been devoted to modeling, stability analysis and control design of NCSs due to the existence of different network-induced imperfections which need to analysis and synthesis methods including all these imperfections. Unfortunately,

much of the available literature research on NCSs has considered some types of network imperfections and ignored the other types where it is important to consider that the design of a typical NCSs requires trade-offs between the different imperfections types. For example, reducing quantization errors leads to larger transmission delays this means that it is necessary to do the trade-offs between these two imperfections to get a good quality of service (QoS).Therefore, it is important to improve and integrate the available results to obtain a framework in which all the network-induced imperfections can be studied simultaneously with the possibility of trade-offs made. This section provides a general overview of the literature on NCSs with a focus on methods for modeling, stability analysis, and control design that incorporate different communication imperfections.
On the same line, the available literature on modeling and stability analysis of NCSs can be categorized on two classes, the first is based on the modeling and analysis approach adopted to study the stability and stabilization of the NCSs under the network imperfections which will be shown in subsection 1.1.2. As for the second is based on the basis of the types of network-induced imperfections like network-induced delay, packets loss, and quantization error which will be display in subsection 1.1.1.

1.1.1 Different methods to model NCSs and network-induced imperfections

: There exists two different ways to model network-induced imperfections, the first one is the deterministic method which assumes a bounded model on the time delays, the number of packet loss and sampling intervals where all possibilities of random processes are ignored. The second class of methods use the stochastic information nature of the network-induced imperfections to model the NCSs.
Several NCSs models appeared in the literature resulting from these two methods (eg., switched system approach, impulsive system approach, sampled data approach, and stochastic approach). Let us focus first on the deterministic method, where there are many systematic approaches modeling the communication channels imperfection in order to analyse the stability of NCSs by considering one or more of these network-induced challenges in the same time. Among them the authors of [23, 85, 144, 136] where they are present different approaches to address the effects of quantization on the stability and performance of NCSs, where [23] addresses the analyze of linear scalar systems with stabilizing quantified feedback control where a general trade-off relation between the number

of quantization intervals, quantifying the information flow, and the convergence time is established,and based on the chaotic behavior of piece-wise affine maps, an alternative stabilization method was proposed. In [85], the authors presents an H_∞ controller design for a linear networked control system where an innovative model is proposed describing both the network conditions and the state quantization of NCSs, a quantified state feedback strategy is developed for global and asymptotical stabilization of the NCSs. Another approach appears in [144] studying the effect of quantization on event-triggered NCSs, where an event-triggered dynamic quantifiers with zooming strategy are adopted to design parameters of the quantifiers under the event-triggered control framework. The advantage of using a zooming scheme is that it only has a finite number of quantization levels but acts the same as a quantizer of infinite quantization levels. Recently, in[136], the authors focuses on the design of a decentralized quantified controller for networked control systems subject to communication constraints and limited bandwidth to study the stability of NCSs, where a uniform quantizer is adopted to deal with limited bandwidth and to switches between "zooming out" and "zooming in" modes, for that, they use a variable called proportionality coefficient of saturation value to handle with the complex coupling relationship between system states and quantified variables.

The studies in [46, 156, 108, 55, 159, 101] addresses the effect of time-varying transmission intervals and delays on NCSs stability: The authors of [156, 55] examine the stability of Networked Control System with time-varying delay, where the time delay area are divided into *'m'* pieces of sections unevenly and use Lyapunov-Krasovskii functional with the help of free-weighting matrices approach to derive the stability conditions, while in [108, 46] a novel delay-distribution approach for a continuous-time networked control system with time-varying transmission delays is presented, based on an input-delay approach, where the NCSs is represented as an hybrid system with multiple input delay subsystems and, the distribution of input delays is modeled as a continuous-dependent and non-identically distributed (d.n.d.) process. On the other hand, the papers in [159] studied the stability and stabilization for a networked control system with additive time-varying delay components' controller, where the sensor-to-controller delay and controller-to-actuator delay are modeled independently (not combine together as one time delay) with a novel approach with free parameters and input-output method is extended to study the stability problem for the NCSs. Authors of [101] treats the linear networked control systems as a sampled-data model with both input delay and input saturation, where the delay-induced non-linearity is over approximately modeled as polytopic inclusion, the aim is to determine a robust model predictive controller (MPC) that asymptotically stabilizes the uncertain system at the origin with a certain level of quadratic performance [72]. Concerning the effect of packet loss, we fined several studies addressing this imperfection as in [124]; [155]; [123]; [122]; [68]; [134]; [164]; [80, 162, 91]: authors of [68] investigates the problem of linear-quadratic Gaussian feedback control over unreliable communication channels, they focuses on the lossy communication networks, where data loss occurs due to the unreliable nature of the link, transmission errors, and/or congestion or long time delays, they derives stability conditions based on the linear-quadratic (LQ) formalism, whereas in [134], the problem of controller design for such network-based iterative learning control (ILC) processes is treated considering data dropout occurring during transfers from the remote plant to the ILC controller. Study in [162] investigate the adaptive event-triggered dynamic output feedback fuzzy control problem for nonlinear NCSs subject to packet loss. The predictive control of network control system with packet dropouts in the feedback and forward channels is presented in [80], introducing data-based networked predictive control method in which a sequence of control increment predictions are calculated in the controller, where the number of consecutive packet dropouts in both channels are assumed to be bounded.

In the case of considering two network-induced imperfection, there is many research in the literature, among them : [93, 95, 16, 54, 142, 65], where in [135, 128, 127] a study of the effect of quantization error and packet dropout on the stability of NCSs is addressed. The authors of [95] focuses on the linear minimum mean square estimator for a networked discrete time-varying linear system subject to data quantification and communication constraints. While in [54, 24, 105] treat the problem of stability and control design of NCSs subject to network-induced delay and packet dropout, Especially the paper in [105] focuses on the mean-square stabilization problem for discrete-time networked control systems, where the control signal is sent to plant over a lossy communication channel, where a network-induced delay and packet dropout occur simultaneously. The stabilization condition is developed in terms of the unique positive-definite solutions to some coupled algebraic Riccati equations (CAREs) . Furthermore, The paper in [151, 37] investigate both the negative effect of quantization error and time transmission delay to derive the stability and stabilization condition of NCSs. The work in [16] addresses the stability of networked control systems with aperiodic sampling and time-varying network-induced delay where the sampling intervals are assumed to vary within a known interval and the transmission delay is assumed to belong to a given interval.

Moreover, when we talk about the effect of more than two network-induced imperfection to study the stability of NCSs, the work in [114] treat the effect the quantization effects, packet dropouts, time-varying transmission intervals, time-varying transmission delays, and communication constraints on the stability of NCSs, and the work in [133]studies the robust H_∞ control problem of networked linear time-delay systems with discrete distributed delays, involving random packet dropout and quantization. and the work in [30] is concerned with the H_∞ performance analysis for networked control systems with transmission delays and successive packet dropouts under aperiodic sampling.

Now, lets present some literature's concerning the second modeling method which is the use of stochastic behavior of network-induced imperfections and their effects on the stability analysis and performances of NCSs, between them, we mention the papers in [121, 87, 18, 116, 154, 92, 132, 109]; [22, 97, 111].

Where in [22], the authors studied the stability of uncertain discrete-time networked control systems subject to random varying network-induced delay. Indeed, they modeled both the measurement and actuation delays as two independent Bernoulli distributed white sequences. While [116] investigates the problem of robust H_∞ fuzzy control for a class of uncertain nonlinear Markovian jump systems with time-varying delay where the goal is to design a fuzzy state-feedback controller such that the closed-loop system is stochastic stable with a prescribed H_∞ performance of disturbance attenuation for all admissible parameter uncertainties. On the other hand, the papers in [111] employ the hybrid-system-with-memory formalism to attain transmission intervals and delays that provably stabilize Networked Control Systems where the authors consider nonlinear time-varying plants and controllers with variable discrete and distributed input, output and state delays along with non constant discrete and distributed network delays. In addition, the papers in [132] present a novel stochastic optimal adaptive event-sampled controller scheme is proposed in the application layer for each physical system or agent expressed as an uncertain linear dynamic system.

To finish this subsection, it is worth noting some papers that study the other models of

NCSs such as observer-based control problem as in [137], [120] [1],[52], [118], where in [120], the authors are propose a new model for an observer-based continuous-time networked control system with network-induced delays and packet dropouts by proposing a linear estimation-based delay compensation method. While the authors in [52] investigates the problem of observer-based robust H_∞ output feedback control for networked control systems (NCSs) with randomly occurring uncertainties, dynamic quantization, and packet loss. On the other hand, the studding of dynamic output feedback control for networked control systems are presented in [117], [161], [109], [115]. First, the authors in [117] study a dynamic output feedback (DOF) H_∞ control for continuous-time networked control systems (NCSs) subject to packet dropouts and network-induced delays where a linear estimation-based method is proposed to estimate the measurement output, and the non-uniform distribution characteristic of the measurement output arrival instant is taken into full consideration to establish a new model for the considered NCSs. Then the authors in [115] treat the problem of reliable event-triggered H_∞ control for networked control systems by using retarded dynamic output feedback by considering the randomness of actuators failures where is modeled by a stochastic variable in a Markov jump model framework.

The tracking Control for Networked Control Systems is investigated in [59], [66], [79], [150], where in [59], the authors investigates the observer-based H_∞ tracking problem of networked output-feedback control systems with consideration of data transmission delays, data-packet dropouts, and sampling effects. While in [66], the H_∞ output tracking control for networked control systems (NCSs) subject to network-induced short time delays is investigated.

1.1.2 Literature concerning the stability analysis of NCSs

: The study of the stability problems of NCSs is a crucial objective in the control field where the main goal is to reduce the effect of the network-induced imperfections on stability by ensuring maximum allowable imperfections bound (MAIB) guaranteeing the stability of the closed-loop and achieve a good performance(eg., guarantee a maximum allowable delay bound, the maximum packet dropouts, the maximum quantization error). Consequently, the MAIB becomes a key performance index to reduce the conservatism of the stability conditions. To achieve this goal, different interesting methods have been proposed to reduce the conservatism of the stability criterion mainly by constructing a new Lyapunov-Krasovskii and use or develop lemmas to estimate its time derivative.

On the other hand, The developed stability criteria are generally classified into two categories according to their dependence on the size of the network imperfections: imperfections-dependent(eg., delay-dependent), and imperfections-independent (eg., delay-independent) stability criteria where at the beginning of studying the stability of NCSs the researchers depended on Lyapunov-Razumikhin functional (due to its simplicity to handle with)to derive an imperfections-independent stability condition [96]; [143]; [29]; [112]; [31]. The information about communication imperfections in imperfections-independent criteria doesn't get into account when deriving the stability condition which leads consequently to a conservative result. While in the last few years the scientists move to study the stability of imperfections-dependent ones which based on the way of using the Lyapunov-Krasovskii functional to derive the stability condition which takes into account the maximum information of negative network-induced effects which leads often to less conservative than imperfections-independent ones [124]; [169]; [78]; [5]; [158]; [163]; [97].

In addition, the recently developed papers in the literature focus on how to get free results to reduce the conservatism mainly by constructing new Lyapunov-Krasovskii functionals to include the maximum information about network-induced imperfections in the deriving of the stability conditions. However, it is well known that LKFs is a powerful tool for studying delayed systems stability ([4]; [126]) but there exist some difficulties to deal with, among which: treating the cross-terms appearing in the time derivatives of LKFs, finding an LKFs depending on the whole system states which is not an easy task even for a linear time-delay system.

Hence, among the recent papers that investigate the choice of new LKFs the papers in [43, 42, 18, 156, 159] where the authors focuses in the construction of an appropriate LKFs with double, triple and quadruple-integral terms to provide larger delay bounds, and introduce several augmented functional which include single integral terms $\left(\int_{t-\tau_m}^{t} x(s)ds\right)$ and $\left(\int_{t-\tau_m}^{t-\tau_M} x(s)ds\right)$, double integral terms $\left(\frac{1}{\tau_m}\int_{-\tau_m}^{0}\int_{t+\beta}^{t} x(s)dsd\beta\right)$ and $\left(\frac{1}{\tau_M-\tau_m}\int_{-\tau_M}^{-\tau_m}\int_{t+\beta}^{t-\tau_m} x(s)dsd\beta\right)$ and triple integral term $\left(\frac{2}{\tau_m^2}\int_{-\tau_m}^{0}\int_{\phi}^{0}\int_{t+\beta}^{t} x(s)dsd\beta d\phi\right)$ (where τ_m and τ_M are the lower and upper bound of network-induced delay) to involve more cross terms between the elements of the resulting augmented vector.

the constructing of an augmented vector in LKFs are presented in several new research such [57, 90] where the authors include simple and double and triple integral terms in an augmented vector.

As we mentioned in the above paragraph that the NCSs developers used several new Lyapunov-Krosovskii functional to derive the NCSs stability conditions, but when they used the LKFs include more than single integral term, they fell in the problem of estimation of the LKFs time derivative (how getting the exact estimation of LKFs time derivative), this problem set an attraction to the mathematics and NCSs researchers to derive the good estimation, Among the papers issued in this regard, past and present, the papers in [47], [35], [156], [57] [159], [152] use of Jensen inequality to estimate simple and double integral terms of Lyapunov time derivative where in [156] and [159] the authors use Jensen inequality to estimate both simple and double integral terms, while in [35]the extended Jensen inequality are used to estimate simple integral terms.In [152] the authors use Jensen inequality to estimate simple, double and triple integral terms.

The authors in ([104, 63, 83, 24, 100, 157, 15]) use Wirtinger-based inequalities to estimate the cross term (single, double and triple integrals forms) in deriving process. where in [157, 39, 56, 90] the authors use an extended Wirtinger's integral inequality which includes the celebrated Wirtinger-based integral inequality as a special case.

When we discuss on the free-weighting matrix technique the authors in ([41, 60, 24, 149]) use free-weighting matrix to handle the cross terms in the derivative of LKFs, wherein the model transformation and bonding techniques are avoided. In ([82, 45, 55, 15]), the convex approach is employed to handle the cross-terms without enlargement, In the same way, In order to add systems parameters in the stability conditions the use of Finsler's lemma are involved([13, 12]).

At the end of this section, it is worth noting some papers use other methods to modeling the NCSs and studying the stability and stabilization among them the predictive control problem for networked systems cited in [129, 139, 125, 11], [165, 86, 131], where in [129] the authors treat the problem of data-driven predictive control for networked control systems (NCSs), which is designed by applying the subspace matrices technique, obtained directly from the input / output data transferred from networks. while the paper in [139] study the problem of predictive output feedback control for NCSs with random communication delays. which the goal of the paper are compensate for the network-induced delay

by employ a new networked-predictive-control scheme. In [125] the authors propose a new robust control scheme that contains an enhanced smith predictor to adaptively estimate and identify control law on line to reply to the time-varying network induced delays. The other control technique investigate the problems of modilization and stabilization of NCSs is adaptive control [119, 17, 153, 53], [51], where the paper in [153] studies an adaptive model-based event-triggered control of an uncertain continuous system with external disturbance where the proposed framework incorporates two important control techniques for reducing communication burden and regulating the states of the system online in control network, that is, adaptive model-based networked control system and event-triggered control (ETC).The paper in [53] studies the static output-feedback control in a class of networked control systems, where the transmission of control signals is based on a novel adaptive event-triggered scheme and the adaptive thresholds depend on the dynamic error of the system rather than predetermined constants as the traditional ones.
At the end, the paper in [61, 48] investigate the sliding mode control technique, where the paper in [61] focuses on the problem of sliding mode control for networked control systems with semi-Markovian switching and random measurement, where the measurement channel is assumed to suffer from random sensor delays and the paper in [48]investigate a discrete-time sliding mode control problem for the NCSs under fading channels.

1.2 Objectives and contributions

As we mentioned previously, the networked control system suffers from several imperfections due to the presence of communication channels, many researchers in the field of control system acts daily to reduce the impact of these imperfections on the performance and stability of networked systems. In the same way, in this thesis, we introduces a novel contribution in the side of minimizing the impact of these imperfections. The main contributions can be summarized briefly as follows:

•. In literature, most researches adopted the problem of random network-induced delay modeling it by several stochastic processes like Bernoulli and Markov process [87, 130, 97, 73, 74], [49, 138, 34], the main results guaranteed the stability of NCSs subject to the largest possible network-induced delay imperfections, this is mainly achieved by introducing a new model of network-induced delay or considering a new Lyapunov Krasovskii Functional including the maximum information of the network-induced delay effects on the system. This leads to reduce the conservatism of the stability and stabilization conditions. In this thesis, a random network-induced delay is introduced and modeled as Bernoulli process, then, less conservative stability conditions are derived by the introduction of a new Lyapunov-Krasovskii functional (LKFs) formulated using the probabilistic information of both lower and upper bounds of the time-varying network-induced delay. Also, the LKFs time's derivatives are estimated using novel integral inequalities lemma (Wirtinger-based integral inequalities).

• When looking at the NCSs structure, we can see that the NCSs is a kind of sampled-data system, they share and transmit the signals of the closed-loop by same manner (the data are sampled and quantified before being transmitted, the controller is implemented in a discreet form), the only difference is that the NCSs contains a communication channel between its components. Therefore, there exist several studies addressing the problem of stability and control design of NCSs as sampled data, among them [57, 60, 50], the main goal is to obtain a maximum allowable network-induced delay bound guarantying the stability and stabilization of NCSs. In this thesis, we used the idea of addressing NCSs as a sampled-data system by constructing new LKFs including simple, double, and triple integral terms and also new augmented LKFs vector. We also used a novel lemmas (Wirtinger and extended reciprocally lemmas) to estimate the LKFs vector derivatives.

• These contributions permits to improve the maximum allowable network-induced delay bound guarantying the stability of NCSs compared with other methods results, a robust state feedback controller is also derived based on the obtained stability conditions.

1.3 Thesis organization

The thesis is organized as follows:

Chapter 1 starts with a general introduction about NCSs, this introduction includes an overview of the different aspect of the networked control systems and traditional point-to-point communication which present the advantages of the implementation of NCSs instead of point-to-point and presented also the challenges to implementing this new structure of control loops. After that we find a section that presents the literature review and recent trends of NCSs which is divided itself into two subsections, the first subsection discusses the different points to model the NCSs and network-induced imperfections. While the second one presents a literature review of NCSs stability analysis and controller design methods. The contributions made within the framework of this thesis are presented in the contribution section. Finally, this chapter is concluded by the thesis organization section to summarize and facilitate the presentation of this work for readers.

Chapter 2 discusses the fundamental definitions, Architecture, Topologies, Modeling, and Control of NCSs, the chapter is divided into five sections. The first one presents a definition of different components of NCSs and the role of each one on the signals processing of the control-loop, while the second section presents the different structures of real-time NCSs with the distribution of the elements of closed-loop in the process area. Two NCSs structures are presented : the direct and hierarchical ones with the advantage of each one and when to use each of them. The third section provides the different topologies of networked control systems explaining the schematic description of the arrangement of various nodes (sensors, controllers, and actuators) in the two NCSs structures discussed in the previous section and illustrates how data flows between the devices of the control loop connected between each other via this network. In section four, the different models of NCSs existing in literature are presented including different NCSs modeling methods: time-varying delays, time-varying sampling intervals, and packet dropouts, we also discusses both discrete-time and continuous-time NCSs models, linear and non-linear NCSs mode, and uncertain and disturbed NCSs model. In the last section, we present the different real protocols used in the real NCSs, we introduce the structure of the network based on the International Organization for Standardization (OSI) and explain what is the layer concerned when studying the control over network and the protocols responsible to manage this processes.

In Chapter 3, the stability analysis and control design techniques for NCSs are discussed, we focus on the Lyapunov stability technique which is a most common and useful approach to analyze the stability of linear and nonlinear networked and non-networked systems. In the beginning, the fundamental concept of the Lyapunov method is presented with with its two forms : the direct and indirect one. Then we presents the different definitions and notations related to the method of Lyapunov stability analysis, we define both an equilibrium point at which the system remains stable in the absence of external interference and the region of attraction using Lyapunov function. Then we define the different Lyapunov stability aspects from Lyapunov asymptotic stability to Lyapunov uniform and uniform asymptotic stability to Lyapunov global and global asymptotic stability to Lyapunov exponential stability. Then when finishing a discussion of the different aspect related to the Lyapunov method, we turn to the concept of stability by the Lyapunov-Krasovskii Functional Approach where we needed it when we talk about the networked delay by specifies the future evolution of the system beyond *'t'*, and also when we talk about the delay-dependent stability condition. Next, we discuss the delay-independent stability condition using the Lyapunov-Razumikhin Functional Approach (LRF). The chapter is concluded by the robust control technique section discussing the robustness of the uncertain systems and the different ways the control systems subject to perturbations and uncertainties, the robust H_∞ technique is especially discussed to derive a robust and non-fragile controller for NCSs.

Chapter 4 presents the first contribution for this thesis where a networked control systems with stochastic time-varying network-induced delays in the given interval with known lower and upper bounds is considered, the NCSs is modeled as a linear system with probabilistic time-varying delay distribution using Bernoulli process. The non-uniform distribution of network-induced delays which simulate the real delay in the real network are fully introduced in the modeling phase. Next,a Lyapunov-Krasovskii functional (LKFs) is formulated using probabilistic information of both lower and upper bounds of the time-varying network-induced delay, then Wirtinger-based integral inequalities are used to estimate the accuracy of the resulting time derivatives and also to reduce conservatism of the stability conditions. Afterward, stability condition based on a H_∞ method is expressed to guarantee the attenuation of external disturbances. The resulting stability conditions are expressed in terms of a set of linear matrix inequalities (LMIs), and Finsler's lemma is used to relax this stability conditions by adding slack decision variables and decoupling the systems matrices from those of Lyapunov-Krasovskii. This procedure permits to obtain a robust state feedback controller by the mean of a simple change in the variables of LMI-stability conditions. Finally, a maximum allowable upper bound of the network-induced delay and state feedback controller gains are calculated by resolving the above relaxed LMIs' convex optimization problem. Practical numerical examples are provided to validate the proposed approach; the results show that the negative effects of the unpredictable network-induced delays are compensated and the stability of NCSs with high disturbance attenuation level is guaranteed.

In chapter 5, the second contribution for this thesis is presented, the problem of the stability analysis and state feedback control design of networked control systems (NCSs's) subject to time-varying network-induced delays are investigated using a sampled-data system with network-induced delay NCSs modeling method. Less conservative stability conditions in the form of linear matrix inequality (LMIs) are then derived by constructing new augmented Lyapunov-Krasovskii functional (LKFs) with double and triple integral terms with a novel augmented vector. Wirtinger-based inequalities and extended reciprocally convex approach are also used to estimate the time derivative of LKFs. The obtained stability condition gives maximum allowable bound (MAUB) of network-induced delay that guarantees the stability of an NCSs and compared it with other existing methods in the literature. Then, a state feedback controller is also derived from the proposed stability condition. Finally, numerical examples clearly show the superiority of the proposed method in terms of conservativeness and control performances over large delay bound.

Chapter 6 summarizes the contributions of this thesis in the field of the NCSs stability analysis and stabilization, and concludes this thesis by some interesting potential future research directions.

Chapter 2

Networked Control Systems (NCSs) : Architectures, Topologies, Modeling and Control

2.1 Introduction

As mentioned in the previous chapter, the loop signals (control and feedback signals) are exchanged among all components of the system (sensors, controllers, and actuators) through a common limited digital communication network. This paradigm brought new challenges to the controlled system such as imperfections induced by the network which include varying delays, packet dropouts, etc.
In this chapter, several models of networked control systems are presented where, it includes different models of the imperfections induced by the network and both discrete-time and continuous-time NCSs models, linear and non-linear NCSs model, uncertain and disturbed NCSs model. After that, the different structures, and topologies of NCSs are presented which include the different structures between the components of closed-loop interconnected through the network and the main and important types of NCSs topologies. finally, several techniques and approaches are presented to analyze the stability and synthesis controller for NCSs while considering one or more of the imperfections will be discussed in the next section.

2.2 Networked control systems components

From the NCSs structure shown in figure 1.1, the typical NCSs consists of the following four basic elements:

2.2.1 Sensors :
In the beginner, it must address the general definition of sensors, where it is the simple equipment in the control loop which its role is to measure the different physical quantities of systems and then convert it to electrical or pneumatic signals formula. In a networked system which the communication channel replaces the wires, which means that the physical quantities do not convert to analog signal but into a numerical signal, from that, it can conclude that the sensor in the networked system plays the role of sensing, sampling and quantifying the measurement signal before transmitted through a network to the controller, this also means that the sensor plays an important role in networked systems, and they are usually worked with limited sensing, computing and communication capabilities (The number of sensors in the industrial zone is usually the largest, hence they have to be its cost is cheap, this led to have a limitation in its roles). for this reason, it should be taken these limitations into account in the analysis and design of NCSs.

2.2.2 Controllers :
The networked controller is the important element in the closed-loop for many researchers, which receives the measurement signal from the sensors and manipulates it subject to the inputs needed to drive the process variable toward the desired set-point, where the controller manipulates the set-point changes defined by operators as well as disturbances and systems uncertainties to the process variable caused by internal and external forces. This operation is repeated over and over until the process variable matches the set-point. In networked control systems, the controller is generally implemented in a discrete-time format (computer or processor) and it differs from the sensor in that it is aperiodic the treatment of signals, it waits the event to do its work (event-driving). The signal of control (output of the controller) is transmitted to the next equipment in the control loop in numerical format through the same communication channel. The networked controller cannot process signals at the same moment which the sensor is sending the information due to the existence of different control node used the communication channel(congestion) where it needs waits its turn to transmit its signal,

this lead to occur a delay in the reception of signals and maybe a loss or disordering in received signals. This phenomena induced by the network degrade the performance and maybe lead to instability of the networked control systems which implies solving these imperfections when designing the control system loops.

2.2.3 Zero Order Hold (ZOH) :

The Zero-Order Hold (ZOH) is a device or algorithm responsible for converting a discrete-time signal to a continuous-time signal by hold each sample value for a specific sample interval. The role of ZOH in the networked control loop is converting the numerical signal coming from the controller to an analog signal to manipulate the physical continuous system. In NCSs, the ZOH receive the discrete-signal at time $t_k + \tau_k$ then convert it to the continuous form by hold it until the next sampling time $t_{k+1} + \tau_{k+1}$ (where $k \in \mathbb{N}$), where it is driven by the event, not by the time (the event is when the controller transmits the signal, generally, the event appear after one sampling time plus an arbitrarily delay $t_k + \tau_k$). The accuracy of ZOH depends on the sampling frequency,where when $(t_{k+1} + \tau_{k+1}) - (t_k + \tau_k) = h$ (where h is the sensor sampling time) the output of ZOH approaches to the exact continuous-time signal.

2.2.4 Actuators :

An actuator is a component of the control loop that is responsible for moving the system subject to its limits safety states or working interval, for example by opening a valve and, the actuators need a source of energy and control signal, the control signal is received from the controller and in NCSs, the control signal is received exactly from the ZOH, where it is a low current or voltage. When the actuators receive a control signal, their response is converting the signal energy into mechanical motion (limited movements or position), where the response can be linear or rotary.

In the linear motion the actuators convert energy into straight-line motions typically for positioning applications, and usually have a push and pull function. On the other hand, rotary actuators convert energy to provide rotary motion, typical use is the control of various valves such as ball valves or butterfly valves.

2.2.5 Communication network :

The implementation of a networked system largely depends on the performance parameters of the communication network, which include quantization, transmission rate, delays, packet losses, and so on. When transmitting a signal over a digital network, we need to sample and encode it in a digital format then transmitted over the networks to the next equipment of closed-loop. the data received

need to be decoded at the receivers, which induce delays and packet loss in the feedback loop. On the other hand, the random distribution of the elements of the control loop affect the propagation properties of the communication channels or even block the communication links which cause packet loss in transit through the communication network, the long delay in transmission caused by node competition also lead to a packet loss this means that the reliable transmission protocols such as TCP are not always appropriate for NCSs since data that are re-transmitted are outdated and may not be useful. Thus, it is fundamentally important to investigate the effect of the network that closes the control loop in NCSs. The communication networks can be also categorized based on the medium of transport data into two groups: wired and wireless networks where wired network offers many advantages like high reliability, high bandwidth, and high security whereas it decreases flexibility and mobility of networks.
On the other hand, wireless networks make remarkable flexibility and mobility in networks. However, the limited bandwidth and reduced reliability are the disadvantages of wireless networks.

2.3 Architectures of networked control systems

The NCSs architecture is a distribution method of the closed-loop components in the industrial area and it is divided into two types. The direct structure and hierarchical structure, where the hierarchical form is a hybrid system and can be used to study the interconnection of different plants, whereas the direct form is a standalone control application.

2.3.1 Direct structure : The direct structure is shown in Fig.2.1, where it compose of one node includes the controller, sensor, and actuator (the actuator includes the ZOH). Each component of the node uses a shared access link to form a closed-loop feedback control system. It must mention that the network has other nodes that have the same structure shares the same communication channel and the network use buffers and several protocols to organize the data flow to avoid congestion, long delay, and loss of information. The controllers and actuators were chosen generally as event-driven mode and the sensors as a time-driven mode.

The NCSs single loop shown in the figure Fig.2.1 is sufficient to study the effect of sampling, delays, and packet dropout in NCSs as it captures the important features. Three

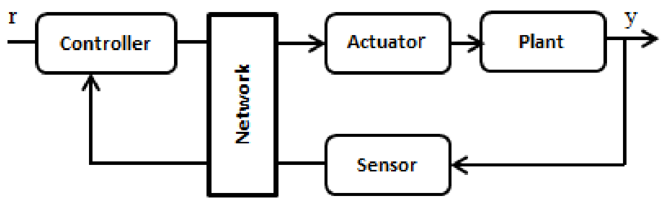

Figure 2.1: Direct structure of an NCSs.

different control architectures are covered by the single feedback loop depending on the presence and absence of delays and packet dropouts of different channels.

2.3.2 Hierarchical structure

: On the other hand, the NCSs hierarchical structure is shown in Fig.2.2, the network connects the distributed sub-systems. Each sub-system is a complete closed-loop control system including the self-contained controller, sensors, and actuators. The main controller computes and sends the reference signal in a frame or a packet via a network to the remote system and the remote system then processes the reference signal to perform local closed-loop control and returns to the sensor measurement to the main controller for networked closed-loop control.

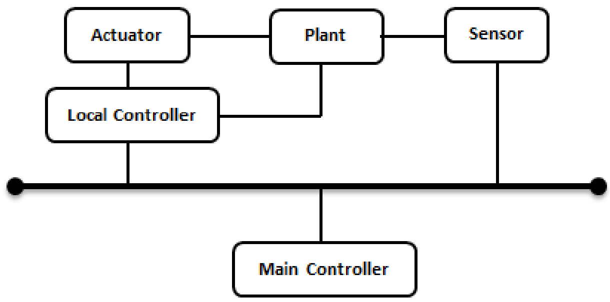

Figure 2.2: Hierarchical structure of an NCSs.

2.4 Topologies of networked control systems

NCSs Topology is the schematic description of the arrangement of the various nodes (sensors, controllers, and actuators) in the control loop closed through a communication network in addition to how data flows between the devices of the control loop connected between each other via this network where the first is called physical NCSs topology and the second called logical topology of NCSs.

In identical networks with have the same topology, the physical interconnections, signal types, transmission rates, or distances between nodes may differ between them, wherein practice, there are about five different topologies among them:

2.4.1 Star topology

: A star topology is a topology where a central node links all different nodes in the networked closed-loop which every component in the control loop is directly connected to this central node and be indirectly connected to every other node. The central node has the responsibility of managing data transmissions across the network between all components of the control loop.Fig.2.3 show the type of star topology.

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Star topologies are very used to link the components of networked systems where it can

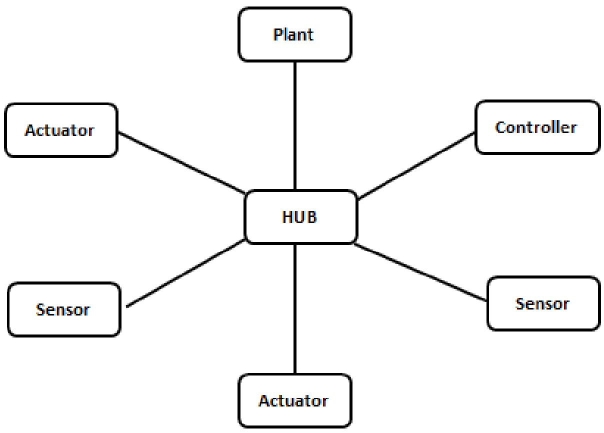

Figure 2.3: Star Topology of an NCSs.

manage the entire network from one location and If an extreme node goes down then the networked closed-loop will remain work, whereas the networked closed-loop fail when the central node fails.

In physical structure, star topologies require fewer cables compared with other topology types, which makes them simple to set and manage over the long-term. As a consequence, performance faults are easier to detect and correct but on another hand they are not cheap and hard to set up and use.

2.4.2 Bus topology : Another type of network topology is the bus topology or often referred to as line topology (Fig.2.4) in which every component of closed-loop is connected to a single cable, where the data is transmitted in a single direction between elements of the control loop. If the bus topology has two endpoints then it is referred to as a linear bus topology.

Bus topologies are generally used in the not complicated networked control loop, this

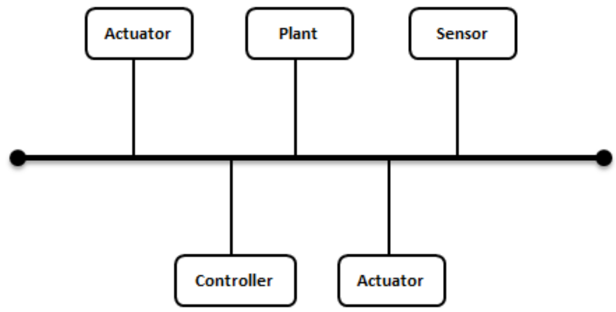

Figure 2.4: Bus Topology of an NCSs.

leads to keep the layout simple, which all devices are connected to a single cable therefore not need to manage a complex topological setup or expensive cost.

However, the reliance on one cable does mean that bus topologies have a single failure point which is if the common cable fails this leads to the entire networked control loop goes down. And if you are use lots of components the performance of the control loop will decrease significantly as all the data will be traveling through one cable. As a consequence, the transmission speeds are going to be slower between the nodes.

2.4.3 Ring topology : In Ring topology shown in Fig.2.5, the components of the networked control loop are connected to each other in a circular format, which causes that the packets need to travel through all nodes on the way to their destination, and you must choose one node to configure the network and monitor the other devices. Ring topology is either half-duplex or full-duplex but when you made full-duplex you would need to have two links between network nodes to form a Dual Ring Topology. Ring topologies can support large networks much more effectively than bus and star topologies, and data can flow in a clockwise or counterclockwise direction. The configuration of ring topology is based mainly on the choice of one node as a monitor node that performs the operations between the entire networked control loop. In Ring topology, the station which needs to

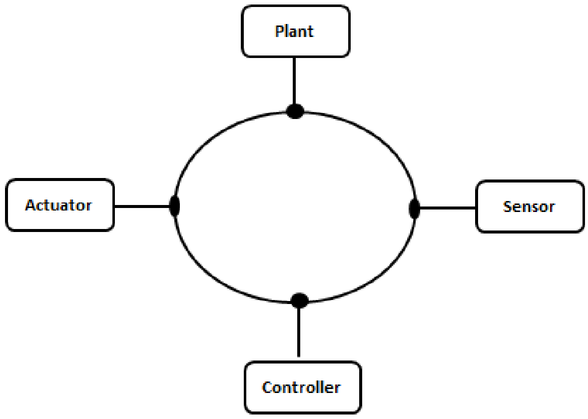

Figure 2.5: Ring Topology of an NCSs.

transmit the data have to hold the token and after the finish of the transmission is done the token is to be released for other stations to use. When no node is transmitting the signal, then the token will circulate in the ring where there are two types of token release techniques: One is *Delay Token Release* which releases the token after the acknowledgment is received from the receiver node, the other is *Early Token Release* which releases the token just after the transmitting the signal.

The risk of packet collisions in a Ring topology is very low due to the use of the protocols of token-based, which is allowed for only one station to transmit data at a given time with high speeds, on the other hand, the failure of one node can lead to a complete networked control loop failure, this implies that it is necessary to ensure good health to all nodes in the networked closed-loop and the transmission lines constantly.

If there is a need to perform maintenance for one of the nodes, it must turn off the entire control loop. This is not ideal as you'll need to factor in downtime every time you want to make a change to the topological structure.

2.4.4 Hybrid topology

: A hybrid topology is a topology that consists of several different topologies as this new topology inherits all the advantages and disadvantages of the topologies that form it. Fig.2.6 show the scheme of hybrid topology.

One of the most important reasons for using hybrid topologies is flexibility where it can incorporate several topologies as buses, rings, and star topologies into one hybrid setup. So, hybrid topology has few constraints on the structure which makes it scalability and well-suited to larger networks. On the other hand, hybrid topologies can be quite complex

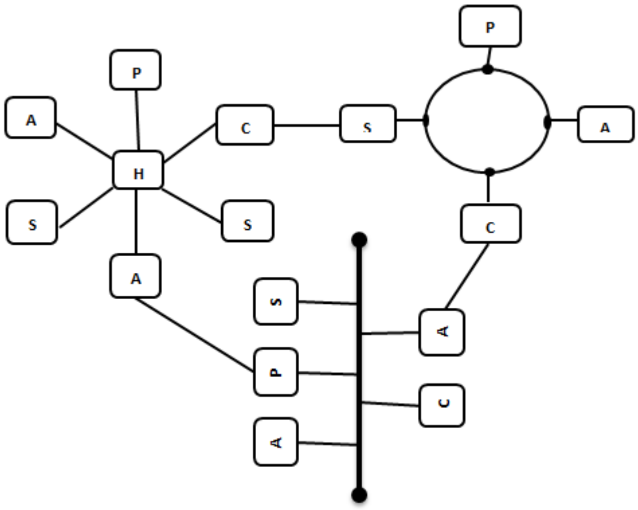

Figure 2.6: Hybrid Topology of an NCSs.

and depending on the topologies that you decide to use which makes the engineering more difficult and the installation can be quite costly.

2.4.5 Tree topology of an NCSs :

Tree topology is a network structure that has a root node that is connected hierarchy with other nodes, where it has three levels of the hierarchy. Tree topology is used within a large control loop to guarantee lots of spread-out devices. Fig.2.7 show the scheme of tree topology.

Tree topologies are used when a bus, star, or ring topologies can't satisfied the requirement of the large networked control loop wherein this topology can easily add more nodes to the network when need to extend the size of the networked control loop. The scheme of tree topology allows us to find errors and check the performance issues systematically, where if the root node fails then all of its sub-trees become partitioned but it stays partially

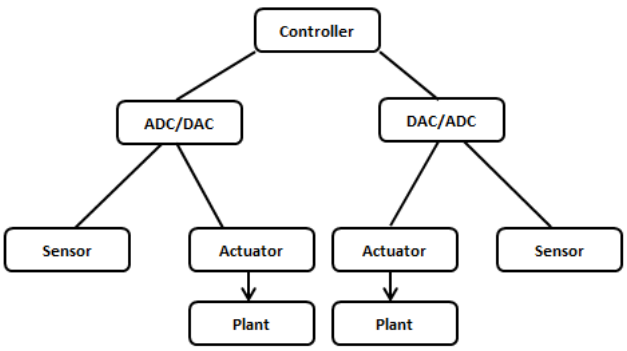

Figure 2.7: Tree Topology of an NCSs.

connected with the other components in the networked control loop. Another problem of the tree topology is the number of cables you required to connect every device throughout the hierarchy which makes the layout more complex when compared to the simple topologies.

2.5 Modeling of networked control systems (NCSs)

In this section, the modeling procedure of networked control systems (NCSs) is presented, where there is an extensive literature is available in the modeling of NCSs, including the different NCSs imperfections such, time-varying delays, time-varying sampling intervals, and packet dropouts, modeling NCSs as discrete-time and continuous-time NCSs models, linear and non-linear NCSs model, uncertain and disturbed NCSs model.

2.5.1 The continuous-time NCSs models :

As shown in Fig. (1.1), the components of NCSs are connected via a communication network, where notes that the plant is a continuous-time system and the controller is in a discrete-time system manner (engendering computer) where it generates a discrete-control signal in its output, this discrete control signal is transmitted to the plant but before that, it must be reconstructed this discrete-time output of the controller to a continuous signal form by zero-order hold (ZOH) where this latter held the control signal constant until the next sampling instant.
Therefore and based on what we mentioned, it can describe the NCSs model as a linear continuous-time model by the following closed-loop linear continuous state-space model:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t) + Du(t). \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector; $u(t) \in \mathbb{R}^m$ is the input vector; $y(t) \in \mathbb{R}^q$ is the output vector; A, B, C and D are real constant matrices with appropriate dimensions.
As we mentioned at the top of this section that the components of NCSs are interconnected with communication network and this connection produces new imperfections and constraints that have to be considered in modeling the complete networked system.
Therefore, it is important before begin with the modeling procedure, discuss the modeling of those imperfections and constraints in NCSs where it can be classified into five types: network-induced delays, packet dropouts, quantization errors, variable sampling/Transmission Intervals, bandwidth Limitation, and other communication constraints.

At first, we present the modeling of different network-induced imperfections, including the constant, the random, and stochastic ones.

2.5.1.1 Network-induced delay : Network-induced delay is the most important imperfection produced by the network control systems which change randomly as observed by numerous researchers and from Fig. (2.1) we notice that there are two kinds of time delays, the first is a delay from sensor-to-controller which represents the time between sampling the signal from sensors and receiving it by the controller, the second is a delay from controller-to-actuator, which represents the time between generating the control signal and its availability on the actuator which, the sources of these delays is generally a result of limited data bandwidth, network traffic, and network protocols [120]. However, these two kinds of delays are not treated separately in most cases and only the round trip delay is considered. In practice, there are different types of communication networks being used in NCSs which leads to varying in characteristics of the network-induced delay vary as follows; bounded constant delays in cyclic service networks (e.g., Toking-Ring, Toking-Bus) [8]; Random and unbounded delays in random access networks (e.g., Ethernet, CAN) [70]; unbounded delays for those with lower priority and bounded delays for the data packets with higher priority in priority order networks (e.g., DeviceNet) [69].
The networked-induced delays can be obtained by using the time-stamp technique and have been modeled as constant delays, random but bounded delays, independent stochastic delay, and random delays governed by a Markov process. The modeling of NCSs with constant time delay has presented in many existing works where delay normally equal to the maximum network-induced delay in the system (sensor to controller delay or controller to actuator delay) and it can be achieved by introducing a buffer longer than the worst-case delay [81]. Furthermore, the assumption of constant delays produces a good model of NCSs only when the introduced delays are much shorter than the processing time constant but, the study of the case where the sampling period is longer than delays is more complicated. And hence, the NCSs can be treated as a deterministic system that can apply many deterministic control methods on this NCSs. The closed-loop on NCSs with constant delay is represented as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t-\tau), \\ y(t) = Cx(t) + DKx(t-\tau), \\ x(t) = \phi(t), \forall t \in [0, \tau]. \end{cases} \tag{2.2}$$

where $\tau(t) = \tau_{sc} + \tau_{ca} + \tau_c$ with τ is the total network-induced delay, τ_{sc}, τ_{ca} and τ_c are the sensor-to-controller, controller-to-actuator and the processing delays, respectively.
The equation $|sI - A - BKe^{-\tau s}| = 0$. is used to return to the transfer function of the system, this equation is transcendental, which makes it very difficult to solve. Therefore, it would

be convenient to get rid of the delay in the characteristic equation, so that conventional analysis and design techniques could be applied.

2.5.1.1.1 Mutually stochastic delay model : In NCSs, The network-induced delay is affected by many stochastic factors such as network load, nodes competition, and network congestion hence, delay tends normally to be stochastic. so, the constant delay and the corresponding deterministic control methodologies could hardly satisfy the performance requirement of the system. The stochastic delay model can be divided into two categories: the model in which delays are probabilistically dependent and the model in which delays are mutually independent. The mutually independent stochastic delay model is often applied to the modeling and control of NCSs with random delays when the probabilistic dependence is unknown[26].
It can model the stochastic network-induced delay with the help of signal distribution using a Bernoulli process where it describes as follows:
The stochastic network-induced delay is divided into two delays and its interval into to sub-interval:

$$\tau(t) = \tau_1(t) + \tau_2(t). \tag{2.3}$$

where:

$$\begin{cases} \tau_1(t) = \sigma(t)\tau(t), \\ \tau_2(t) = (1 - \sigma(t))\tau(t). \end{cases} \tag{2.4}$$

with $\tau(t) \in [\tau_{min}, \tau_{max}]$ and $(\tau_{max} \geq \tau_{min} > 0)$, and where τ_{max} and τ_{min} is a given the upper and the lower of the network-induce delay, respectively, and $\sigma(t)$ is Bernoulli process describe the delay distribution describe as:

$$\sigma(t) = \begin{cases} 1 \text{ if } t \in \Omega_1 = \{t : \tau(t) \in [\tau_{min}, \tau_{med})\}, \\ 0 \text{ if } t \in \Omega_2 = \{t : \tau(t) \in [\tau_{med}, \tau_{max}]\}. \end{cases} \tag{2.5}$$

with $\tau_{med} \in [\tau_{min}, \tau_{max}]$ a parameter to be chosen in order to take benefit of distribution of the delay. From (2.5), we can notice that the network-induced delay $\tau(t)$ changes randomly and the probability of $\tau(t) \in \Omega_1$ and $\tau(t) \in \Omega_2$ can be known. Moreover, it is straightforward that $\Omega_1 \cup \Omega_2 = [\tau_{min}, \tau_{max}]$ and $\Omega_1 \cap \Omega_2 = 0$. Therefore, he closed-loop dynamics expressed as:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)BKx\,(t-\tau_1(t)) + (1 - \sigma(t))\,BKx\,(t-\tau_2(t)). \\ y(t) = Cx(t) + \sigma(t)DKx\,(t-\tau_1(t)) + (1 - \sigma(t))\,DKx\,(t-\tau_2(t)). \\ x(t) = \phi(t), \forall t \in [-\tau_{max}, -\tau_{min}]. \end{cases} \tag{2.6}$$

2.5.1.1.2 Markov chain model : Sometimes, stochastic delays are not always mutually independent such, there are some probabilistic dependency relationships among the delays, such as Markov chain. According to the relation between the current time delay and previous delays in the real network environment, reasonably modeling the stochastic network-induced delays as Markov chains and the delay could be modeled in two ways, one chain to represent the sum of two delays (full round trip delay); two chains to represent the two delays separately, one for the sensor-to-controller delay and one for controller-to- actuator delay.

Markov chain model for stochastic time delays provides another advantage that the packet dropouts can be included naturally.

Markov chain is described as follows: There is a set of $S = \{s_1, s_2, ..., s_r\}$, the process

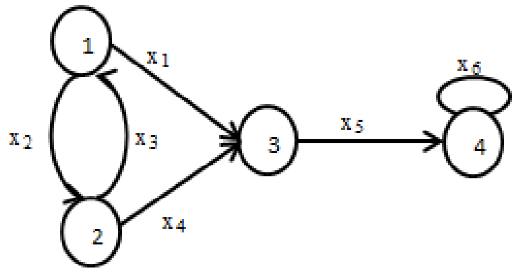

Figure 2.8: Markov chain model.

starts in one of these states and moves successively from one state to another, each move is named step. For clarification If the chain is currently in state s_i, then it moves to state s_j at the next step with a probability denoted by p_{ij}, and this probability does not depend upon which states the chain was in before the current state. An initial probability distribution (defined on S) specifies the starting state. Usually, this is done by specifying a particular state as the starting state.

2.5.1.1.3 Hidden Markov model : The stochastic factors induce by the communication network (network congestion, load in the network, and the competition of nodes) can be grouped into a hidden variable and defined it as a network state governed the distribution in delays; this network state could not be directly observed, only rather it estimated by observing network delays and use a hidden Markov model to represent the relation between it and the network-induced delay.

2.5.1.2 Packet dropouts : Due to the use of the network to connect the components of NCSs, It is required before sending the signals of the systems to be sampled and grouped, each group called a “packet”, the size of the packet depends on the network used. The transmission of packets could be either single or multiple, the sending of signals in single packet transmission is being when all data from sensors or controllers are sampled and grouped together. In a second way, the data are transmitted in several packets which causing non-simultaneous arriving of data to the actuators or controllers due to limited size of the network and the distribution of sensors and actuators practically over a large area

which make it difficult to lump data into one network packet.
When there are packets collisions, network congestion or buffer overflows the packet dropout occurs and creates another critical problem for NCSs, leads this dropout in packets a degradation in system performance or even makes unstable. To this end, researchers take into account the problem of packet loss in the design of the networked controller for NCSs, by design a controller to withstand the upper bound of dropouts in the system where they consider the packet dropout as a random process and model it as a Bernoulli distribution or a Markov process or they considered as kind of time delay because if packet dropout occur the receiver (actuator or controller) wait time for receiving another packet (the lost packet has expired) this waited time is modeled as .

$$\tau_p(t) = (\eta(t) + 1) * h. \tag{2.7}$$

where $N \in \mathbb{N}$ is the number of packets loss and h is the sampling time, and the number of packet loss can be denoted by

$$\eta(t) = \frac{t_{k+1} - t_k}{h} - 1. \tag{2.8}$$

where $[t_1, t_2, \ldots, t_k, t_{k+1}]$ are the sampling instants of data transmitted from the sensor to the ZOH.
According to the previous model of packet dropout, it can lumped the network-induced delay and the packet dropout as one input delay system as

$$\tau(t) = \tau_d(t) + \tau_p(t). \tag{2.9}$$

And the closed-loop system formulated as an input delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ y(t) = Cx(t) + DKx(t - \tau(t)), \\ x(t) = \phi(t), \forall t \in [\underline{\tau}, \bar{\tau}]. \end{cases} \tag{2.10}$$

with $\underline{\tau} = \inf_{k \in \mathbb{N}}(\tau_k) = \tau_{min}$ and $\bar{\tau} = \sup_{k \in \mathbb{N}}(\tau_{k+1}) = \tau_{max} + (\bar{\eta} + 1)h$ and $\bar{\eta}$ is the maximum number of consecutive packet dropout.
In some reliable transmission protocols (eg., TCP), the lost packet is re-transmitted until correct data are received. However, this solution not very useful for NCSs. thus, the packet dropout still an important issue in NCSs design.

2.5.1.3 Quantization : In information theory, Quantization is a phenomenon occurring in all data networks, and is considered as information encoders and thus as an integral part of the whole system where a real-valued signal is transformed into a piece-wise constant signal. By definition, a quantizer is a function that maps a real-valued function into a piece-wise constant function taking on a finite set of values. In the literature, there are two common types of quantization and they are the uniform quantizer and logarithmic ones, for the first kind maps the real-valued function to a finite number of quantization regions with the rectilinear shape []or arbitrary shape. The study of a system affected by uniform quantizer is always basing on a "zoom" strategy, which is usually composed of two stages, i.e. "zooming-out" and "zooming-in". In the first stage, the range of quantizer is increased to guarantee the states of a system can be adequately measured. In a second stage, the quantization error is decreased to drive the states to the origin. When a system is affected by logarithmic quantizer, in which the quantization levels are linear in logarithmic scale, the simple classical approach to analysis and mitigation of quantization effects is to treat the quantization error as uncertainty or non-linearity and bound it using a sector bound. The logarithmic quantizer is expressed as follows

$$q_L(v) = \begin{cases} sgn(v)\mathcal{V}_i & \text{if } \frac{\mathcal{V}_i}{1+\Delta} < |v| < \frac{\mathcal{V}_i}{1-\Delta}, \\ 0 & \text{if } v = 0. \end{cases} \tag{2.11}$$

where $\mathcal{V}_i = \rho^i \mathcal{V}_0,\ i = \pm 1, \pm 2, \pm 3, \ldots,\ 0 < \rho < 1$, $\mathcal{V}_0$ is a positive scaling constant, $\Delta = \frac{1-\rho}{1+\rho}$ and $sgn(.)$ is a sign function satisfying

$$sgn(v) = \begin{cases} 1 & \text{if } v > 0, \\ -1 & \text{if } v < 0, \\ 0 & \text{if } v = 0. \end{cases} \tag{2.12}$$

The corresponding quantization error satisfies:$q_L(v) = \delta v$. where $\delta \in \begin{bmatrix} -\Delta & \Delta \end{bmatrix}$.
Logarithmic quantizer provide a sector bound method to deal with the quantization errors, and have been widely used in various control and filtering problems.
The other typical quantizer is the uniform quantizer, which is described as follows

$$q_U(v) = \begin{cases} sgn(v)\frac{\lfloor 2^{N-1}v\rfloor + 0.5}{2^{N-1}} & \text{if } |v| < \bar{v}, \\ sgn(v)(1 - \frac{0.5}{2^{N-1}})\bar{v} & \text{if } |v| = \bar{v}. \end{cases} \tag{2.13}$$

where $\bar{v} > 0$ is the given constant quantizer limitation, N is the number of given quantization bits and $\lfloor x\rfloor = \{max\, z \in \mathbf{N}, z \leq x\}$.
The corresponding quantization error is obtained as $|q_U(v) - v| \leq \frac{\bar{v}}{2^{N-1}}$.
The main problem with quantization is to find a quantified feedback control law to stabilize the given system which can be stabilized by linear time-invariant feedback.

2.5.1.4 Variable sampling / Transmission intervals : Before grouped the signals of NCSs and transmitted them through the network it needs to be sampled, which in the conventional systems the sampling period are usually fixed due to their simplicity in design and analysis and called "periodic sampling", but in the recent NCSs, the sampling period is varying since there is awaiting in a queue before the transmission process which will be based on the availability of the network and the protocol used and called "Aperiodic sampling". It was proved recently that aperiodic sampling may have better performance than sampling at fixed intervals.

2.5.1.5 Bandwidth limitation : Any communication network has limited bandwidth which is the number of bits that can be transmitted per second, this limitation of the bandwidth depends not only on the physical bandwidth but also on the efficiency of encoding the data into packets, how efficiently the network operates in terms of inter-frame times (long or short), and whether network time is wasted due to message collisions. Which imposes significant constraints on the operation of NCSs.
For this purpose, the researchers made tremendous efforts in determining the minimum bit rate that is needed to achieve the control objectives, Among these research the involving of **Minimum bit rate** necessary to stabilize an LTI system has been derived, **Average bit rate** where is a measure of how infrequent feedback information is needed (in digital networks) to guarantee that the system remains stable, **Intermittent feedback** in which the open-loop is closed for certain fixed or time-varying periods, leading to opportunistic situations where the sensor sends bursts of information when the network is available, this helps in taxing the network less. **Quantified feedback** which is provided in the digital system implementation of NCSs, and if the open-loop system is unstable, only then can we determine the minimum average bit rate to process feedback information. Further research on communication constrained feedback channels is establishing a connection between stabilizability and an inequality relating feedback channel data to open-loop eigenvalues.

2.5.2 The discrete-time modeling approach

: The discrete-time system of networked control system equivalent to the continuous-time system in equation (2.1) is obtained by sampled periodically and assuming that the continuous-time control input is reconstructed via zero-order hold. The general solution of the system when the total time delay in the system τ is less than one sampling period h $(0 < \tau \geq h)$ is:

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-s)}Bu(s-\tau)ds. \tag{2.14}$$

When sampling time the system, the state of the plant is given as

$$\begin{aligned} x((k+1)h) &= e^{A_c h} + \int_{kh}^{kh+h} e^{A_c(kh+h-s)}B_c u(s-\tau)ds \\ &= e^{A_c h} + u(kh+h)\int_{kh}^{kh+\tau} e^{A_c(kh+h-s)}B_c ds + u(kh)\int_{kh=\tau}^{kh+h} e^{A_c(kh+h-s)}B_c ds. \end{aligned} \tag{2.15}$$

The control input is held constant typically over one sampling period but this fact is not true always due to the time delay.

Then, we define

$$\begin{aligned}
\Psi(\tau) &= e^{A_c h}, \\
\Gamma_0(\tau) &= \int_0^{h-\tau} e^{A_c s} ds B_c, \\
\Gamma_1(\tau) &= e^{A_c(h-\tau)} \int_0^{\tau} e^{A_c s} ds B_c.
\end{aligned} \tag{2.16}$$

The sampled version of (2.1) can be written as

$$x((k+1)h) = \Psi x(kh) + \Gamma_0 u(kh-\tau) + \Gamma_1 u(kh). \tag{2.17}$$

The state-space form of the sampled version of the system in Equation (2.1) is thus given as:

$$\begin{cases}
\begin{bmatrix} x((k+1)h) \\ u(kh) \end{bmatrix} = A_s(\tau) \begin{bmatrix} x(kh) \\ u(kh-h) \end{bmatrix} + B_s(\tau) u(kh), \\
y(kh) = C_s(\tau) \begin{bmatrix} x(kh) \\ u(kh-h) \end{bmatrix}.
\end{cases} \tag{2.18}$$

where

$$A_s = \begin{bmatrix} \Psi & \Gamma_1 \\ 0 & 0 \end{bmatrix}, B_s = \begin{bmatrix} \Gamma_0 \\ 0 \end{bmatrix}, C_s = \begin{bmatrix} C & 0 \end{bmatrix}.$$

The dependence of A_s, B_s, and C_s on τ is due to the dependence of Ψ, Γ_0, and Γ_1 as in (2.16).

In the second case when the time delay τ is longer than the sampling period, h, one may receive zero, one, or more than one control sample in a single sampling period. The analysis needs a little adaptation. Decompose τ, such that

$$\tau = (\sigma-1)h + \tau_0, \quad 0 < \tau_0 \leq h. \tag{2.19}$$

where σ is an integer. The analysis is modified to

$$x((k+1)h) = \Psi x(kh) + \Gamma_0 u(kh-(\sigma-1)h) + \Gamma_1 u(kh+\sigma h). \tag{2.20}$$

The corresponding state-space description is

$$\begin{bmatrix} x((k+1)h) \\ u(kh-(\sigma-1)h) \\ \vdots \\ u(kh-h) \\ u(kh) \end{bmatrix} = \begin{bmatrix} \Psi & \Gamma_1 & \Gamma_0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} x(kh) \\ u(kh-\sigma h) \\ \vdots \\ u(kh-2h) \\ u(kh-h) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix} u(kh). \tag{2.21}$$

In the following, we present the different control approaches of networked control systems and denoted the benefit of each one when used to analyze the stability and synthesis of a new control law for NCSs.

2.6 Control approaches of networked control system

As we all know that there are advantages and challenges of a system controlled over a network such as a scalability (adding sensors connected through the network at different locations), low maintenance cost, and so on. However, is difficult to find the NCSs controller makes the closed-loop stable and guarantee the system performance. for this fact, the researchers have given us precise and optimum control strategies emerging from classical control theory, starting from PID control, optimal control, adaptive control, robust control, intelligent control and many other advanced forms of these control algorithms. Applying all these control strategies over a network, however, becomes a challenging task where manifested in ensuring stability and good performance of systems. To deal with the stability problem of NCSs the research community develops several methods when considering one or more of the imperfections discussed in the previous section, where the main issue is to guarantee the stability or other performance objectives of NCSs subject to these imperfections. In this section, we discussion on each of these approaches which include the input delay approach, switched approach, Markovian approach, impulsive approach, stochastic approach ...etc.

2.6.1 Input delay system approach :

In input delay approach, the NCSs is modeled as a system with time-varying delay where the time-varying delay includes the delay from the sensor-to-controller, the delay from controller-to-actuator, the computation delay, and the packets dropout represented as a delay. This approach was developed also to solving the synchronization problem of the complex network by considering the signal sampling. The main and important issue in this approach is determining the maximum allowable upper bound (MAUB) of transmission delay which guarantees the stability and the good performance of NCSs. Also, the determination of maximum allowable upper bound of transmission delay are important factors for practical applications Therefore, the NCSs closed-loop was described by the following time-varying system with input delay:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ x(t) = \phi(t), \quad \tau(t) \in [-\tau_M, -\tau_m). \end{cases} \tag{2.22}$$

with

$$\begin{cases} 0 \le \tau_m \le \tau(t) \le \tau_M \text{ or } (\tau_M + (\eta + 1)h \text{ if packet dropout}), \\ \tau_m > 0, \tau_M > 0, \eta \ge 0. \end{cases} \tag{2.23}$$

where τ_m, τ_M, η are the upper value and lower value and number of packet loss, respectively.
A sufficient condition of stability of this NCSs is derived based on a proper Lyapunov function and presented using the LMI method in Chapter 5.

2.6.2 Stochastic system approach

: In general, the constraints of communication in NCSs such time delay and packet dropout are stochastic in nature, so the application of conventional stochastic control approaches to NCSs is possible by using Bernoulli distribution or Markov chain. This system model simulates the NCSs model in realities and the closed-loop of NCSs was described by the stochastic time distribution delay system with random delay using a Bernoulli distribution sequence or Markov chain. Firstly, the stochastic system using a Bernoulli distribution sequence where the closed-loop of NCSs was described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)\bar{B}Kx\,(t{-}\tau_1(t)) + (1 - \sigma(t))\,\bar{B}Kx\,(t{-}\tau_2(t))\,, \\ x(t) = \phi(t), \quad \tau(t) \in [-\tau_{max}, -\tau_{min})\,. \end{cases} \tag{2.24}$$

where $\sigma(t)$, is describe in (2.5) and

$$\begin{cases} 0 < \tau_{min} \le \tau_{med} \le \tau_{max}, \\ \tau_{min} > 0, \tau_{med} > 0, \tau_{max} > 0. \end{cases} \tag{2.25}$$

with τ_{med} a parameter to be chosen in order to take benefit of distribution of the delay and the network-induced delay $\tau(t)$ changes randomly along the interval $[\tau_{min}, \tau_{max}]$ and its probability is be known. A sufficient condition of stability and controller design of this model of NCSs is derived based on a Layapunov Krasovskii functional and formulated as LMI conditions in Chapter 4.

2.6.3 Markovian system approach

: The NCSs modeled as Markovian jump systems generally consist of a finite number of subsystems and a jumping law governing the active or deactivate mode switches among these subsystems. These subsystems are typically modeled as differential equations, and the jumping law is a Markov chain. Markovian jump systems are a powerful modeling tool in Networked control systems. The NCSs can be modeled as having different dynamics at the abrupt changes, and the changes are typically memoryless, thus resulting in a Markovian jump system. The NCSs as Markovian jump linear systems are expressed as follow:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))u(t), \\ y(t) = C(r(t))x(t) + D(r(t))u(t). \end{cases} \tag{2.26}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^p$ are the state, input, and output, respectively. The parameter $r(t)$ denotes a continuous-time Markov process on a probability space, which takes values in a finite set $S := \{1, 2, \ldots, N\}$ with transition probabilities given by

$$Pr((t+\delta) = j|r(t) = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & if\ i \neq j \\ 1 + \pi_{ii}\sigma + o(\delta). & if\ i = j \end{cases} \tag{2.27}$$

where $\delta > 0$, and π_{ij} denotes the transition probability rate from mode i to mode j when $i \neq j$. Furthermore, for all $i \in S$, π_{ij} satisfies $\pi_{ij} > 0, (i \neq j)$ and $\pi_{ij} = -\sum_{j \in S, i \neq} \pi_{ij}$.
The Markov process $\{r(t), t > 0\}$ is assumed to have an initial process $r(0) = (\mu_1, \mu_2, \ldots, \mu_n)$. The matrices $A(r(t))$, $B(r(t))$, $C(r(t))$ and $D(r(t))$ are contained in $\{A_1, A_2, \ldots, A_N\}$, $\{B_1, B_2, \ldots, B_N\}$, $\{C_1, C_2, \ldots, C_N\}$ and $\{D_1, D_2, \ldots, D_N\}$, respectively, and if $r(t) = i \in S$, we have $A(r(t)) = A_i$, $B(r(t)) = B_i$, $C(r(t)) = C_i$, and , $D(r(t)) = D_i$. When studying the NCSs as a markovian jump system, generally the jumping parameters are often assumed to be precisely known, this assumption is usually incorrect in practice, while the system states can often be observed. So, the *Wonham filter* is used to estimate the jumping parameters using the given system matrices. On a parallel line, in analog systems, imprecise measurements are often present and the quantization error sometimes can not be ignored which makes the implementation of control almost impossible. In addition to that and which make it worse, the poor stability margins of systems will be present if not implemented the robust control such as common techniques H_∞, L_2 or μ synthesis, etc.

2.6.4 Switched system approach :

A switched linear system is a hybrid system that consists of several linear subsystems and a rule that orchestrates the switching among them. Switched linear systems provide a framework that bridges the linear systems and the complex and/or uncertain systems. On one hand, switching among linear systems may produce complex system behaviors such as chaos and multiple limit cycles. On the other hand, switched linear systems are relatively easy to handle as many powerful tools from the linear and multilinear analysis is available to cope with these systems. Moreover, the study of switched linear systems provides additional insights into some long-standing and sophisticated problems, such as intelligent control, adaptive control, and robust analysis and control. In a switched system approach, the NCSs is represented as a switched system that is typically used by modeling different network conditions in NCSs as different system modes with a finite number of subsystems. This approach is able to easily deal with network constraints such as transmission delay and packet loss. The use of this approach is difficult and the difficulty to use it grounded on how well we

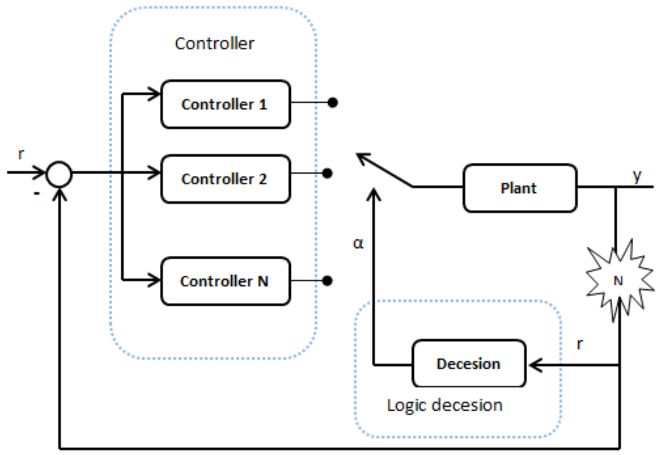

Figure 2.9: Switched System Architecture.

understand the properties of the changes in the network conditions. The continuous-time switched linear networked control system represented as following:

$$\dot{x}(t) = Ax(t) + BK_{\sigma}x(t - \tau_{\sigma}) + E\omega(t). \tag{2.28}$$

initialized at $x(t_0) = x_0$ with input u and switching signal σ. Suppose that the switching signal is well-defined and its switching sequence is

$$\{x_0, (t_0, i_0), (t_1, i_1), \dots, (t_l, i_l)\}.$$

As the i_0 the subsystem is active during $[t_0, t_1)$, we have

$$\dot{x}(t) = Ax(t) + BK_i x(t - \tau_i) + E\omega(t), \quad x(t_0) = x_0 \;\; t \in [t_0, t_1). \tag{2.29}$$

Definition: The system (2.28) is exponentially stable under the switching signal σ if there exist positive constants γ andα such that the solution of $x(t)$ of the system (2.28) satisfies

$$||x(t)|| \leq \gamma ||x(t_0)||_e e^{-\alpha(t-t_0)}, t \geq t_0.$$

where

$$||x(t_0)||_e \triangleq sup_{-h_{M+1} \leq \theta \leq 0}\{||x(t+\theta)||, ||\dot{x}(t+\theta)||\}.$$

For a switched linear NCS, the system permits a unique solution for the forward time-space, hence, any discrete-time switched NCS is well-posed.
The state transition matrix is multiple multiplications of matrices. Accordingly, the properties of matrix multiplication play an important role in analyzing the switched NCS.
Finally, for both continuous-time and discrete-time switched linear NCS, the state trajectory possesses several nice properties under mild conditions.

2.6.5 Impulsive system approach

: Between the most important kind to represent the NCSs model is linear impulsive systems where the reason is that the linear impulsive systems are a class of hybrid systems where the state propagates according to linear continuous-time dynamics except for a countable set of times at which the state can change instantaneously, as is the case in NCSs where the plant is a linear continuous-time system and the controller is a linear discrete-time system. In impulsive systems, the impulsive effects can be event-driven and/or time-driven.
The linear impulsive systems can be represented as:

$$\begin{cases} \dot{x}(t) = A_C x(t) + B_C u(t) + E_C \omega(t), \quad t \in R\,\mathcal{T}, \quad x(t_0) = x_0, \\ x(t_k) = A_I x(t_k{}^-) + B_I u[k] + E_I \omega[k], \quad t \in Z^+ \\ Z(t) = C_C x(t), \\ y(t_k) = C_I x(t_k{}^-). \end{cases} \tag{2.30}$$

Where $x(t)$ is the continuous-time state, $u(t)$ is a continuous-time control input, $\omega(t)$ is a continuous-time exogenous input, $u[k]$ is a discrete-time control input, $w[k]$ is a discrete-time exogenous input, $z(t)$ is a continuous-time output to be controlled, and $y[k]$ is a discrete-time measurement, $\mathcal{T} = \{\tau_1, \tau_2, \dots\}$ is a set of impulse times assumed to contain a finite number of elements on any finite time interval. The state space for (2.30) is denoted by $\mathcal{X}$. For a function of time $f(\Delta)$ that is continuous from the right but possibly discontinuous at some t,$f(t^-)$ denotes $\lim\limits_{\varepsilon \to 0} f(t - \varepsilon)$, assuming that the limit exists. To distinguish continuous-time signals from discrete-time signals, using the parentheses surrounding a real-time argument to identify the quantities that evolve in continuous time, and the square brackets surrounding an integer index to represent the quantities that are available or computed only at the impulse instants.
In the event-driven case, we can represent the impulsive system with the same minor or formulate but occur at a countable number of events when the state vector crosses a given hyper-surface of the state space. In such a case, impulses can still be indexed by time, and the set of impulse times can be expressed as $\{\tau_i(x) | i \in N\}$ [140], [141].

2.7 Networked control systems in Real-Time (Protocol used in the implementation of NCSs)

The real-time network can be categorized into two groups. Data networks and control networks where the data networks are designed for communication networks to send large data packets with high throughput. On the other hand, control networks are designed where a large amount of small packets have to be transmitted at a high frequency between a large number of nodes and meet the real-time constraints. The main difference between the two networks is the capability of control networks to meet real-time constraints. Based on the International Organization for Standardization (OSI), the network is described in seven layers, Each layer has its own protocols which it is carefully designed. With this, developers can focus on the design of one layer and still the overall network keeps working because of the fixed interface definitions between the individual layers. Since only one node in NCSs can send a message over the shared network at any given time and as several nodes are connected to the network, access to the network has to be controlled. In the Data Link layer, three main types of medium access control are used in control networks: random access with prioritization for collision avoidance (e.g., Controller Area Network (CAN)), random access with re-transmission when collisions occur (e.g., Ethernet and most wireless mechanisms) and time-division multiplexing (such as master-slave or token-passing). Within each of these three categories, there are Media Access Control (MAC) protocols that have been defined and used which, plays an important role in determining the characteristics of delays and packet dropouts. It is therefore important to also consider the MAC protocol in the analysis of an NCSs. Common examples of the MAC protocols are the Time Division Multiple Access (TDMA) protocol, the Carrier Sense Multiple Access (CSMA) protocol, and the Token passing protocol.

2.7.1 Ethernet (CSMA) :

The Ethernet is well known as cheap and easy in installation and is wide usage in industrial networks. the main disadvantage of Ethernet is the collision between the data of nodes when the transmission where, If two nodes try to transmit simultaneously, the collision appears and both nodes wait for a random time to re-transmission. Generally, the Ethernet uses the carrier sense multiple access (CSMA) with CD or CA mechanisms to avoid the problem of a collision during the communication. In the industrial installation, there is three types of Ethernet: the hub-based Ethernet, the switched Ethernet, and the wireless Ethernet, where the first type is widely implemented and used in office environments and the second type is used in manufacturing and control environments.

2.7.1.1 Hub-based Ethernet(CSMA/CD) :

In Hub-based Ethernet, the devices of the network are interconnected using a hub(s) interface where the hub role is to broadcast the packets that come to it to all other hub interfaces. Hence, all of the devices on the same network receive the same packet simultaneously, wich causing a collision. Hub-based Ethernet protocol operates as follows: the node she wants to transmit a message listening to the network to know if the network is busy or idle. When the network is busy the node waits until the network is idle, otherwise, it transmits the message immediately. If there is more than one node listening to an idle network and decided to send its messages simultaneously, the messages of these transmitting nodes collide and the messages are corrupted. The nodes which transmit the message need to detect the message collision while transmitting. When a collision between two or more messages is detected, transmitting node stops send and waits a random length of time to retry its transmission.this

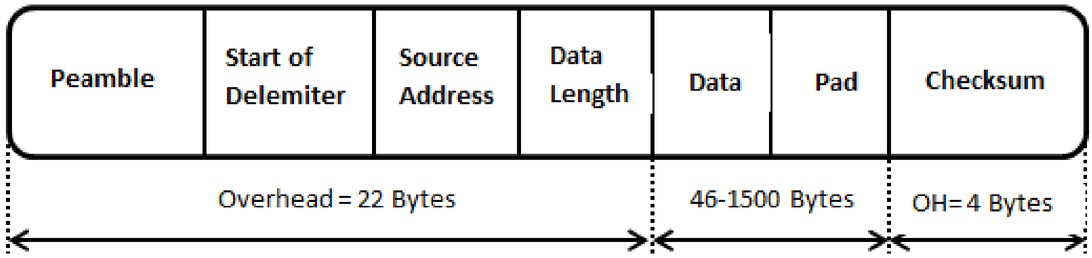

Figure 2.10: Format of Ethernet (CSMA/CD) Frame.

steps are repeated until the node transmitting the message successfully or sends a failure report back to the node microprocessor after 16 attempts. This random time is determined by the standard binary exponential back-off (BEB) algorithm where the re-transmission time is randomly chosen between 0 and (2^{i-1}) slot times, where i denotes the *ith* collision event detected by the node, and one slot time is the minimum time needed for the round-trip transmission. However, when the node is arrived to detect 10 collisions, the interval is fixed at a maximum of 1023 slots. After 16 collisions, the node stops attempting to transmit and report failure back to the node microprocessor [106].

Fig,2.10 show the Ethernet frame format where the total overhead is 26 bytes. The data packet frame size is between 46 and 1500 bytes where the standard states that valid frames must be nonzero and its length is at least 64 bytes long, from destination address to checksum (72 bytes including preamble and start of delimiter). If the data portion of a frame is less than 46 bytes, the pad field is used to fill out the frame to the minimum size. The reasons for this limitation return to:
First, when transceiver detects a collision, it truncates the current frame, which is mean that the stray bits and the pieces of frames frequently appear on the cable, this lead to facilitate to distinguish valid frames from "garbage." Second, it prevents a node from completing the transmission of a short frame before the first bit has reached the far end of the cable, where it may collide with another frame. For a $10 - Mbps$ Ethernet with a maximum length of $2500m$ and four repeaters, the minimum allowed frame time or slot time is 51.2 us(micro-second), which is the time required to transmit 64 bytes at 10 Mbps [106].
According to the low medium access overhead, Ethernet uses a simple algorithm to operate the network which leads to no delay at low network loads. What also distinguishes the Ethernet is no communication bandwidth is used to gain access to the network compared with other protocols (bus or token ring protocol) In the Ethernet used as a network to close the control loop commonly uses the 10 Mbps standard (e.g., Modbus/TCP).
Moreover, Ethernet has many disadvantages where it is a non-deterministic protocol and not support the prioritization at the transmission of messages, Also do not forget the main problem that is the message collisions caused by the greatly affect data throughput, which

greatly affects the time delay to become unlimited. In Ethernet, the node transmitting the message transmits packets exclusively for a long time despite other nodes waiting for transmitting its messages, which cause unfairness and substantial performance degradation [94].

According to the BEB algorithm, the message to be sent may be ignored after a series of collisions, which means that end-to-end communication is not guaranteed. Because of the required minimum valid frame size, Ethernet uses a large message size to transmit a small amount of data. However, to use this form of Ethernet protocol, researchers proposed several solutions, for example, every message could be time-stamped before it is sent, but this solution cause requires of clock synchronization which is not easy to accomplish. Although, this can be achieved at the expense of inferior performance to CSMA/CD at low to moderate channel utilization in terms of delay throughput [40]. Among the solutions also try to prioritize CSMA/CD (e.g., LonWorks) to improve the response time of critical packets [75].

2.7.1.2 Switched Ethernet (CSMA/CA) : The switched Ethernet protocol format utilizes switches device to connect the components of the Networked control system this leads to avoiding collisions, improving determinism, increasing network efficiency. The main difference between switch Ethernet and hub Ethernet protocol is the intelligence of forwarding packets, where the hub(s), transfer the incoming traffic from any port to all other ports, Although there are ports not concerned with this message. thus, the collision appears, whereas switches learn the topology of the network and forward packets to the destination port only, this means that the message which arrived at the switch, it forwards to the destination component only. for example, In a network star topology, every node is connected with a single cable to the switch. Thus, collisions can no longer occur on any network cable.

To forward packets from one port to another, switches use the cut-through or store-and-forward technique, where it utilizes the per-port buffers to do the operation, this implies that the packets wait a short time before being sent to its concerned destination. Switches with cut-through technique read first the MAC address and then transfer the packet to its destination according to this MAC address and the forwarding table found on the switch. Whereas, the store-and-forward technique examines the complete packet first using the cyclic redundancy check (CRC) method in order to make sure that the frame has been correctly transmitted before forwarding the packet to its destination. If there is an error during this verification the frame will be ignored. that is, the message corrupted is not forwarded. this operation leads to slower sending which resulting in a significant delay and packets loss. In switched Ethernet, the message collision is avoided successfully but this protocol creates another new challenge to NCSs which is the congestion that appears inside the switch when one port suddenly receives a large number of packets from the other ports which cause fullness in the buffer.

This issue can be solved by implement three main principles for queueing, First is the First-In-First-Out (FIFO) queue, where is simple and fair, however, the quality of service (QoS) cannot be guaranteed if the network traffic is heavy. the second method is the priority queueing scheme, where the network manager reads some of the data frames to distinguish which queues will be more important. therefore, the packets are classified into different levels of queues. The third method is the per-flow queue, where the queues are assigned in different levels of priority (or weights) then all queues are processed one by one according to priority, the queues with higher priority will generally have higher performance and could potentially block queues with lower priority.

2.7.1.3 Wireless Ethernet (CSMA/CA) : In a wireless Ethernet protocol format, the wired connection between the components of closed-loop is replaced by wireless connection where space plays the physical medium for transfer the data and based on the OSI model , wireless Ethernet protocol implements the two lowest layers such wired Ethernet whereas the main difference between them lies in the physical layer exactly in the medium access control and the connection media.

As we mentioned in the previous Ethernet protocols that collision is an important issue trying to solve it, wireless Ethernet uses a collision avoidance mechanism but cannot entirely prevent it because the wireless stations cannot "hear" the collision where when the destination node receives the packet successfully it sends a short acknowledgment packet (ACK) back to the original sender. If the sender does not receive an ACK packet, it assumes that the transmission was unsuccessful and re-transmits. We can summarize the wireless Ethernet collision avoidance mechanism as follow:

When a node wants to send a message while the network is busy, it firstly sets its back-off counter to a randomly chosen value, then when the network is idle, the node waits for an inter-frame space and then back-off time for this wait before attempting to send. If another node accesses the network during that time, it must wait again for another idle interval, which means that the node with the lowest back-off time sends first, for example, Acknowledgment message (ACK) has a higher priority because it has a shorter inter-frame space. According to the random nature of the back-off time, the collision always happens where it could be two nodes have the same back-off time.

To improve wireless Ethernet protocol, several ideas were proposed where nodes can be reserve the network either by sending a request to send (RTS) message or by breaking a large message into many smaller messages (fragmentation), where can be sent each successive message after a smallest inter-frame time. If there is a single master node on the network, the master can poll all the nodes and effectively create a TDM contention-free network [58].

2.7.2 Time-Division Multiplexing (TDM) networks

Time-division multiplexing is one of the widely used types of media access control (MAC) protocols in networked control systems where can be accomplished in one of two ways, the master-slave network or single master polls multiple slaves. In a master-slave network, the slaves can only send data over the networks when requested by the master which leads to avoiding the collision, since the master node schedule carefully the transmissions data. Where, in a token-passing network, the node that has currently the token is allowed to send data and when the sending data is finished or the maximum token holding time has expired, the node that owns the token passes it to the next logical node on the networks, if this node has no message to send, it just passes the token to the successor node. This kind of TDM leads to no collision of data, as only one node can transmit at a time. The maximum time between network accesses for each node is a factor guaranteed by most token-passing protocols where this kind has provisions to regenerate the token if the token holder stops transmitting and does not pass the token to its successor. Token-passing protocols distinguished by the ability to add nodes dynamically to the bus and can request to be dropped from the logical ring.

Among several Typical examples of master-slave networks include Bitbus, Interbus-S, and ASI, while ControlNet and Profibus are typical examples of token-passing networks. In the ControlNet network, the nodes are arranged logically into a ring, where each node in the ring knows the address of its predecessor and its successor, the node with the token transmits data frames until either it runs out of data frames transmitting or when the time of holding token reaches the limit, whereas the Profibus network, each peer node can also behave like a master and communicate with a set of slave nodes during the time it holds the token.

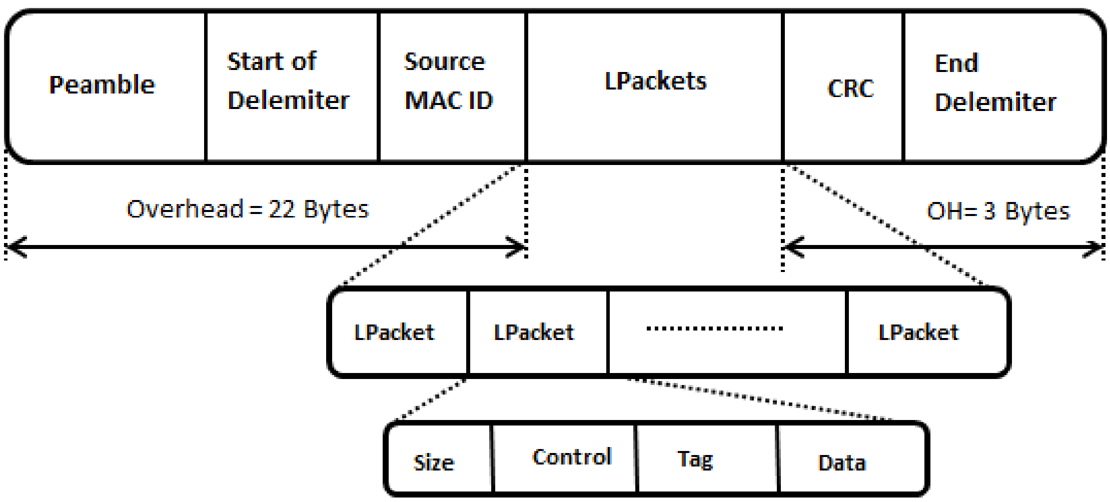

Figure 2.11: ControlNet message frame format.

As we mentioned that the time-division multiplexing has several typical examples in industrial and manufacturing fields grouped in two main kinds, master-slave network, and

token-passing network.To do an in-depth study, we take an example of the token-passing network, it is ControlNet, and specialize it with more interpretation and explanation.
Fig.(2.11) shows the message frame format of ControlNet where the total overhead is 7 bytes including preamble, start delimiter, source MAC ID , CRC , and an end delimiter and data packet frame called link packet (Lpacket) with total frame size between 0 and 510 bytes, which is may compose of several Lpackets that contain size, control, tag and data fields where the tag field is responsible for the individual destination address, whereas the size field specifies the number of byte pairs contained in an individual Lpacket (from 3 to 255) including the link data fields, size, tag, and control.
The ControlNet protocol is based in its work on an implicit token-passing mechanism where it defines a unique MAC ID to each node or device and its work philosophy is as follow :
The node which has the token can send data as in token passing buses. Instead, each node reads the source MAC ID of each message frame received, at the end of a message frame, each this node sets an "implicit token register" to the received source MAC ID+1, the node which has the token can now transmit messages if its MAC ID equals the implicit token register. all the nodes in the ControlNet network have the same value in their implicit token registers, this leads to avoiding collisions on the medium, on the other hand, if this node has no data to send, It only has to send a message with an empty Lpacket field.
In general, the length of a data cycle is divided into three major parts: scheduled, unscheduled, and guard band, the data cycle in ControlNet is called the network update time (NUT) whereas in general token buses called Token Rotation Time (TRT).
At the end of speak on the token bus protocol, we can conclude that the token bus protocol is a deterministic protocol that leads to providing excellent throughput and efficiency at high network loads, the token bus protocol can add nodes or remove it from the network easily and dynamically. In ControlNet protocol, scheduled and unscheduled segments make a suitable solution for both time-critical and non-time-critical messages.
Instead, the token bus protocol cannot compete the transmission performance compared with other contention protocols at low channel traffic also a large percentage of the network time is used in passing the token between nodes -when data traffic is light - caused by there are many nodes in one logical ring.

2.7.3 CAN-Based networks- DeviceNet :

Another kind of real NCSs protocol is the serial communication protocol 'CAN' where is mainly developed for applications of the automotive industry, after that, it was used in time-critical industrial applications due to its capability to offering a good performance.
The CAN protocol is oriented to transport the short messages where it uses a CSMA/arbitration on message priority (AMP) medium access method. In the case of simultaneous transmission, each message has a specific priority that is used to arbitrate access to the bus where the bit-stream of transmission is synchronized on the start bit, and the arbitration

is performed on the following message identifier, in which a logic zero is dominant over a logic one ([58]). When the bus is free the node that wants to transmit a message starts to send the identifier of its message bit by bit.
The problem of conflicts for access to the bus can be solved during transmission by an arbitration process at the bit level of the arbitration field, which is the initial part of each frame, so, if two or more devices want to send messages at the same time, the first continues to send the message frames and then listen to the network.
When one of these nodes wants to transmit and it receives a different bit to the one it sends out then it deprived the right to continue sent and the other node wins the arbitration. This method allows for continuous and never corrupted in transmission.

2.8 Conclusion

Networked control systems are a wide field of control systems, in this chapter, we presented the major components of NCSs with the main role of each one, then we introduced different real structures and topologies with their benefits and disadvantages. After that, we focused on a very important part of this thesis concerning the modeling of NCSs including continuous and discrete models, and the network-induced imperfections (time delays, packet dropouts .etc.) with theirs effects on the stability of NCSs. On the other hand, we presented several techniques of analyzing stability and designing controllers for NCSs, and finally, we introduced some Protocols used in the implementation of NCSs with their characteristics, advantages and inconvenient.

Chapter 3

Stability analysis and robust control of NCSs

3.1 Introduction

The stability analysis plays an important role in engineering and industrial systems, mainly when analyzing the behavior of systems and synthesizing controllers where it is known that the stability of the system is mandatory to perform the required process performances. The stability analysis can be defined as the technical methods that deal with the stability of the differential equations solutions and of the trajectories of dynamical systems under the small disturbances of initial conditions.
In NCSs, The stability analysis problems are a crucial objective which the problem is about how to determine the maximal range of network imperfections (delays, packet loss, quantization error ... etc) that keeps the system stable with the best possible performances, for example, the network-induced delay is an important factor to evaluate conservatism of the stability criteria in the networked systems, it is defined by the critical value of network delay at which a system becomes unstable.
The study in this thesis is mainly based on the Lyapunov-Krasovskii stability theorem [166], it is the most recent stability analysis technique, we investigate the approach based on reducing conservatism of stability criteria by constructing an appropriate Lyapunov-Krasovskii functional LKF candidate from the selection of LKFs including maximum information about the network-induced imperfections.
On the other hand, controlling NCSs requires dealing with different uncertainties acting on the system including the network-induced imperfections, for that, the robust control techniques [32] are the most suitable for the NCSs control. Our approach consists to take advantages of the stability analysis phase and derive a robust controller from the obtained stability criteria. In the sections below, we give further explanations on the adopted stability analysis methods and robust control technique's of NCSs.

3.2 Lyapunov stability of NCSs

When looking at the NCSs stability analysis literature, we find that most of the papers adopt the Lyapunov stability theorem to derive the stability conditions criteria instead of all other approaches studying the stability of the linear and nonlinear systems, this is due to its capacity to assess the stability of equilibrium points of a system without solving the differential equations that describe the system. The study of stability in Lyapunov's sense is a general mathematical theory applicable to all differential equations and to differences. The principle of stability as adopted by Lyapunov is based on the energy of the dynamic system. In this sense, we speak of a 'dissipative system' if this energy (which is usually scalar) is constantly dissipated, hence we can also say that the system tends towards a point of equilibrium. Lyapunov's idea was to examine the variation of a scalar function for the study of the variation in the energy of a given system, thus, the Lyapunov method consists mainly of two steps:

- Constructing a suitable scalar function denoted as a Lyapunov function
- Evaluating its first-order time derivative along the trajectory of the system where if the first-order time derivative of the Lyapunov function is decreasing along the system trajectory as time increases, then the system energy is dissipating and the system will finally stabilize.

Notice that there is no method for the choice of the Lyapunov functions, it is generally based on trial and error and mathematical/physical insight is often used with other methods such as the variable gradient method [38] and the sum of squares decomposition [84, 81].

By coming back to the control of NCS, the stability of NCSs is considered the main aspect treated by the engineers in real systems, it about determining the maximum range of the network-induced disadvantages on the stability of systems, this is a critical objective to determine the extent to which the system can be implemented in the reality, where the critical value of the resulting network-induced imperfection at which a system becomes unstable is used to derive less conservative criteria guaranteeing the stability of NCSs.

Among all the disadvantages of using control via networks, the network-induced delay is the most important indexes to evaluate the conservatism of stability criteria. Hence, many researchers in the control field tried to develop and improve the stability conditions of NCSs by ensuring the maximal allowable upper bound of network-induced delay (MAUB). Most researches are based on two main fundamental key points: the choose of the Lyapunov function (especially the Lyapunov Krasovskii Functional) and the use of some mathematical tools properly in deriving the stability conditions (especially the estimations of integral terms of LKFs time derivatives). First, we will present some fundamental notions and concepts relating to the stability of NCSs in the sense of Lyapunov, while paying particular attention to the temporal approaches linked to the second Lyapunov method, namely Lyapunov - Krasovskii approach and Lyapunov- Razumikhin approach. Note that the results that we will develop in chapters 3 and 4 will mainly rely on these approaches in order to demonstrate the stability properties used subsequently. To clarify the formulations and definitions to be introduced in the current section, we consider an ordinary NCSs described by the general model of the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ x(t) = \phi(t), \forall t \in [t_0 - \bar{\tau}, t_0 - \underline{\tau}]. \end{cases} \tag{3.1}$$

where $x(t)$ is the state vector, A, B and are real matrices of appropriate dimensions, K non-networked control gain matrix $\tau(t)$ represents the network-induced delay.
Note that the function $\phi(t) \in \mathbf{C}([t_0 - \bar{\tau}, t_0 - \underline{\tau}], \mathbb{R}^n)$ ($\phi(t)$ is a function belonging to $\mathbf{C}$ the set of functions $[t_0 - \bar{\tau}, t_0 - \underline{\tau}]$ to $\mathbb{R}^n$) we will define the continuous norm $\|.\|$ for the delay system (3.1) as follows:

$$\|\phi(t)\|_c = \max_{t_0 - \bar{\tau} < \theta < t_0 - \underline{\tau}} (\|\phi(\theta)\|) \tag{3.2}$$

In this definition, the norm $\|.\|$ represents the norm L_2 ($\|.\|_2$).
In what follows, we will discuss the main definitions and tools related to the stability of NCSs in sense of Lyapunov, we will assume that the NCSs linear model admits a single solution.

3.2.1 Stability notions and preliminaries :

This subsection addresses the different notions used in stability analysis and needed to study the stability of NCSs include the definition of the different types of stability analysis and the Lyapunov, Lyapunov Krasovskii and Lyapunov Razumikhin methods used to analyze the stability of NCSs, and it can begin with these notions by:

3.2.1.1 Stability (equilibrium point) : The trivial $x(t) = 0$ solution of the system (3.1) is said to be stable for all $t_0 \in \mathbb{R}$ if for all $\epsilon > 0$, there exists $\delta = \delta(t_0, \epsilon)$ such that $\|x_{t_0}\|_c < \delta$ implies $\|x_t\| < \epsilon$ for $t \leq t_0$, or simply [27]:

$$\forall \epsilon >, \exists \delta = \delta(t_0, \epsilon), \text{ such that } \|x_{t_0}\|_c < \delta \implies \|x(t)\| < \epsilon, \forall t \leq t_0 \tag{3.3}$$

3.2.1.2 Asymptotic and global Asymptotic stability : The Lyapunov asymptotic stability can be defined with the help of geometric interpretation shown in (3.2) where we assume that the initial state x_0 is located in a closed ball region with equilibrium state x_e of its center and its arbitrary radius δ when time t tends to infinity, the system state $x(t)$ can be converged to equilibrium state x_e, then the system is in Lyapunov asymptotic stability. To more illustration, the mathematical definition of Lyapunov Asymptotic Stability is presented as follow:

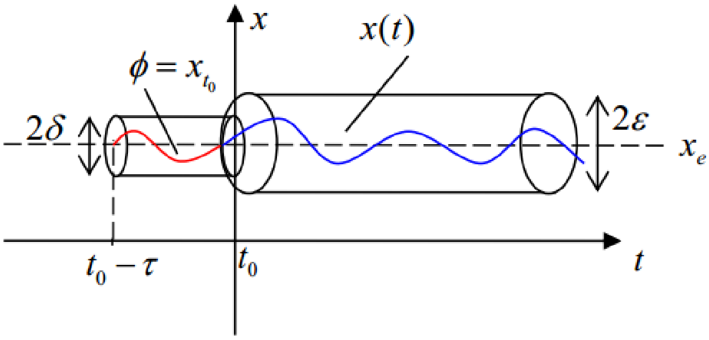

Figure 3.1: Stability in the sense of Lyapunov for an equilibrium point x_e.

Definition 3.1 *The trivial solution $x(t) = 0$ of the system (3.1) is asymptotically stable if it is stable and attractive and in addition:*

$$\forall \epsilon > 0, \exists \delta_a = \delta(t_0, \epsilon), \text{ such that } \|x_{t_0}\|_c < \delta_a \implies (\|x(t)\| < \epsilon) \text{ and } (\lim_{t\to\infty} \|x(t)\| = 0), \forall t \le t_0 \tag{3.4}$$

It is globally stable if it is stable and globally attractive.

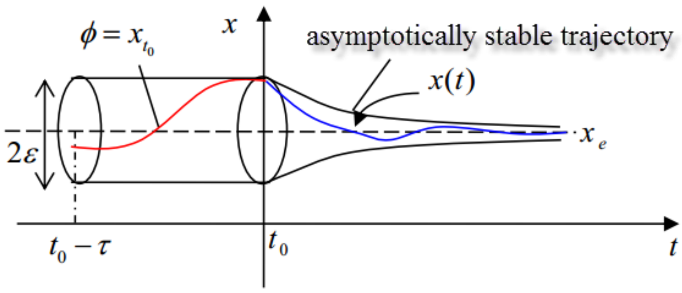

Figure 3.2: Lyapunov asymptotic stability trajectory.

3.2.1.3 Uniform and uniform asymptotic stability : The trivial solution $x(t) = 0$ is called a uniformly stable if it is stable and δ can be chosen independently of t_0, For clarification, the uniform stability means that the equilibrium point is not getting progressively less stable with time [99].

Definition 3.2 *The trivial solution $x(t) = 0$ of the system (3.1) is uniformly asymptotically stable if it is uniformly stable and attractive, so if there is δ_a as for all $\eta > 0$ it exists $T = T(\delta_a, \eta)$ such as $\|x_{t_0}\|_c < \delta_a$ implies $\|x_t\| < \eta$ for all $t \le t_0 + T$ and $t_0 \in \mathbb{R}$ or simply:*

$$\exists \delta_a > 0, \text{ such that, } \forall \eta > 0, \exists T = T(\delta_a, \eta), \text{ such that, } \|x_{t_0}\|_c < \delta_a \implies \|x(t)\| < \eta, \forall t \le t_0 + T \tag{3.5}$$

Remark 3.1 *It is worth mentioning that the definition of the asymptotic stability do not express the speed of convergence of trajectories to the origin, which is exponential in time-*

invariant linear systems either to or from the origin. But, for time-varying and nonlinear systems, the rate of convergence can be different according to different system types.

3.2.1.4 Exponential and global exponential stability :

Lyapunov exponential stability includes both the uniform and the asymptotic stability, it is particularly necessary when studying stability of uncertain systems and advanced control algorithms like adaptive ones.

Definition 3.3 *The trivial solution $x(t) = 0$ of the system* (3.1) *is exponentially stable if it is stable and if there are three strictly positive constants $\lambda_1 > 0$, $\lambda_2 > 0$, $\delta > 0$, such as for any initial condition $x_{t_0} = \phi$ and for all $t \leq t_0$*

$$\|x_{t_0}\|_c \leq \delta \implies \|x(t)\| < \left(\lambda_1 \|x_{t_0}\| e^{\lambda_2 (t-t_0)} \|\right), \forall t \geq t_0 \tag{3.6}$$

In this case, the constant λ_2 is called the "rate of decay" or the "rate of exponential convergence".

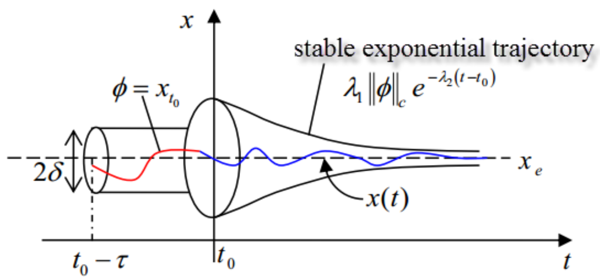

Figure 3.3: Lyapunov exponential stability trajectory.

Remark 3.2 *We said that the system is globally exponentially stable around an equilibrium state if the bound in equation* (3.6) *holds for all $x(t) = 0 \in \mathbb{R}^n$.*
The uniform and asymptotic stability of a linear system with uncertain coefficients/delays or time-varying does not imply the exponential stability. To search the decay rate and the constant λ_2 of exponential stability in (3.6)*, the Lyapunov–Krasovskii method is the most recommended to give sufficient conditions.*

3.2.1.5 Robust stability :

We say that an uncertain system is robust asymptotically stable if it is asymptotically stable for all allowable uncertainties on the model parameters and we say also that an uncertain system is H-robustly stable or stable with respect to space H if its equilibrium is asymptotically stable for all the admissible uncertainties on the parameters of the model and for any delay function $\tau(t)$ defined in space H [9].
Note that Lyapunov second method relies on the existence of a positive definite function

$V > 0$ so that along the trajectories of the system (3.1) we have $\dot{V} < 0$. This method is valid only for a limited class of systems delay, because V depends on the past values x_t. This is why it is very difficult to apply in the general case of delay systems.
Two techniques based on extensions of Lyapunov second method were developed by Krasovskii and Razumikhin respectively in the context of delay systems. In what follows, we will briefly present these two extensions.

3.2.2 Stability by Lyapunov-Krasovskii Functional approach (LKFs)

: The Lyapunov approach is an effective method to study the stability of systems without any parameter variations like time-delay, it requires the construction of a Lyapunov function, $V(t, x(t))$, which can be viewed as a measure of how much the state $x(t)$ deviates from the trivial solution 0, only the state $x(t)$ is required to specify the future evolution of the system beyond t. In the same line, when studying the NCSs or time-delay system, we also need the value of $x(t)$ in the interval $[t - \tau_{min}, t - \tau_{max}]$ (that is, x_t). So, it is natural to expect that, for a networked control system, the Lyapunov function becomes a functional $V(t, x_t)$ that depends on x_t and indicates how much x_t deviates from the trivial solution 0. This type of functional is called a Lyapunov-Krasovskii functional. In fact, it is an extension of the second Lyapunov method dedicated to the stability analysis of functional differential equations, the main idea is based on how we can construct a Lyapunov Krasovskii functionals (energy functionals) including all the information of the effect of network-induced delay or packet dropout on the stability of NCSs i.e. (functions of the state $x(t)$ in the form $V(t, x_t)$, that are positive definite function with its first-time derivative negative), this means that the LKFs is decreasing along the trajectories of system (3.1).
Before mentioning the Lyapunov-Krasovskii Theorem, let first introduce the Krasovskii method :
- Let $V : R \times C\,[-\tau, 0] \rightarrow \mathbf{R}$ be a continuous functional, and let $x_\tau(t, \phi)$ be the solution of (3.1) *at time $\tau \geq t$ with the initial condition $x_t = \phi$. We define the right upper derivative $\dot{V}(t, \phi)$ along* (3.1) *as follows:*

$$\dot{V}(t, \phi) = lim\ sup_{\Delta t \rightarrow 0} + \frac{1}{\Delta t}\left[V(t + \Delta t, x_{\Delta t \rightarrow 0}(t, \phi)) - V(t, \phi)\right]. \tag{3.7}$$

Intuitively, a non-positive $\dot{V}(t, x_t)$ indicates that x_t does not grow with t, meaning that the system under consideration is stable.

3.2.2.1 Lyapunov-Krasovskii theorem : In addition to constructing the appropriate LKFs, the other main idea of the Lyapunov-Krasovskii theorem is that it is not necessary to guarantee the negative definiteness of $V(t, x(t))$ along all the trajectories of the system (the time derivative of LKFs). Indeed, it is sufficient to ensure its negative definiteness only for the solutions that tend to escape the neighborhood of $V(t, x(t)) \leq c$ of the equi-

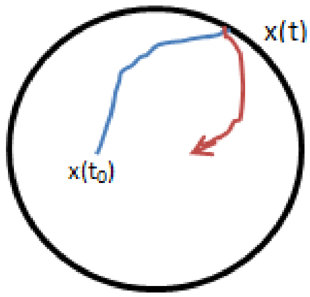

Figure 3.4: Razumikhin approach.

librium. This idea is formalized in the following theorem.

Theorem 3.1 *Suppose $f : R \times C[-\tau, 0] \to \mathbf{R}$ maps $R\times$ (bounded sets in $C[-\tau, 0]$) into bounded sets of $\mathbf{R}^n$ and that u, v and $w : R_+ \to R_+$ are continuous nondecreasing functions, $u(s)$ and $v(s)$ are positive for $s > 0$, and $u(0) = v(0) = 0$. The trivial solution of* (3.1) *is uniformly stable if there exists a continuous functional $V : R \times C[-\tau, 0] \to \mathbf{R}^+$, which is positive definite*

$$u(|\phi(0)|) \leq V(t, \phi(t)) \leq v(|\phi|_c) \tag{3.8}$$

such that its derivative along (3.1) *is non-positive in the sense that*

$$\dot{V}(t, \phi) \leq -w(|\phi(0)|) \tag{3.9}$$

If $w(s) > 0$ for $s > 0$, then the trivial solution is uniformly asymptotically stable. If in addition $\lim_{s\to\infty} u(s) = \infty$, then it is globally uniformly asymptotically stable.

3.2.3 Stability by Lyapunov-Razumikhin Functionals approach (LRF)

: Sometimes, it is difficult to construct or select an appropriate Lyapunov-Krasovskii functional because it requires the state variable $x(t)$ in the interval $[t - \tau_{min}, t - \tau_{max}]$, it can only be obtained by manipulating the LKFs. This difficulty can be circumvented by using the Razumikhin theorem, this alternative involves only functions, but no functionals. The idea of Razumikhin method can be explained geometrically as follow: for the typical choice of (quadratic) Lyapunov functions of the form $V(x) = x^T P x$; $P > 0$, if a solution start inside the ellipsoid $V(x(t_0 + \delta)) = x^T(t_0 + \delta)Px(t_0 + \delta) \leq \theta, \forall\theta \in [-\tau, 0]$ and is about to leave this ellipsoid at time $t \geq t_0$, then

$$x^T(t_0 + \theta)Px(t_0 + \theta) \leq x^T(t)Px(t), \forall\theta \in [-\tau, 0] \tag{3.10}$$

So, the solution will not leave the ellipsoid $x^T(t)Px(t) \leq \delta$ if $\frac{d}{dt}V(x(t) \leq 0$ along $\dot{x}(t) =$

$f(t, x_t)$ for all $x_t(\theta) = x_t(t+\theta), \theta \in [-\tau, 0]$ satisfying the Razumikhin condition (3.10):

$$V(x(t+\theta)) \leq V((x(t)), \forall\theta \in [-\tau, 0] \tag{3.11}$$

(and not for any $x(t+\theta)$). This guarantees the stability of the system. To give a precise formulation of Razumikhin method, let consider a differentiable function $V : \mathbf{R}^n \times \mathbf{R} \rightarrow \mathbf{R}_+$ and define the derivative of V along the solution $x(t)$ of (3.1) as

$$\dot{V}(t, x(t)) = \frac{d}{dt}V(t, x(t)) = \frac{\partial V(t, x(t))}{dt} + \frac{\partial V(t, x(t))}{dx} f(t, x_t) \tag{3.12}$$

3.2.3.1 Lyapunov-Razumikhin theorem : Suppose $f : R \times C[-\tau, 0] \rightarrow \mathbf{R}$ maps $R\times$ (bounded sets in $C[-\tau, 0]$) into bounded sets of $\mathbf{R}^n$ and that u, v and $w : R_+ \rightarrow R_+$ are continuous nondecreasing functions, $u(s)$ and $v(s)$ are positive for $s > 0$, and $u(0) = v(0) = 0$. The trivial solution of (3.1) is uniformly stable if there exists a continuous functional $V : R \times C[-\tau, 0] \rightarrow \mathbf{R}^+$, which is positive definite

$$u(|\phi(0)|) \leq V(t, \phi(t)) \leq v(|\phi|_c) \tag{3.13}$$

such that its derivative along (3.1) is non-positive in the sense that

$$\dot{V}(t, \phi) \leq -w(|\phi(0)|),\ if V(t+\theta, x(t+\theta)) \leq V(t, x(t)), \forall\theta \in [-\tau, 0] \tag{3.14}$$

in addition, $w(s) > 0$ for $s > 0$, and there exists a continuous non-decreasing function $\rho(s) > s$ for $s > 0$, such that condition (3.15) is strengthened to

$$\dot{V}(t, \phi) \leq -w(|\phi(0)|),\ if V(t+\theta, x(t+\theta)) \leq \rho(V(t, x(t))), \forall\theta \in [-\tau, 0] \tag{3.15}$$

Then the trivial solution of (3.1) is uniformly asymptotically stable. If, in addition, $\lim_{s\rightarrow\infty} u(s) = \infty$, then it is globally uniformly asymptotically stable.
In the next subsection, we will present the different lemmas and properties that are useful and important for helping us to develop a less conservative stability conditions for stability analysis and controller design of NCSs.

3.3 Properties and useful lemmas

Let us recall the following lemmas, which will be considered to derive the proofs of the proposed theorems coming in the next chapters.

3.3.1 Schur's lemma (Schur's Compliment) :

Let $M(x)$ be a symmetric matrix with:

$$M(x) = M^T(x) = \begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} \tag{3.16}$$

where $Q(x) \in \mathbb{R}^{n\times n}$ and $R(x) \in \mathbb{R}^{n\times n}$ are square matrices and $S(x) \in \mathbb{R}^{n\times m}$ is any other matrix.
The following inequalities are equivalent:

$$M(x) < 0 \implies \begin{cases} R(x) < 0 \\ Q(x) - S(x)R^{-1}S^T(x) < 0 \end{cases} \implies \begin{cases} Q(x) < 0 \\ R(x) - S(x)R^{-1}S^T(x) < 0 \end{cases} \tag{3.17}$$

3.3.2 Finsler's lemma

: [102] Let $\xi \in \mathbb{R}^n$, $\mathcal{G} \in \mathbb{R}^{m\times n}$ and $Q = Q^T \in \mathbb{R}^{n\times n}$ such that $rank(\mathcal{G}) < n$. The following statements are equivalent.

$$\xi^T Q\xi < 0, \quad \forall \xi \in \{\xi \in \mathbb{R}^n : \xi \neq 0, \mathcal{G}\xi = 0\}. \tag{3.18}$$

$$\exists \mathcal{R} \in \mathbb{R}^{n\times m} : Q + \mathcal{H}(\mathcal{R}\mathcal{G}) \prec 0 \tag{3.19}$$

3.3.3 Inequality lemma (Square matrices)

: [146] Suppose that $\tau_i(t), (i \in \mathcal{I}_2)$ are defined as previously and ϕ_i, Ψ_i, $(i \in \mathcal{I}_2)$ $\Omega = \Omega^T$ are some constant matrices with appropriate dimensions, then:

$$\Omega + \frac{\tau_2 - \tau_1(t)}{\tau_2 - \tau_1}\phi_1 + \frac{\tau_1(t) - \tau_1}{\tau_2 - \tau_1}\phi_2 + \frac{\tau_3 - \tau_2(t)}{\tau_3 - \tau_2}\Psi_1 + \frac{\tau_2(t) - \tau_2}{\tau_3 - \tau_2}\Psi_2 \leq 0\,. \tag{3.20}$$

If the following inequalities hold:

$$\begin{aligned} &\Omega + \phi_1 + \Psi_1 \leq 0\,, \\ &\Omega + \phi_2 + \Psi_2 \leq 0\,, \\ &\Omega + \phi_1 + \Psi_2 \leq 0\,, \\ &\Omega + \phi_2 + \Psi_1 \leq 0\,. \end{aligned}$$

3.3.4 Wirtinger's inequality lemma

: [83] For any positive matrix Z, real scalars a and b satisfying $a < b$, the inequalities (3.21), (3.22), (3.23) hold for any continuously differentiable function $x : [a, b] \to \mathbb{R}^n$.

$$-\int_a^b \dot{x}^T(s)Z\dot{x}(s)ds \leq -\frac{1}{b-a}\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ * & 3Z \end{bmatrix}\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \tag{3.21}$$

$$-\int_a^b \int_\lambda^b \dot{x}^T(s)Z\dot{x}(s)dsd\lambda \le -\begin{bmatrix}\psi_3\\ \psi_4\end{bmatrix}^T \begin{bmatrix}2Z & 0\\ * & 4Z\end{bmatrix}\begin{bmatrix}\psi_3\\ \psi_4\end{bmatrix}. \tag{3.22}$$

$$-\int_a^b \int_a^\lambda \dot{x}^T(s)Z\dot{x}(s)dsd\lambda \le -\begin{bmatrix}\psi_5\\ \psi_6\end{bmatrix}^T \begin{bmatrix}2Z & 0\\ * & 4Z\end{bmatrix}\begin{bmatrix}\psi_5\\ \psi_6\end{bmatrix}. \tag{3.23}$$

where

$$\begin{aligned}
\psi_1 &= x(b) - x(a)\\
\psi_2 &= x(b) + x(a) - \tfrac{2}{b-a}\textstyle\int_a^b x(s)ds\\
\psi_3 &= x(b) - \tfrac{1}{b-a}\textstyle\int_a^b x(s)ds\\
\psi_4 &= x(b) + \tfrac{2}{b-a}\textstyle\int_a^b x(s)ds - \tfrac{6}{(b-a)^2}\int_a^b\int_\lambda^b x(s)dsd\lambda\\
\psi_5 &= x(a) - \tfrac{1}{b-a}\textstyle\int_a^b x(s)ds\\
\psi_6 &= x(a) - \tfrac{4}{b-a}\textstyle\int_a^b x(s)ds + \tfrac{6}{(b-a)^2}\int_a^b\int_\lambda^b x(s)dsd\lambda
\end{aligned}$$

3.3.5 Reciprocally convex lemma :

[44] For any vectors x_1, x_2, constant matrices S, $Q_i, (i \in \mathcal{I}_4)$, and real constant scalars $\alpha > 0$ and $\beta > 0$ satisfying $\alpha + \beta = 1$, the following inequality holds:

$$-\frac{1}{\alpha}x_1^T Q_1 x_1 - \frac{\beta}{\alpha}x_1^T Q_3 x_1 - \frac{1}{\beta}x_2^T Q_2 x_2 - \frac{\alpha}{\beta}x_2^T Q_4 x_2 \le -\begin{bmatrix} x_1\\ x_2\end{bmatrix}^T \begin{bmatrix} Q_1 & S\\ S^T & Q_2\end{bmatrix}\begin{bmatrix} x_1\\ x_2\end{bmatrix}$$

subject to:

$$\begin{bmatrix} Q_1 + Q_3 & S\\ S^T & Q_2 + Q_4\end{bmatrix} > 0$$

3.3.6 Newton-Leibniz lemma:

[87] For any constant matrix $Q \in \mathbb{R}^{n\times n}$, $Q > 0$, scalars $\tau_1 \le \tau(t) \le \tau_3$ and vector function $\dot{x} : [-\tau_3, -\tau_1] \to \mathbb{R}^n$ such that the following integration is well defined, then it holds that:

$$\begin{aligned}
&-(\tau_3 - \tau_1)\int_{t-\tau_3}^{t-\tau_1} \dot{x}(s)^T Q\dot{x}(s)ds \le\\
&-[x(t-\tau_1) - x(t-\tau(t))]^T Q[x(t-\tau_1) - x(t-\tau(t))]\\
&-[x(t-\tau(t)) - x(t-\tau_3)]^T Q[x(t-\tau(t)) - x(t-\tau_3)]\,.
\end{aligned} \tag{3.24}$$

3.3.7 Free-Weighting Matrix FWM:

The free weighting matrix method plays an important role in the analysis of delay dependent stability and the synthesis of controllers from various classes of delay systems. In the literature, this method is based either on the structure of the model, or on the Newton-Lepinez formula. From the Newton-Leibniz formula, the following equations are true for all matrices M_1, M_2 and M_3, and of appropriate dimensions.

$$2\left[x^T(t)M_1 + x^T(t-\tau(t))M_2 + \dot{x}^T(t)M_3\right] \times \left[x(t) - x(t-\tau(t) - \int_{t-\tau(t)}^{t} \dot{x}(t)\right] \qquad (3.25)$$

The matrices M_1, M_2 and M_3, and indicate the relations between the terms of the Newton-Leibniz formula. The left side of this equation is added to the derivative of the LKFs function.

3.4 Robust control techniques

There is always a gap between the behavior of a real physical system and the system mathematical model, this mainly due to several factors such as identification errors, system parameters variations, non-linearities, simplifying assumptions, presence of delays, etc. Physical phenomena cannot be completely translated by mathematical models, indeed, real systems has essentially a nonlinear dynamic behaviour which is approximated by means of several linear models around several points of equilibrium. These factors are called uncertainties.

The control field that synthesize controllers which not only ensures the desired performances but also maintains them despite the uncertainties mentioned above is called "Robust Control". This field has grown rapidly in the industrial environment where it is an invaluable tool for the analysis and design of control systems due to its applied nature and its relevance to the practical problems of an automation engineer. Indeed, The control is usually designed based on an idealized and simplified model of the real system, and to function properly, it must be robust to the imperfections of the model, the uncertainty of systems, and the external disturbances. Therefore, Engineers have to make control laws that satisfy the two requirements mentioned above. Any synthesis of a robust control law must therefore take into account, during its development, not only the nominal model but also a set of uncertain models in order to guarantee the stability and have the desired performances in closed loop for all these models. This task can sometimes be complicated because there are several types of uncertainties (structured or unstructured) that can act on the real system at several points of the loop (input, output or others). We will then say that a system is robust if it remains stable and maintains the desired performance in the presence of these various uncertainties.

3.4.1 $\mathcal{H}_\infty$ Control : Between the robust controller design method, the $\mathcal{H}_\infty$ technique is certainly the most used and confirmed one, in this method, the controller is obtained by minimizing the $\mathcal{H}_\infty$ norm of a criterion in the frequency plane taking into account the robustness requirements in terms of stability and performance. This approach was developed during the 1980s thanks to the work of researchers such as[147],[148],

[20]. The $\mathcal{H}_\infty$ norm can be explained simply as the maximum energy amplification overall input signals, where for a single input single output system, the $\mathcal{H}_\infty$ norm $||G||_\infty$ equals the peak magnitude in the Bode diagram of its transfer function.
In $\mathcal{H}_\infty$ control, the performance to be optimized is defined in terms of minimizing the $\mathcal{H}_\infty$ norm of the closed-loop transfer function, this type of optimization problem is also called min-max problem (the $\mathcal{H}_\infty$ seeks to minimize the worst-case scenario).

3.4.2 Formulation of the standard $\mathcal{H}_\infty$ problem :

The configuration of the standard problem $\mathcal{H}_\infty$ is represented schematically by figure 3.5. where ω is a vector

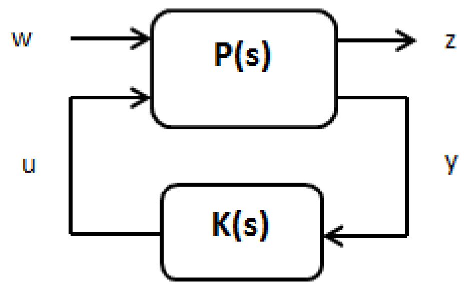

Figure 3.5: Configuration of the standard $\mathcal{H}_\infty$ problem.

of signals representing external inputs such as reference signals, disturbances, noise, u is the vector of commands, z is the cost vector. These elements are signals representing the outputs of the weighting functions (or matrices) chosen to characterize the correct operation of the system, y are the vectors of the output signals or measurements available to generate the commands.
The aim is to synthesize a control law $u = K(s)y$ which minimizes the effect of the signals ω on the signals of the vector z (problem of rejection of disturbances). These two sets of signals being assumed to be of finite energy, the impact of ω on z can be measured by: $\frac{||z||_2}{||\omega||_2}$, and in the worst case this report will therefore be: $\sup\limits_{w\neq 0} \frac{||z||_2}{||w||_2} = ||T_{zw}||_\infty$ where T_{zw} is the transfer between z and w.
Note that, according to the diagram in figure 3.5, the transfer T_{zw} can be expressed in the form of a lower Linear Fractional Transformation LFT such that $F_l(P, K)$.
The optimal $\mathcal{H}_\infty$ problem consists in finding a stabilizing controller by minimizing the $\mathcal{H}_\infty$ norm of the T_{yz} transfer or in the following mathematical form:

$$\min_{K\,stabilizing} ||F_l(P, K)||_\infty \tag{3.26}$$

There is no analytical formula for solving this problem and its solution is not unique. This is why this problem is treated in its sub-optimal form expressed as follows:

Given $\gamma > 0$, find a stabilizing controller that ensures:

$$\|F_l(P, K)\|_\infty < \gamma \tag{3.27}$$

This problem is solved by different methods, the most common of which remains the approach using Riccati's equations or the approach by affine inequalities (LMI) introduced recently.

3.5 Conclusion

The notion of stability is very important in control systems, it has been the subject of many theories. In this chapter, we focused on the systems stability in the sense of Lyapunov, where we presented different related definitions . Next, we presented two approaches frequently used for the stability analysis of NCSs, namely : the imperfections-independent using Lyapunov Razumikhin method and the imperfections-dependent approach using Lyapunov Krasovskii method . Finally, we recalled the standard problem of $\mathcal{H}_\infty$ to deal with different disturbances and uncertainties attaining NCSs, and to get robust stability and control.
In the next chapter, we will focus on the stability and control problems of NCSs with a stochastic model subject to distributed network-induced delays.

Chapter 4

Robust stability and stabilization of networked control systems with stochastic time-varying network-induced delays

4.1 Introduction

As we mentioned in the chapters "General Introduction" and "NCS Modeling" that network control systems are modeled or designed as deterministic or random/stochastic methods depending on the type of network or protocol used. In this chapter, we focus on the second method which is random ones, as this method has been widely used in many previous and recent works such as these in [87, 108, 107] where the authors employ the delay-distribution characteristics of the network-induced delay to model the stochastic delay. and the works in [22] are focused on the random varying network-induced delay to modeled both the measurement and actuation delays as two independent Bernoulli distributed white sequences and use it to analyze the stability and design a networked controller. Our contribution in this thesis to improving the stability analysis and controller synthesis for NCSs using the stochastic method illustrated in using the network-induced delay as a stochastic delay model in a given interval with known lower and upper bounds and then the modeled the NCSs as a linear system with probabilistic time-varying delay distribution, where the main goal is to reduce the effect of the network-induced delay on stability of NCSs by ensuring maximum allowable delay bound (MADB) guaranteeing the stability of the networked closed-loop and achieve a good performance. The Lyapunov-Krasovskii functional (LKF) method formulated using probabilistic information of both lower and upper bounds of the probabilistic time-varying network-induced delay is used to derive a new less conservative stability conditions and state feedback controller design in the form of linear matrix inequality, then a novel integral inequalities such as Wirtinger integral inequality are used to estimate the accuracy of integral terms of the time derivatives of LKFs and also to reduce conservatism by introducing some new cross terms. Next, a set of LMIs stability conditions is constructed based on the H_∞ disturbance rejection level. Finsler's lemma is then used to relax these set of LMI's by adding slack decision variables and decoupling the system's matrices from those of Lyapunov-krasovskii, this will facilitate the design of state feedback controller by the mean of simple variables change. Then, a maximum allowable upper transmission delay bound and an optimal state feedback controller gains are calculated by the resolution of the above relaxed LMI's convex optimization problem. Finally, five numerical illustrative examples representing different NCSs with time-varying network-induced delay, as well as a comparison with other existing methods in terms of conservativeness and robustness are provided.

Notations 4.1 *In the following, stars * in matrices denote bloc transpose quantities. For a matrix M of appropriate dimension, one denotes* $\mathcal{H}(M) = M + M^T$. *A finite set of r positive integers is denoted* $\mathcal{I}_r = \{1, ..., r\}$. *Also,* $\forall j \in \mathcal{I}_{12}$, *we denote the block entry matrices* $e_j = \begin{bmatrix} 0_{n\times(j-1)n} & I_{n\times n} & 0_{n\times(12n-j)n} \end{bmatrix}^T \in R^{12n\times n}$, *for example:* $e_4 = \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

4.2 System description and preliminaries

Let us consider the structure of a typical NCSs of Fig.1.1 with a continuous linear system model described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_w\omega(t), \\ y(t) = Cx(t) + Du(t) + D_w\omega(t), \\ x(t) = \phi(t), \qquad t \in [-\tau_3, -\tau_1]. \end{cases} \tag{4.1}$$

Where $x(t) \in R^n$ is the system state vector. $u(t) \in R^m$ is the control input vector. $y(t) \in R^q$ is the controlled output. $\omega(t) \in R^p$ denotes the external perturbation which is assumed to belong to $L_2[0\ \infty)] \in R^p$. A, B, C, D, B_w and D_w are real constant matrices with appropriate dimensions. $\phi(t)$ is a vector-valued initial function of initial condition for $t \in [-\tau_3, -\tau_1]$.

Assumption 4.1 *The sensor is clock-driven with a fixed sampling period T, the controller, the zero-order holder (ZOH) and the Actuator are event-driven.*

Assumption 4.2 *Assuming that all the state variables are available from measurements and that these data are transmitted to the controller part through the network by single-packets.*

Let first consider the state feedback controller law given by:

$$u(t) = Kx(t - \tau(t)), \qquad \tau(t) \in [\tau_1, \tau_3]. \tag{4.2}$$

Where $K \in R^m$, is the controller gain matrix to be designed and $\tau(t)$ is the network-induced delay that represents the time delay of both the sensor-controller and the controller-actuator channels which satisfies $\tau_1 \leq \tau(t) \leq \tau_3 < \infty$, where $\tau_1 = \min(\tau(t)) > 0$ and $\tau_3 = \max(\tau(t)) < \infty$ are the lower and upper bounds of $\tau(t)$ respectively.
Most of NCSs uses IP network, its nature is considered as multi-fractal with non-uniform distribution with short delays occurring in a high probability and long delays in a low probability [110]. Regarding the results of [87] who analyzes different class of IP based NCSs models using stochastic variable and Bernoulli distributed white sequence, when the delay distribution information of the network induced delay is introduced, the control law (4.2) can be re-written as:

$$u(t) = \sigma(t)Kx(t - \tau_1(t)) + (1 - \sigma(t))Kx(t - \tau_2(t)). \tag{4.3}$$

with :

$$\tau_1(t) = \sigma(t)\tau(t), \;\; \tau_2(t) = (1 - \sigma(t))\tau(t). \tag{4.4}$$

where $\sigma(t)$ is a stochastic variable given by:

$$\sigma(t) = \begin{cases} 1, & t \in \Omega_1 = \{t : \tau(t) \in [\tau_1, \tau_2[\}, \\ 0, & t \in \Omega_2 = \{t : \tau(t) \in [\tau_2, \tau_3]\}. \end{cases} \tag{4.5}$$

From (4.5), one can see that the communication delay $\tau(t)$ changes randomly and the probability of $\tau(t) \in \Omega_1$ and $\tau(t) \in \Omega_2$ can be known.

Remark 4.1 *It can be obviously seen that $\Omega_1 \cup \Omega_2 = R^+$ and $\Omega_1 \cap \Omega_2 = 0$. In addition, from* (4.4)*: if $t \in \Omega_1$, the time-varying delay $\tau_1(t)$ appear. Else if $t \in \Omega_2$, the time-varying delay $\tau_2(t)$ appear.*

Substituting(4.3) into (4.1) yields to the following closed-loop state space expression:

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)BKx\,(t - \tau_1(t)) + (1 - \sigma(t))\,BKx\,(t - \tau_2(t)) + B_w\omega(t), \\ y(t) = Cx(t) + \sigma(t)DKx\,(t - \tau_1(t)) + (1 - \sigma(t))\,DKx\,(t - \tau_2(t)) + D_w\omega(t). \end{cases} \tag{4.6}$$

Furthermore, in order to describe the network QoS, we use the Bernoulli distributed white sequence related stochastic variable $\sigma(t)$ described as follows [87]:

$$\begin{cases} Prob\,\{\sigma(t) = 1\} = \mathbb{E}\,\{\sigma(t)\} = \bar{\sigma}, \\ Prob\,\{\sigma(t) = 0\} = 1 - \mathbb{E}\,\{\sigma(t)\} = 1 - \bar{\sigma}. \end{cases} \tag{4.7}$$

The closed-loop system (4.6) can be rewritten in the following form :

$$\begin{cases} \dot{x}(t) = Ax(t) + \bar{\sigma}BKx\,(t-\tau_1(t)) + (1-\bar{\sigma})\,BKx\,(t-\tau_2(t)) \\ \qquad + (\sigma(t)-\bar{\sigma})\,BK\,[x\,(t-\tau_1(t)) - x\,(t-\tau_2(t))] + B_w\omega(t), \\ y(t) = Cx(t) + \bar{\sigma}DKx\,(t-\tau_1(t)) + (1-\overline{\sigma})\,DKx\,(t-\tau_1(t)) \\ \qquad + (\sigma(t)-\bar{\sigma})\,DK\,[x\,(t-\tau_1(t)) - x\,(t-\tau_2(t))] + D_w\omega(t). \end{cases} \tag{4.8}$$

Remark 4.2 *When $\sigma(t) = 1$ or 0, the control law* (4.3) *is reduced to the standard one in* (4.2)*, consequently, the closed loop system is also reduced to the following common time-delay systems form:*

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t-\tau(t)) + B_w\omega(t), \\ y(t) = Cx(t) + DKx(t-\tau(t)) + D_w\omega(t), \\ x(t) = \phi, t \in [-\tau_3, 0]. \end{cases} \tag{4.9}$$

The purpose of this work is first to propose a new relaxed LMI-based stability conditions for the closed-loop dynamics (4.8) (and consequently for (4.9) too) when considering the gain K known. Then, a convexification procedure will be proposed to design the state feedback controller K when it is a priori unknown. Moreover, the considered NCSs is subject to external disturbances $\omega(t) \in L_2[0,\infty)$, thus, for robustness purpose, the proposed approach have to satisfy the following requirements:

• The NCSs closed-loop dynamics (4.8), without external disturbances ($\omega(t) = 0$), is asymptotically stable.

• Under zero initial conditions and with $\omega(t) \neq 0$, the NCSs model (4.8) satisfies the following H_∞ constraint:

$$\| y(t) \|_2 < \gamma \| \omega(t) \|_2 \ . \tag{4.10}$$

Then, the H_∞ disturbance rejection criterion can be expressed as:

$$J = \int_0^\infty \left(y^T(s)y(s) - \gamma^2\omega^T(s)\omega^T(s) \right) ds < 0\,. \tag{4.11}$$

where the scalar $\gamma > 0$ denotes the disturbance attenuation level (performance index) to be minimized.

4.3 Main results

In this section, we will develop a robust H_∞ stability criterion as well a state feedback controller design procedure for NCSs (4.8) with stochastic time delay distribution expressed by the following theorem.

4.3.1 NCSs stability analysis.

Theorem 4.1 *For given scalars $\tau_i, (i \in \mathcal{I}_3)$, such that $\tau(t) \in [\tau_1, \tau_3]$ with $\gamma > 0$ and known feedback gain K, the considered NCSs* (4.8) *is robustly stochastic asymptotically stable with the H_∞ performance index γ if there exist positive definite symmetric matrices P, $R_i, Q_i (i \in \mathcal{I}_3)$, $S_j (j \in \mathcal{I}_2)$ and real matrices $M_l (l \in \mathcal{I}_4)$ with appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \sum_{i=1}^{4} \Phi_i + \mathcal{H}_e(\mathcal{M}\mathcal{G}) + \Pi_1 + \Pi_3 & \tilde{D}^T \\ * & -I \end{bmatrix} < 0 \ , \tag{4.12}$$

$$\begin{bmatrix} \sum_{i=1}^{4} \Phi_i + \mathcal{H}_e(\mathcal{M}\mathcal{G}) + \Pi_1 + \Pi_4 & \tilde{D}^T \\ * & -I \end{bmatrix} < 0 \ , \tag{4.13}$$

$$\begin{bmatrix} \sum_{i=1}^{4} \Phi_i + \mathcal{H}_e(\mathcal{M}\mathcal{G}) + \Pi_2 + \Pi_3 & \tilde{D}^T \\ * & -I \end{bmatrix} < 0 \ , \tag{4.14}$$

$$\begin{bmatrix} \sum_{i=1}^{4} \Phi_i + \mathcal{H}_e(\mathcal{M}\mathcal{G}) + \Pi_2 + \Pi_4 & \tilde{D}^T \\ * & -I \end{bmatrix} < 0 \ . \tag{4.15}$$

where:

$$\mathcal{G} = \begin{bmatrix} A & 0 & \bar{\sigma}BK & 0 & (1-\bar{\sigma})BK & 0_{(1\times5)} & -I & B_w \end{bmatrix},$$

$$\tilde{D} = \begin{bmatrix} C & 0 & \overline{\sigma}DK & 0 & (1-\overline{\sigma})DK & 0_{(1\times6)} & D_w \end{bmatrix},$$

$$\Phi_1 = 2e_1Pe_{11}^T + e_1(Q_1 + Q_2 + Q_3)e_1^T - e_2Q_1e_2^T - e_4Q_2e_4^T - e_6Q_3e_6^T, \tag{4.16}$$

$$\begin{aligned}\Phi_2 =& e_{11}\left(\tau_1^2R_1 + \tau_2^2R_2 + \tau_3^2R_3\right)e_{11}^T - (e_1 - e_2)R_1(e_1 - e_2)^T - (e_1 - e_3)R_2(e_1 - e_3)^T \\ & - (e_3 - e_4)R_2(e_3 - e_4)^T - (e_1 - e_5)R_3(e_1 - e_5)^T - (e_5 - e_6)R_3(e_5 - e_6)^T,\end{aligned} \tag{4.17}$$

$$\begin{aligned}\Phi_3 = & e_{11}\left((\tau_2 - \tau_1)^2S_1 + (\tau_3 - \tau_2)^2S_2\right)e_{11}^T - (e_2 - e_3)S_1(e_2 - e_3)^T - (e_3 - e_4)S_1(e_3 - e_4)^T \\ & - (e_4 - e_5)S_2(e_4 - e_5)^T - (e_5 - e_6)S_2(e_5 - e_6)^T - 3(e_2 + e_3 - 2e_7)S_1(e_2 + e_3 - 2e_7)^T \\ & - 3(e_3 + e_4 - 2e_8)S_1(e_3 + e_4 - 2e_8)^T - 3(e_4 + e_5 - 2e_9)S_2(e_4 + e_5 - 2e_9)^T \\ & - 3(e_5 + e_6 - 2e_{10})S_2(e_5 + e_6 - 2e_{10})^T ,\end{aligned} \tag{4.18}$$

$$\Phi_4 = -\gamma^2 e_8 e_8^T, \tag{4.19}$$

$$\Pi_1 = -(e_2 - e_3)S_1(e_2 - e_3)^T - 3(e_2 + e_3 - 2e_7)S_1(e_2 + e_3 - 2e_7), \tag{4.20}$$

$$\Pi_2 = -(e_3 - e_4)S_1(e_3 - e_4)^T - 3(e_3 + e_4 - 2e_8)S_1(e_3 + e_4 - 2e_8), \tag{4.21}$$

$$\Pi_3 = -(e_4 - e_5) S_2 (e_4 - e_5)^T - 3(e_4 + e_5 - 2e_9) S_2 (e_4 + e_5 - 2e_9), \tag{4.22}$$

$$\Pi_4 = -(e_5 - e_6) S_2 (e_5 - e_6)^T - 3(e_5 + e_2 - 2e_{10}) S_2 (e_5 + e_6 - 2e_{10}). \tag{4.23}$$

Proof 4.1 *Let us consider the following LKFs candidate*

$$V(t) = V_1(t) + V_2(t) + V_3(t). \tag{4.24}$$

where :

$$V_1(t) = x^T(t)Px(t) + \int_{t-\tau_1}^{t} x(s)^T Q_1 x(s)ds + \int_{t-\tau_2}^{t} x(s)^T Q_2 x(s)ds + \int_{t-\tau_3}^{t} x(s)^T Q_3 x(s)ds. \tag{4.25}$$

$$\begin{aligned} V_2(t) = {} & \tau_1 \int_{-\tau_1}^{0} \int_{t+s}^{t} \dot{x}(s)^T R_1 \dot{x}(s)\, ds\, dv + \tau_2 \int_{-\tau_2}^{0} \int_{t+s}^{t} \dot{x}(s)^T R_2 \dot{x}(s)\, ds\, dv \\ & + \tau_3 \int_{-\tau_3}^{0} \int_{t+s}^{t} \dot{x}(s)^T R_3 \dot{x}(s)\, ds\, dv. \end{aligned} \tag{4.26}$$

$$V_3(t) = (\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+s}^{t} \dot{x}(s)^T S_1 \dot{x}(s)\, ds\, dv + (\tau_3 - \tau_2) \int_{-\tau_3}^{-\tau_2} \int_{t+s}^{t} \dot{x}(s)^T S_2 \dot{x}(s)\, ds\, dv. \tag{4.27}$$

The LKFs candidate (4.24) *is positive if P, Q_1 Q_2, Q_3, R_1, R_2, R_3, S_1 and S_2 are all positive definite matrices. In this case, the NCSs system with network-induced delay is stochastic asymptotically stable if:*

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) < 0. \tag{4.28}$$

Let first focus on the time derivative of $V_1(t)$, one can write:

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t)P\dot{x}(t) + x^T(t)(Q_1 + Q_2 + Q_3)x(t) \\ &\quad - x^T(t-\tau_1)Q_1x(t-\tau_1) - x^T(t-\tau_2)Q_2x(t-\tau_2) - x^T(t-\tau_3)Q_3x(t-\tau_3) \\ &= \zeta^T(t)\Phi_1\zeta(t). \end{aligned} \tag{4.29}$$

with Φ_1 is expressed by (4.16) *and $\zeta(t)$ is defined as:*

$$\begin{aligned} \zeta(t) = \Big[\ & x(t) \quad x(t-\tau_1) \quad x(t-\tau_1(t)) \quad x(t-\tau_2) \quad x(t-\tau_2(t)) \quad x(t-\tau_3) \quad \tfrac{1}{(\tau_1(t)-\tau_1)} \int_{t-\tau_1(t)}^{t-\tau_1} x(s)ds \\ & \tfrac{1}{(\tau_2-\tau_1(t))} \int_{t-\tau_2}^{t-\tau_1(t)} x(s)ds \quad \tfrac{1}{(\tau_2(t)-\tau_2)} \int_{t-\tau_2(t)}^{t-\tau_2} x(s)ds \quad \tfrac{1}{(\tau_3-\tau_2(t))} \int_{t-\tau_3}^{t-\tau_2(t)} x(s)ds \quad \dot{x}(t) \quad \omega(t)\ \Big]^T. \end{aligned} \tag{4.30}$$

Now, let focus on the time derivative of $V_2(t)$. From (4.26)*, one has:*

$$\begin{aligned}\dot{V}_2(t) =& e_{11}\left(\tau_1^2 R_1 + \tau_2^2 R_2 + \tau_3^2 R_3\right) e_{11}^T - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds - \tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds \\ &- \tau_3 \int_{t-\tau_3}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds\,.\end{aligned} \tag{4.31}$$

Applying Lemma 3.3.6 to all integral terms in (4.31)*, it yields :*

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds \le -(e_1 - e_2) R_1 (e_1 - e_2)^T, \tag{4.32}$$

$$-\tau_2 \int_{t-\tau_2}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds \le -(e_1 - e_3) R_2 (e_1 - e_3)^T - (e_3 - e_4) R_2 (e_3 - e_4)^T \tag{4.33}$$

and

$$-\tau_3 \int_{t-\tau_3}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds \le -(e_1 - e_5) R_3 (e_1 - e_5)^T - (e_5 - e_6) R_3 (e_5 - e_6)^T. \tag{4.34}$$

Considering (4.32), (4.33) *and* (4.34)*, we have:*

$$\begin{aligned}\dot{V}_2(t) <& e_{11}\left(\tau_1^2 R_1 + \tau_2^2 R_2 + \tau_3^2 R_3\right) e_{11}^T - (e_1 - e_2) R_1 (e_1 - e_2)^T \\ &- (e_1 - e_3) R_2 (e_1 - e_3)^T - (e_3 - e_4) R_2 (e_3 - e_4)^T \\ &- (e_1 - e_5) R_3 (e_1 - e_5)^T - (e_5 - e_6) R_3 (e_5 - e_6)^T \\ \le& \zeta^T(t) \Phi_2 \zeta(t)\,.\end{aligned} \tag{4.35}$$

with Φ_2 is expressed by (4.17)*.*
Finally, let us focus on time derivative of $V_3(t)$ in (4.24)*, one has:*

$$\begin{aligned}\dot{V}_3(t) =& \dot{x}^T(t)\left((\tau_2 - \tau_1)^2 S_1 + (\tau_3 - \tau_2)^2 S_2\right)\dot{x}(t) - (\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) S_1 \dot{x}(s) ds \\ &- (\tau_3 - \tau_2)\int_{t-\tau_3}^{t-\tau_2} \dot{x}^T(s) S_2 \dot{x}(s) ds.\end{aligned} \tag{4.36}$$

Using similar method in [57], the integral terms in (4.36) *can be re-written as:*

$$\begin{aligned}-(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) S_1 \dot{x}(s) ds =& -(\tau_2 - \tau_1)\int_{t-\tau_2}^{t-\tau_1(t)} \dot{x}^T(s) S_1 \dot{x}(s) ds \\ &- (\tau_2 - \tau_1)\int_{t-\tau_1(t)}^{t-\tau_1} \dot{x}^T(s) S_1 \dot{x}(s) ds.\end{aligned} \tag{4.37}$$

and

$$
\begin{aligned}
-(\tau_3-\tau_2)\int_{t-\tau_3}^{t-\tau_2}\dot{x}^T(s)S_2\dot{x}(s)ds = &-(\tau_3-\tau_2)\int_{t-\tau_3}^{t-\tau_2(t)}\dot{x}^T(s)S_2\dot{x}(s)ds \\
&-(\tau_3-\tau_2)\int_{t-\tau_2(t)}^{t-\tau_2}\dot{x}^T(s)S_2\dot{x}(s)ds\,.
\end{aligned}
\tag{4.38}
$$

Then, we can rewrite the previous equality as follows:

$$
\begin{aligned}
&-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)S_1\dot{x}(s)ds = \\
&-(\tau_2-\tau_1(t))\int_{t-\tau_2}^{t-\tau_1(t)}\dot{x}^T(s)S_1\dot{x}(s)ds-(\tau_1(t)-\tau_1)\int_{t-\tau_1(t)}^{t-\tau_1}\dot{x}^T(s)S_1\dot{x}(s)ds \\
&-(\tau_1(t)-\tau_1)\int_{t-\tau_2}^{t-\tau_1(t)}\dot{x}^T(s)S_1\dot{x}(s)ds-(\tau_2-\tau_1(t))\int_{t-\tau_1(t)}^{t-\tau_1}\dot{x}^T(s)S_1\dot{x}(s)ds.
\end{aligned}
\tag{4.39}
$$

Since $\tau_2-\tau_1(t)\le\tau_2-\tau_1$ *and* $\tau_1(t)-\tau_1\le\tau_2-\tau_1$*, we can write:*

$$
\begin{aligned}
&-(\tau_1(t)-\tau_1)\int_{t-\tau_2}^{t-\tau_1(t)}\dot{x}(s)^TS_1\dot{x}(s) = \\
&-\frac{(\tau_1(t)-\tau_1)}{\tau_2-\tau_1}\int_{t-\tau_2}^{t-\tau_1(t)}(\tau_2-\tau_1)\dot{x}(s)^TS_1\dot{x}(s)\le-\frac{(\tau_1(t)-\tau_1)}{\tau_2-\tau_1}\int_{t-\tau_2}^{t-\tau_1(t)}(\tau_2-\tau_1(t))\dot{x}(s)^TS_1\dot{x}(s).
\end{aligned}
\tag{4.40}
$$

and

$$
\begin{aligned}
&-(\tau_2-\tau_1(t))\int_{t-\tau_1(t)}^{t-\tau_1}\dot{x}(s)^TS_1\dot{x}(s) = \\
&-\frac{(\tau_2-\tau_1(t))}{\tau_2-\tau_1}\int_{t-\tau_1(t)}^{t-\tau_1(t)}(\tau_2-\tau_1)\dot{x}(s)^TS_1\dot{x}(s)\le-\frac{(\tau_2-\tau_1(t))}{\tau_2-\tau_1}\int_{t-\tau_1(t)}^{t-\tau_1}(\tau_1(t)-\tau_1)\dot{x}(s)^TS_1\dot{x}(s)\,.
\end{aligned}
\tag{4.41}
$$

Next, applying Lemma 3.3.4 to (4.40) *and* (4.41)*, the equality* (4.39) *yields:*

$$
\begin{aligned}
&-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)S_1\dot{x}(s)ds\le \\
&-(e_2-e_3)S_1(e_2-e_3)^T-3(e_2+e_3-2e_7)S_1(e_2+e_3-2e_7)-(e_3-e_4)S_1(e_3-e_4)^T \\
&-3(e_3+e_4-2e_8)S_1(e_3+e_4-2e_8)-\alpha(e_2-e_3)S_1(e_2-e_3)^T \\
&-3\alpha(e_2+e_3-2e_7)S_1(e_2+e_3-2e_7)-\beta(e_3-e_4)S_1(e_3-e_4)^T \\
&-3\beta(e_3+e_4-2e_8)S_1(e_3+e_4-2e_8)\ .
\end{aligned}
\tag{4.42}
$$

where $\alpha=\left(\frac{\tau_1(t)-\tau_1}{\tau_2-\tau_1}\right)$ *and* $\beta=\left(\frac{\tau_2-\tau_1(t)}{\tau_2-\tau_1}\right)$.

Similarly for (4.38)*, since* $\tau_3 - \tau_2(t) \le \tau_3 - \tau_2$ *and* $\tau_2(t) - \tau_2 \le \tau_3 - \tau_2$*, we get:*

$$\begin{aligned}
&-(\tau_3 - \tau_2)\int_{t-\tau_3}^{t-\tau_2} \dot{x}^T(s) S_2 \dot{x}(s) ds \le \\
&-(e_4 - e_5) S_2 (e_4 - e_5)^T - 3(e_4 + e_5 - 2e_9) S_2 (e_4 + e_5 - 2e_9) - (e_5 - e_6) S_2 (e_5 - e_6)^T \\
&-3(e_5 + e_6 - 2e_{10}) S_2 (e_5 + e_6 - 2e_{10}) - \alpha_1 (e_4 - e_5) S_2 (e_4 - e_5)^T \\
&-3\alpha_1 (e_4 + e_5 - 2e_9) S_2 (e_4 + e_5 - 2e_9) - \beta_1 (e_5 - e_6) S_2 (e_5 - e_6)^T \\
&-3\beta_1 (e_5 + e_6 - 2e_{10}) S_2 (e_5 + e_6 - 2e_{10}) \; .
\end{aligned} \tag{4.43}$$

where $\alpha_1 = \left(\frac{\tau_2(t)-\tau_2}{\tau_3-\tau_2}\right)$ *and* $\beta_1 = \left(\frac{\tau_3-\tau_2(t)}{\tau_3-\tau_2}\right)$.

Thus, from (4.36)-(4.43)*, the time derivative of* (4.27) *is equivalent to:*

$$\dot{V}_3(t) \le \zeta^T(t)\Phi_3 + \alpha\Pi_1 + \alpha_1\Pi_3 + \beta\Pi_2 + \beta_1\Pi_4\zeta(t) \, . \tag{4.44}$$

where Φ_3*,* Π_1*,* Π_2*,* Π_3 *and* Π_4 *are defined in* (4.18)*,* (4.20)*,* (4.21)*,* (4.22) *and* (4.23) *respectively.*
Now, let us rewrite the NCSs model with network-induced delay (4.8) *as:*

$$\begin{aligned}
&Ax(t) + \bar{\sigma}BKx(t - \tau_1(t)) + (1 - \overline{\sigma})BKx(t - \tau_2(t)) \\
&+ (\sigma(t) - \overline{\sigma})BK(x(t - \tau_1(t)) - x(t - \tau_2(t))) \\
&+ B_w\omega(t) - \dot{x}(t) = 0 \, .
\end{aligned} \tag{4.45}$$

which can be rewritten as:

$$(\mathcal{G} + \mathcal{G}_\sigma(t))\zeta(t) = 0. \tag{4.46}$$

where

$$\begin{aligned}
\mathcal{G} &= \begin{bmatrix} \bar{A} & 0 & \bar{\sigma}BK & 0 & (1-\bar{\sigma})BK & 0_{(1\times5)} & -I & B_w \end{bmatrix}, \\
\mathcal{G}_\sigma(t) &= \begin{bmatrix} 0_{(1\times2)} & (\sigma(t) - \overline{\sigma})\bar{B}K & 0 & (\sigma(t) - \overline{\sigma})\bar{B}K & 0_{(1\times7)} \end{bmatrix} .
\end{aligned}$$

Since $\mathbb{E}\{\sigma(t) - \overline{\sigma}\} = 0$ *(which implies* $\mathbb{E}\{\mathcal{G}_\sigma(t) = 0\}$*). From* (4.29) - (4.44) *and* (4.45)*, and applying lemma 3.3.2, the NCSs model with network-induced delay* (4.8) *is stochastic asymptotically stable if there exists:* $\mathcal{M} \in \mathbb{R}^{4n\times n}$ *such that:*

$$\dot{V}(t) \le \zeta^T(t)\sum_{i=1}^{3}\Phi_i + \mathcal{H}(\mathcal{M}\mathcal{G}) + \alpha\Pi_1 + \alpha_1\Pi_3 + \beta\Pi_2 + \beta_1\Pi_4\zeta(t) < 0 \, . \tag{4.47}$$

Now, let us discuss the H_∞ *performance defined in Section II. It is well known that the* H_∞ *performance and stability analysis can be derived from the following condition :*

$$\dot{V}(t) + y^T(s)y(s) - \gamma^2 w^T(s)w(s) < 0 \, . \tag{4.48}$$

From (4.47)*, the inequality* (4.48) *can be rewritten as:*

$$\zeta^T(t)\sum_{i=1}^{3}\Phi_i + \mathcal{H}_e(\mathcal{MG}) + \alpha\Pi_1 + \alpha_1\Pi_3 + \beta\Pi_2 + \beta_1\Pi_4\zeta(t) + y^T(s)y(s) - \gamma^2 w^T(s)w(s) < 0\,. \tag{4.49}$$

To deal with $\sigma(t)$*, we can express* $y^T(t)y(t)$ *as:*

$$\begin{aligned}
&y^T(t)y(t) = \mathbb{E}\{[Cx(t) + \overline{\sigma}DKx(t-\tau_1(t)) + (1-\overline{\sigma})DKx(t-\tau_2(t))\\
&+(\sigma(t)-\overline{\sigma})DK[x(t-\tau_1(t)) - x(t-\tau_2(t))] + D_w\omega(t)]^T\\
&\times[Cx(t) + \overline{\sigma}DKx(t-\tau_1(t)) + (1-\overline{\sigma})DKx(t-\tau_2(t))\\
&+(\sigma(t)-\overline{\sigma})DK[x(t-\tau_1(t)) - x(t-\tau_2(t))] + D_w\omega(t)]\}\\
&\leq \zeta(t)^T\tilde{D}^T\tilde{D}\zeta(t)\,.
\end{aligned} \tag{4.50}$$

where

$$\tilde{D} = \begin{bmatrix} C & 0 & \overline{\sigma}DK & 0 & (1-\overline{\sigma})DK & 0_{(1\times 6)} & D_w \end{bmatrix}.$$

Therefore, the condition (4.49) *becomes:*

$$\zeta^T(t)\sum_{i=1}^{3}\Phi_i + \mathcal{H}_e(\mathcal{MG}) + \alpha\Pi_1 + \alpha_1\Pi_3 + \beta\Pi_2 + \beta_1\Pi_4 + \tilde{D}^T\tilde{D}\zeta(t) - \gamma^2 w^T(s)w(s) < 0\,. \tag{4.51}$$

Then, applying lemma (3.3.3) *and Schur complement* (3.3.1) *to* (4.51) *with*

$$-\gamma^2 w^T(t)w(t) = \zeta^T(t)\Phi_4\zeta(t)\,.$$

where Φ_4 *is given in* (4.19)*. We obtain the conditions expressed in Theorem 4.1. This completes the proof.*

Remark 4.3 *lemma 3.3.6 is used to estimate the integral terms in the time derivative of* (4.26)*. We did not use lemma 3.3.4 (Wirtinger inequality) to avoid introduction of new cross terms with poor information in the current terms of* $\zeta(t)$*. This lead to more conservatism. Moreover, since the introduction of lemma 3.3.3, the convexity of the matrix equations* (4.42) *and* (4.43) *is employed to derive the criteria without introducing any additional variable.*

The results of Theorem 4.1 can also be reduced to the case of unperturbed NCSs with network-induced delay given by :

$$\begin{cases} \dot{x}(t) = Ax(t) + \sigma(t)BKx(t-\tau_1(t))\,(1-\sigma(t))\,BKx(t-\tau_2(t))\,, \\ x(t) = \phi(t) \qquad t\in[-\tau_3, -\tau_1]. \end{cases} \tag{4.52}$$

Then, we have the following corollary.

Corollary 4.1 *For given scalars $\tau_i, (i \in \mathcal{I}_3)$ such that $\tau(t) \in [\tau_1, \tau_3]$ with known feedback gain K, the considered NCSs in* (4.52) *is stochastic asymptotically stable if there exist positive definite symmetric matrices P, $R_i, Q_i (i \in \mathcal{I}_3)$ and $S_j (j \in \mathcal{I}_2)$, and real matrices $M_l (l \in \mathcal{I}_4)$ with appropriate dimensions such that the following LMIs hold:*

$$\begin{aligned} &\sum_{i=1}^{3} \Phi_i + \mathcal{H}_e\left(\mathcal{M}\bar{\mathcal{G}}\right) + \Pi_1 + \Pi_3 < 0, \\ &\sum_{i=1}^{3} \Phi_i + \mathcal{H}_e\left(\mathcal{M}\bar{\mathcal{G}}\right) + \Pi_2 + \Pi_4 < 0\,, \\ &\sum_{i=1}^{3} \Phi_i + \mathcal{H}_e\left(\mathcal{M}\bar{\mathcal{G}}\right) + \Pi_1 + \Pi_3 < 0\,, \\ &\sum_{i=1}^{3} \Phi_i + \mathcal{H}_e\left(\mathcal{M}\bar{\mathcal{G}}\right) + \Pi_2 + \Pi_4 < 0\,. \end{aligned} \tag{4.53}$$

where $\Phi_1, \Phi_2, \Phi_3, \Pi_1, \Pi_2, \Pi_3, \Pi_4$ are defined in Theorem 4.1 and

$$\bar{\mathcal{G}} = \begin{bmatrix} A & 0 & \bar{\sigma}BK & 0 & (1-\bar{\sigma})\,BK & 0_{(1\times 5)} & -I \end{bmatrix}.$$

Proof 4.2 *From the conditions expressed in Theorem 4.1, by setting $\omega(t) \equiv 0$, we easily obtain the conditions expressed in corollary 4.53.*

4.3.2 Stabilization and H_∞ state feedback controller design.

In this section, based on Theorem 4.1, the robust $H\infty$ controller problem of NCSs (4.8) using the state feedback controller (4.3) will be studied. The main result is proposed in the following theorem.

Theorem 4.2 *For given scalars $\tau_i, (i \in \mathcal{I}_3)$ and $\gamma > 0$ such that $\tau(t) \in [\tau_1, \tau_3]$ the considered NCSs* (4.8) *is robustly stochastic asymptotically stabilized by the NCSs controller* (4.3) *and satisfy the H_∞ performance index γ, if there exist positive definite symmetric matrices X, $\bar{P}$, $\bar{R}_i, \bar{Q}_i (i \in \mathcal{I}_3)$ and $\bar{S}_j, (j \in \mathcal{I}_2)$ with appropriate dimensions and scalars $\varepsilon_r > 0$ $(r \in \mathcal{I}_3)$ such that the following LMIs hold:*

$$\begin{bmatrix} \sum_{i=1}^{4} \bar{\Phi}_i + \mathcal{H}_e(\mathcal{X}) + \bar{\Pi}_1 + \bar{\Pi}_3 & \bar{D}^T \\ * & -I \end{bmatrix} < 0\ , \tag{4.54}$$

$$\begin{bmatrix} \sum_{i=1}^{4} \bar{\Phi}_i + \mathcal{H}_e(\mathcal{X}) + \bar{\Pi}_1 + \bar{\Pi}_4 & \bar{D}^T \\ * & -I \end{bmatrix} < 0\ , \tag{4.55}$$

$$\begin{bmatrix} \sum_{i=1}^{4}\bar{\Phi}_i + \mathcal{H}_e(\mathcal{X}) + \bar{\Pi}_2 + \bar{\Pi}_3 & \bar{D}^T \\ * & -I \end{bmatrix} < 0 \ , \tag{4.56}$$

$$\begin{bmatrix} \sum_{i=1}^{4}\bar{\Phi}_i + \mathcal{H}_e(\mathcal{X}) + \bar{\Pi}_2 + \bar{\Pi}_4 & \bar{D}^T \\ * & -I \end{bmatrix} < 0 \ . \tag{4.57}$$

where:

$$\mathcal{X}=\begin{bmatrix} AX & 0 & \bar{\sigma}B\bar{K} & 0 & (1-\bar{\sigma})B\bar{K} & 0_{(1\times5)} & -X & B_w \\ \varepsilon_1 AX & 0 & \varepsilon_1\bar{\sigma}B\bar{K} & 0 & \varepsilon_1(1-\bar{\sigma})B\bar{K} & 0_{(1\times5)} & -\varepsilon_1 X & \varepsilon_1 B_w \\ \varepsilon_2 AX & 0 & \varepsilon_2\bar{\sigma}B\bar{K} & 0 & \varepsilon_2(1-\bar{\sigma})B\bar{K} & 0_{(1\times5)} & -\varepsilon_2 X & \varepsilon_2 B_w \\ 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times5)} & 0_{(5\times1)} & 0_{(5\times1)} \\ \varepsilon_3 AX & 0 & \varepsilon_3\bar{\sigma}B\bar{K} & 0 & \varepsilon_3(1-\bar{\sigma})B\bar{K} & 0_{(1\times5)} & -\varepsilon_3 X & \varepsilon_3 B_w \\ 0 & 0 & 0 & 0 & 0 & 0_{(1\times5)} & 0 & 0 \end{bmatrix},$$

$$\bar{D} = \begin{bmatrix} CX & 0 & \overline{\sigma}DF & 0 & (1-\overline{\sigma})DF & 0_{(1\times6)} & D_w \end{bmatrix},$$

$$\bar{\Phi}_1 = 2e_1\bar{P}e_{11}^T + e_1\left(\bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3\right)e_1^T - e_2\bar{Q}_1e_2^T - e_4\bar{Q}_2e_4^T - e_6\bar{Q}_3e_6^T, \tag{4.58}$$

$$\begin{aligned}\Phi_2 =& e_{11}\left(\tau_1^2\bar{R}_1 + \tau_2^2\bar{R}_2 + \tau_3^2\bar{R}_3\right)e_{11}^T - (e_1-e_2)\bar{R}_1(e_1-e_2)^T - (e_1-e_3)\bar{R}_2(e_1-e_3)^T \\ & - (e_3-e_4)\bar{R}_2(e_3-e_4)^T - (e_1-e_5)\bar{R}_3(e_1-e_5)^T - (e_5-e_6)\bar{R}_3(e_5-e_6)^T,\end{aligned} \tag{4.59}$$

$$\begin{aligned}\bar{\Phi}_3 =& e_{11}\left((\tau_2-\tau_1)^2\bar{S}_1 + (\tau_3-\tau_2)^2\bar{S}_2\right)e_{11}^T - (e_2-e_3)\bar{S}_1(e_2-e_3)^T - (e_3-e_4)\bar{S}_1(e_3-e_4)^T \\ & - (e_4-e_5)\bar{S}_2(e_4-e_5)^T - (e_5-e_6)\bar{S}_2(e_5-e_6)^T - 3(e_2+e_3-2e_7)\bar{S}_1(e_2+e_3-2e_7)^T \\ & - 3(e_3+e_4-2e_8)\bar{S}_1(e_3+e_4-2e_8)^T - 3(e_4+e_5-2e_9)\bar{S}_2(e_4+e_5-2e_9)^T \\ & - 3(e_5+e_6-2e_{10})\bar{S}_2(e_5+e_6-2e_{10})^T \ ,\end{aligned} \tag{4.60}$$

$$\bar{\Phi}_4 = -\gamma^2 e_8 e_8^T, \tag{4.61}$$

$$\bar{\Pi}_1 = -(e_2-e_3)\bar{S}_1(e_2-e_3)^T - 3(e_2+e_3-2e_7)\bar{S}_1(e_2+e_3-2e_7), \tag{4.62}$$

$$\bar{\Pi}_2 = -(e_3-e_4)\bar{S}_1(e_3-e_4)^T - 3(e_3+e_4-2e_8)\bar{S}_1(e_3+e_4-2e_8), \tag{4.63}$$

$$\bar{\Pi}_3 = -(e_4-e_5)\bar{S}_2(e_4-e_5)^T - 3(e_4+e_5-2e_9)\bar{S}_2(e_4+e_5-2e_9), \tag{4.64}$$

$$\bar{\Pi}_4 = -(e_5-e_6)\bar{S}_2(e_5-e_6)^T - 3(e_5+e_2-2e_{10})\bar{S}_2(e_5+e_6-2e_{10}). \tag{4.65}$$

Proof 4.3 *Based on Theorem 4.1, let X be an invertible matrix with appropriate dimension and* $M_1 = X^{-T}$, $M_{i+1} = \varepsilon_i X^{-T} (i \in \mathcal{I}_3)$. *The slack variables vector* $\mathcal{M}$ *defined in Theorem 4.1 can be re-expressed with the previous change of variables as:*
$\mathcal{M} = \begin{bmatrix} X^{-T} & 0 & \varepsilon_1 X^{-T} & 0 & \varepsilon_2 X^{-T} & 0_{(1\times5)} & \varepsilon_3 X^{-T} & 0 \end{bmatrix}$ *with* $\varepsilon_r (r \in \mathcal{I}_3)$ *are arbitrary scalars.*

Let $D_X = diag\begin{bmatrix} X^T & X^T & X^T & X^T & X^T & X^T & X^T & X^T & X^T & X^T & X^T & I \end{bmatrix}$ *Pre-and-post multiplying* (4.12), (4.13), (4.14) *and* (4.15), *by* D_X *and its transpose respectively and with variables change* $F = KX$, $\bar{P} = X^T PX$, $\bar{R}_i = X^T R_i X$, $\bar{Q}_i = X^T Q_i X, (i \in \mathcal{I}_3)$ *and* $\bar{S}_j = X^T S_j X (j \in \mathcal{I}_2)$, *we obtain the conditions expressed in Theorem 4.2. This complete the proof.*

Remark 4.4 *Using Theorem 4.2 and fixing the Network-induced delay bound, we can obtain the optimal attenuation level* γ *by solving the following constrained optimization problem:*

$$\begin{aligned} & min\ \gamma^2 \\ & s.t \begin{cases} X > 0, \bar{P} > 0, \bar{R}_i > 0, \bar{Q}_i > 0, \bar{S}_j > 0. (i \in \mathcal{I}_3, j \in \mathcal{I}_2), \\ LMI\ (4.54), (4.55), (4.56)\ and\ (4.57). \end{cases} \end{aligned} \tag{4.66}$$

Theorem 4.2 considers the robust H_∞ controller design. Further, when $\omega(t) = 0$, we can easily get the NCSs model in (4.1). Then, we can derive the following corollary:

Corollary 4.2 *For given scalars* $\tau_i, (i \in \mathcal{I}_3)$ *such that* $\tau(t) \in [\tau_1, \tau_3]$, *the considered NCSs* (4.52) *is stochastic asymptotically stabilized by the controller* (4.3), *if there exist positive definite symmetric matrices* X, $\bar{P}$, $\bar{R}_i, Q_i (i \in \mathcal{I}_3)$ *and* $S_j (j \in \mathcal{I}_2)$ *with appropriate dimensions and scalars* $\varepsilon_r > 0$ $(r \in \mathcal{I}_3)$ *such that the following LMIs hold:*

$$\begin{aligned} \sum_{i=1}^{3} \bar{\Phi}_i + \mathcal{H}_e\left(\tilde{\mathcal{X}}\right) + \bar{\Pi}_1 + \bar{\Pi}_3 < 0, \\ \sum_{i=1}^{3} \bar{\Phi}_i + \mathcal{H}_e\left(\tilde{\mathcal{X}}\right) + \bar{\Pi}_2 + \bar{\Pi}_4 < 0, \\ \sum_{i=1}^{3} \bar{\Phi}_i + \mathcal{H}_e\left(\tilde{\mathcal{X}}\right) + \bar{\Pi}_1 + \bar{\Pi}_3 < 0, \\ \sum_{i=1}^{3} \bar{\Phi}_i + \mathcal{H}_e\left(\tilde{\mathcal{X}}\right) + \bar{\Pi}_2 + \bar{\Pi}_4 < 0. \end{aligned} \tag{4.67}$$

where $\bar{\Phi}_1, \bar{\Phi}_2, \bar{\Phi}_3, \bar{\Pi}_1, \bar{\Pi}_2, \bar{\Pi}_3, \bar{\Pi}_4$ *are defined in Theorem 4.2 and*

$$\tilde{\mathcal{X}} = \begin{bmatrix} AX & 0 & \bar{\sigma} B\bar{K} & 0 & (1-\bar{\sigma}) B\bar{K} & 0_{(1\times5)} & -X \\ \varepsilon_1 AX & 0 & \varepsilon_1 \bar{\sigma} B\bar{K} & 0 & \varepsilon_1 (1-\bar{\sigma}) B\bar{K} & 0_{(1\times5)} & -\varepsilon_1 X \\ \varepsilon_2 AX & 0 & \varepsilon_2 \bar{\sigma} B\bar{K} & 0 & \varepsilon_2 (1-\bar{\sigma}) B\bar{K} & 0_{(1\times5)} & -\varepsilon_2 X \\ 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times1)} & 0_{(5\times5)} & 0_{(5\times1)} \\ \varepsilon_3 AX & 0 & \varepsilon_3 \bar{\sigma} B\bar{K} & 0 & \varepsilon_3 (1-\bar{\sigma}) B\bar{K} & 0_{(1\times5)} & -\varepsilon_3 X \end{bmatrix}.$$

4.4 Numerical examples

In this section, many examples are provided to illustrate the effectiveness and the conservatism reduction of the results presented in the previous section. The first three examples are for the robust stability improvement (Theorem 4.1) and the two last ones for the H_∞ state feedback controller design (Theorem 4.2). The purpose is to compare the maximum allowable bounds of time-delay that guaranteed the asymptotic stability of the NCSs.

Example 4.1 *Consider the following NCSs described by*

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, B \times K = \begin{bmatrix} 0 & 1 \\ -0.3750 & -1.1500 \end{bmatrix}, B_w = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D \times K = \begin{bmatrix} -0.3750 & -1.1500 \end{bmatrix}, D_w = 0.$$

Table (4.1) *shows the minimized H_∞ performance index γ by given values of lower and upper bounds of network-induced delay, when different methods are applied. The results show that the stability criterion in Theorem 4.1 gives the best H_∞ performance index.*

Table 4.1: Minimized H_∞ performance index γ when $\bar{\sigma} = 0.5$ and $\tau_2 = (\tau_3 + \tau_1)/2$ of Motor system

Methods	$\tau_3 = 0.8695$		$\tau_3 = 1$		
	$\tau_1 = 0$	$\tau_1 = 0.5695$	$\tau_1 = 0$	$\tau_1 = 0.5$	$\tau_1 = 0.9$
[33]	0.87	0.81	4.05	3.27	2.59
[44]	0.81	0.72	2.95	2.13	1.73
[42]	0.78	0.73	2.61	1.76	1.87
[6](Cor1)	0.63	0.59	2.37	1.61	1.32
[6](Th1)	0.51	0.35	1.97	1.28	0.94
[98]	0.40	0.38	0.66	0.60	0.55
Theorem 4.1	**0.44**	**0.27**	**0.65**	**0.39**	**0.28**

Remark 4.5 *In the next examples, we used the value of τ_2 close to τ_1 more than τ_3, the reason has been mentioned in [110]: the measure of the round-trip time (RTT) delays from different Ethernet network nodes for 24h show that in an interval* $[20, 139ms]$, 90% *of $\tau(t)$ falls into the lower interval* $[20, 53ms]$ *and only* 10% *are within the interval of* $[53, 139ms]$.

Example 4.2 *Consider the following linear system with time-varying delay (example used in many papers):*

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B \times K = \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}.$$

Table (4.2) *lists the maximum allowable upper bounds* τ_3 *that guarantees the asymptotic stability of system* (4.8) *for various values of* τ_1. *It can be seen that the proposed method gives best results compared with others.*

Table 4.2: **The obtained** $maub(\tau_3)$ **with varying lower bound of networked-induced delay** τ_1 **of first order system**

Considered results / τ_1	1	2	3	4	5
[103]	1.6198	2.4884	3.3403	4.3424	5.2970
[167]($m = 4$)	1.7228	2.5608	3.4542	4.3787	5.3228
[62]	1.7753	2.6134	3.5046	4.4271	5.3696
[42]	1.8446	2.6344	3.5124	4.4304	5.3709
Corollary 4.1 ($\sigma = 0.6$)	**2.4600**	**3.1040**	**3.8741**	**4.7112**	**5.5940**
Corollary 4.1 ($\sigma = 0.8$)	**3.5600**	**3.8301**	**4.3400**	**5.0330**	**5.7862**

Now, in order to confirm the asymptotic stability of the system, Fig.4.1 presents a simulations of the states responses of NCSs with network-induced delay profile of Fig.4.2 and stochastic distribution of Fig.4.3. With: $\bar{\sigma} = 0.2$, $\tau_1 = 0.5$, $\tau_2 = 0.7$, $\tau_3 = 1.5251$ *and the initial conditions:* $x(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$, $\forall \tau(t) \in [-\tau_3, -\tau_1]$.

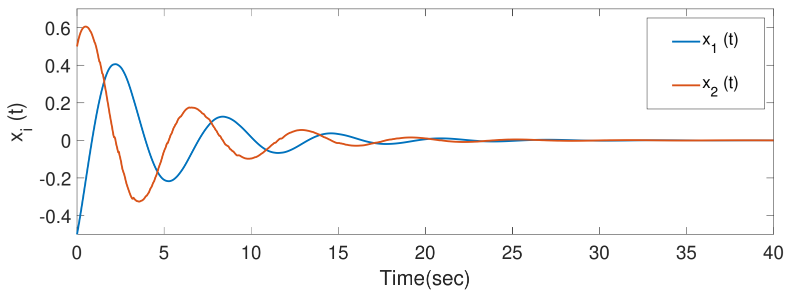

Figure 4.1: States Trajectories in example of First order system.

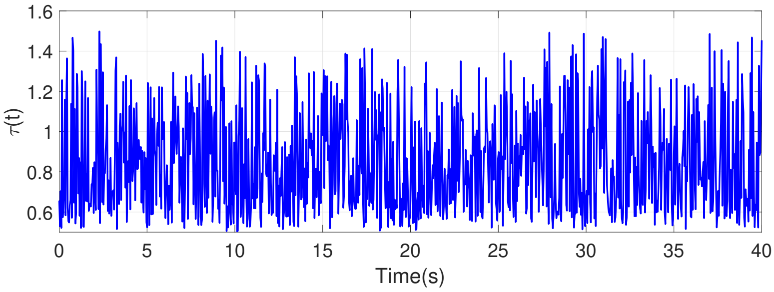

Figure 4.2: Delay profile in the simulation in example of First order system.

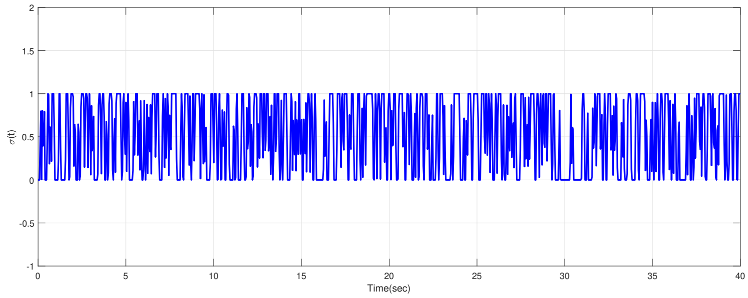

Figure 4.3: Stochastic distribution Delay '$\sigma(t)$'in the simulation of First order system.

Example 4.3 *In this example, we apply Corollary 4.1 to examine the stability of a power system with wide-area damping controllers (2-area, 4-machine) containing 11 nodes and 12 lines, the scheme of this power system is illustrated in Fig.4.4, (for more details, see [67]). The system is represented as follow:*

$$A = \begin{bmatrix} 0 & 0 & 0 & 376.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 376.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 376.9 \\ -0.073 & 0.065 & 0.004 & -0.730 & 0.272 & 0.076 \\ 0.058 & -0.087 & 0.009 & 1.160 & -0.343 & -0.134 \\ 0.008 & 0.011 & -0.082 & -0.020 & 0.047 & -0.554 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.234 & -0.839 & 0.010 \\ 0 & -0.0011 & 0.0010 & -0.348 & -1.362 & -0.138 \\ 0 & 0.0010 & 0 & 0.049 & -0.290 & -0.638 \end{bmatrix},$$

$$B_1 = B \times K.$$

Table 4.3 lists the maximum admissible upper bounds of input delay for various values of $\bar{\sigma}$ with the proposed method (Corollary 4.1) compared to [67] ,[57] and [159].
Fig.4.5 shows the states trajectories of the above power system with $\tau_1 = 0.1$, $\tau_2 = 0.15$, $\bar{\sigma} = 0.3$, $\tau_3 = 0.4032$ and the initial condition $x(0) = \begin{bmatrix} -0.6 & 0.5 & -0.4 & 0.5 & 0.3 & 0.2 \end{bmatrix}^T$.
It can be seen that the system is asymptotically stable.

Example 4.4 *(Robust H_∞ state feedback controller design) : Consider the following*

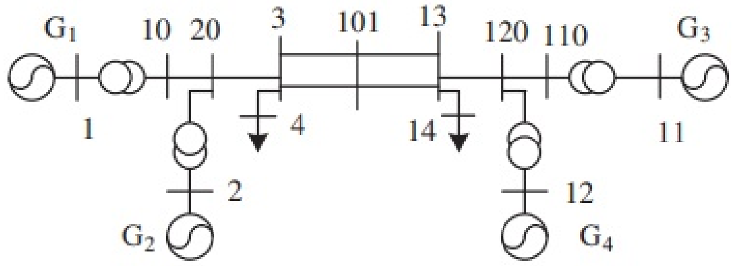

Figure 4.4: 2-area 4-machine system [67].

Table 4.3: The obtained *maub*(τ_3) with subject to varying $\bar{\sigma}$ of 2-area, 4-machine

Considered results / τ_1	0.1
[67]	0.2818
[57] $\sigma = 2$	0.2872
[57] $\sigma = 10$	0.2857
[159] $Th2$	0.3000
[159] $Th1$	0.3346
Corollary 4.1 ($\bar{\sigma} = 0.3$)	**0.4032**
Corollary 4.1 ($\bar{\sigma} = 0.7$)	**0.7400**

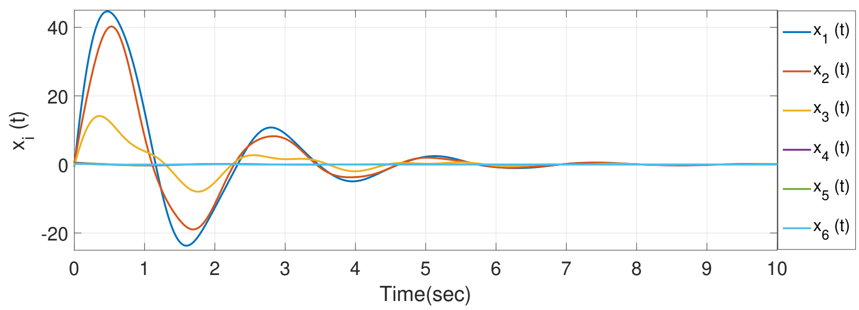

Figure 4.5: States Trajectories of wide-area damping controllers (2-area, 4-machine).

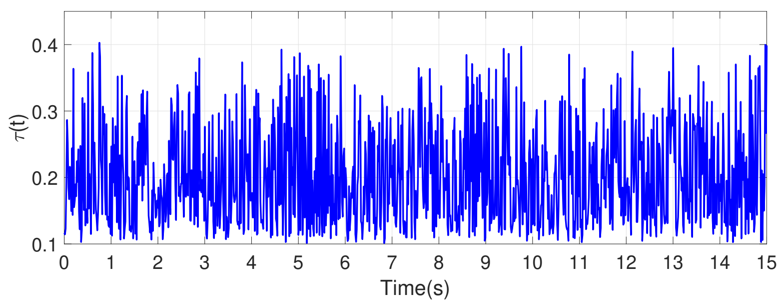

Figure 4.6: Delay profile in the simulation of wide-area damping controllers (2-area, 4-machine).

time-varying system model controlled over network given by:

$$A = \begin{bmatrix} 4 & 0.1 & -0.3 \\ -0.2 & 3 & -0.2 \\ 0.2 & -0.3 & 2 \end{bmatrix}, B = \begin{bmatrix} -4 & 0.2 & 0 \\ 0 & 3 & 1 \\ 0.1 & 0 & 3 \end{bmatrix}, D_w = 0.1$$

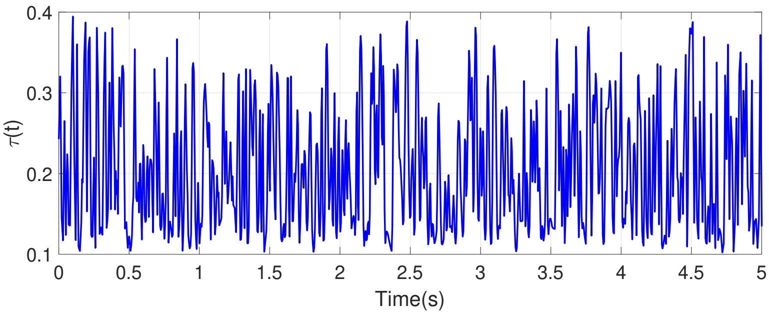

Figure 4.7: Stochastic distribution Delay '$\sigma(t)$' in the simulation of wide-area damping controllers (2-area, 4-machine).

$$C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, B_w = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The main goal is to design a robust state feedback controller guarantying the NCSs stability by the minimization of the H_∞ *disturbance attenuation level* γ*. For that, we use Theorem 4.2 with:* $0.1 \leq \tau(t) \leq 0.4$ *,*$\tau_2 = 0.15$*,* $\varepsilon_1 = \varepsilon_2 = 10^2$ *and* $\sigma = 1$*. After solving the convex optimization problem* (4.66)*, we obtain* $\gamma = 0.3851$ *under the state feedback gains:*

$$K = \begin{bmatrix} 1.4335 & -0.0519 & -0.0537 \\ -0.0495 & -1.3700 & 0.3977 \\ -0.1116 & 0.1323 & -1.0706 \end{bmatrix},$$

Now, in order to show the feasibility of the design procedure proposed above, let plot the state trajectories Fig. 4.8 with the delay introduced in network channel Fig.4.8 and commands trajectories Fig.4.10. The disturbance signal is defined as $[a; b]$ *with* a, b *are uniformly distributed randomly over* $[-0.01\ 0.01]$ *and the initial conditions are assumed to be* $x(0) = \begin{bmatrix} 0.5 & -0.5 & 1.4 \end{bmatrix}^T$*. One can see the good stabilization performances of the obtained controller.*

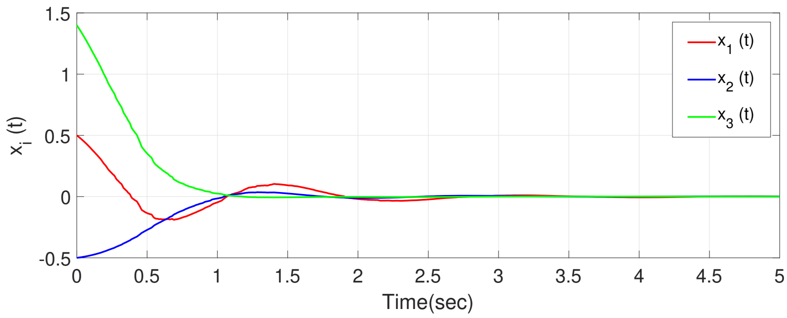

Figure 4.8: States Trajectories of example 4.4.

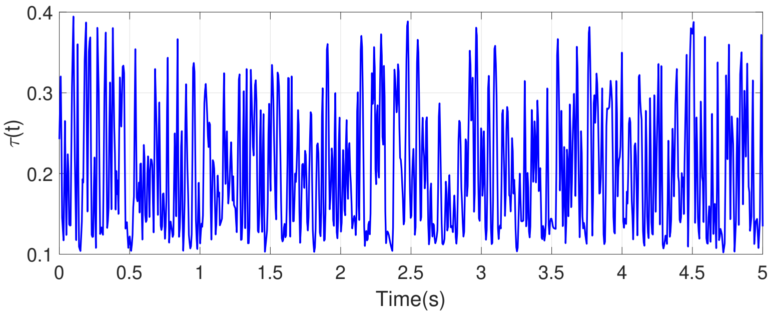

Figure 4.9: Delay profile in the simulation of example 4.4.

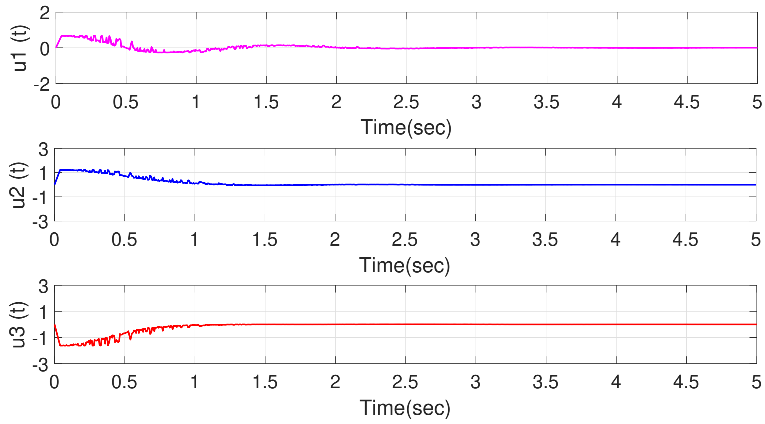

Figure 4.10: Commands Trajectories of example 4.4.

Example 4.5 *(state feedback controller design for inverted pendulum): In this example, we apply the proposed feedback controller design method "Corollary 4.2" to control "T-shape" inverted pendulum system over TCP network (more details on this system can be found in [77]. The linearized model state matrices of the inverted pendulum (without friction) are given by :*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -21.54 & 0 & 14.96 & 0 \\ 0 & 0 & 0 & 1 \\ 65.28 & 0 & -15.59 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 8.10 \\ 0 \\ -10.31 \end{bmatrix},$$

The inverted pendulum NCSs is modelled as continuous-time system with input delay as in [145] :

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx\,(t - \tau(t)) \\ x(t) = \phi, t \in [-\tau_3, -\tau_1] \end{cases} \tag{4.68}$$

For given various value of τ_1*, Table 4.4 lists the maximum allowable upper bound of* τ_3 *using Corollary 4.2, and compare the obtained results with those of [21] who choose lower bound of network-induced delay as* $h_1 = 0$ *and get the maximum allowable value* $h_2 = 0.0768$ *under the state feedback controller* $u(t) = -\begin{bmatrix} 11.1384 & 2.1632 & 3.3258 & 0.9217 \end{bmatrix} x(t)$.

Using Corollary 4.2, for given $\tau_1 = 0$, $\tau_2 = 0.01$, $\bar{\sigma} = 0.7$ *and* $\varepsilon_r = 0.01, r \in \mathcal{I}_3$, *we obtain the maximum allowable value* $\tau_3 = 0.1527$ *under the state feedback controller law* $u(t) = \begin{bmatrix} -13.2864 & -3.6575 & -6.6620 & -1.8834 \end{bmatrix} x(t)$. *It is clear that Corollary 4.2 gives less conservative results than in [21]).*

Table 4.4: The obtained $maub(\tau_3)$ with $\tau_2 = \tau_1 + 0.01$ and varying τ_1 of Inverted pendulum system

.

Considered results / τ_1	0	0.01	0.02	0.03
Corollary 4.2 ($\sigma = 0.7$)	**0.152**	**0.120**	**0.094**	**0.070**
Corollary 4.2 ($\sigma = 0.9$)	**0.570**	**0.425**	**0.280**	**0.140**

The states trajectories of the inverted pendulum system with the obtained state feedback controller under the initial conditions $x(0) = \begin{bmatrix} -0.1 & 0.5 & -0.3 & 0.2 \end{bmatrix}^T$ *and* $0 \leq \tau(t) \leq 0.1527$ *tends to zero as showed by Fig.4.11, under the delay profile Fig.4.12 and stochastic distribution of of stochastic delay Fig.4.13. Hence, the proposed state feedback controller design procedure clearly shows its effectiveness.*

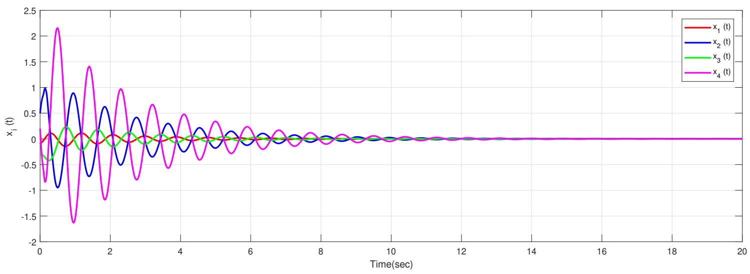

Figure 4.11: States Trajectories of Inverted pendulum system.

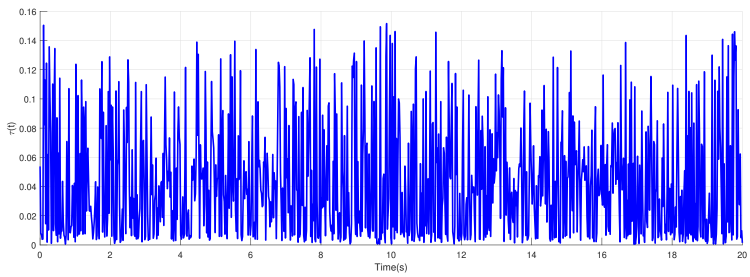

Figure 4.12: Delay profile in the simulation of Inverted pendulum system.

4.5 Conclusion

In this chapter, the problem of the stability analysis and robust state feedback controller design for NCSs with stochastic delay distribution approach have been treated. Lyapunov-

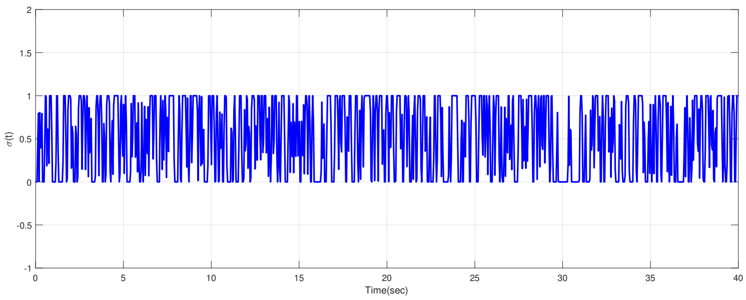

Figure 4.13: Stochastic distribution Delay '$\sigma(t)$' in the simulation of Inverted pendulum system.

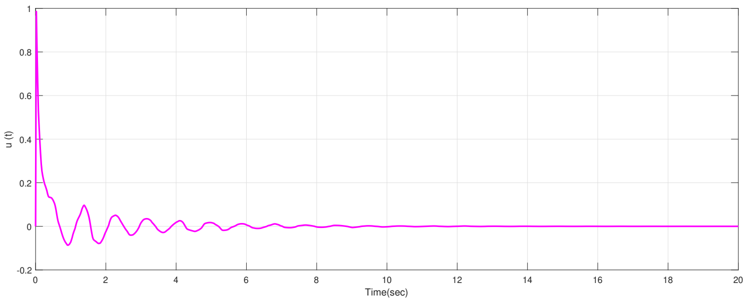

Figure 4.14: Commands Trajectories of Inverted pendulum system.

krasovskii theorem have been used to develop stability condition using probabilistic information of both the lower and upper bounds of the network-induced time delay. The accuracy of the resulting time derivatives has been estimated using Wirtinger-based integral inequalities, and for more conservatism, new cross terms has been introduced. A set of LMIs stability condition is then constructed based on the H_∞ disturbance attenuation level, and Finsler's lemma is used to relax it by adding slack decision variables and decoupling the system's matrices from those of Lyapunov-Krasovskii ones. This procedure greatly simplifies the state feedback controller design with simple variables change. The resolution of such relaxed LMI stability condition provides both the maximum allowable upper network-induced delay bound and optimal state feedback controller gains. The proposed approach offers more conservatism reduction using fewer number of decision variables, which leads to small amount of computing time compared with some existing methods. Several simulation results illustrated the merits of the proposed method. re

Chapter 5

Relaxed stability analysis and controller design for linear networked control systems subject to network-induced delays and packets dropout

5.1 Introduction

The Media Access Control (MAC) protocols based NCSs helps to determine the characteristics of delays and packet dropout, and for categorizing the network imperfections into stochastic (such as "Ethernet") or deterministic (such as "DeviceNet") based on "CAN" protocol as we mention in several places in this thesis. In chapter 4 we focused on the stochastic method to derive a new state feedback controller stabilize the networked system, let us in this chapter switched to the deterministic method to synthesize a new control low and analyze the stability of NCSs used in several works among them the works in [101, 136], [57, 60, 50] where the main motivation of the authors is to obtain a maximum allowable network-induced delay bound and packet dropout guarantee the stability and stabilization of NCSs. As such, the main contribution of this chapter is to derive new less conservative stability conditions and state feedback controller design for NCSs mainly by constructing new augmented LKFs containing double-integral and triple-integral terms and using a novel augmented vector, then the resulting integral terms of the LKFs time derivative are estimated using Wirtinger-based inequalities combined with an extended reciprocal convexity approach resulting in an LMI-based stability condition. For further purpose of reducing conservatism, Finsler's lemma is used to relax the obtained LMI's stability condition by adding slack decision variables and decoupling the system's matrices from the Lyapunov-Krasovskii ones. The resolution of the obtained LMI's convex optimization problem gives a maximum allowable upper network-induced delay bound and maximum number of packet dropout. We can also derive a state feedback control gain. In order to demonstrate the feasibility of the proposed method, we provide numerical academic-used examples representing time-varying systems controlled over network, we also present results comparison with others existing methods in terms of conservativeness and control performances.

Notations 5.1 *In the following, stars * in matrices denote bloc transpose quantities. For a matrix M of appropriate dimension, one denotes $\mathcal{H}(M) = M + M^T$. A finite set of r positive integers is denoted $\mathcal{I}_r = \{1, ..., r\}$. Also, $\forall j \in \mathcal{I}_{13}$, we denote the block entry matrices $e_j = \begin{bmatrix} 0_{n\times(j-1)n} & I_{n\times n} & 0_{n\times(13n-j)n} \end{bmatrix}^T \in R^{13n\times n}$, for example: $e_4 = \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.*

5.2 System description and preliminaries

Let us consider the class of continuous-time linear systems described by the following state space representation:

$$\dot{x}(t) = Ax(t) + Bu(t). \tag{5.1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are respectively the state vector and the input vector, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are real constant matrices. In this paper, we will focus on the control of (5.1) via networks, i.e where the control loop is closed via a communication network. In this context, the structure of a typical Networked Control Systems (NCSs) is depicted in Fig.1.1, with the following assumptions to be accurately described.

Assumption 5.1 *The sensor is clock-driven with a fixed sampling period h, whereas the controller, the Zero-Order Holder (ZOH) and the actuators are event-driven.*

Assumption 5.2 *All the state variables are available from measurements and the data are transmitted from the sensors to the controller and from the controller to the actuators through the network channel with single-packets.*

In the sequel, the sampling period $h > 0$ is assumed to be constant. Let us denote the sampling instants of data transmitted from the sensors to the ZOH by the set $[t_0, t_1, t_2, \dots, t_k]$ where k are positive integers. As shown in Fig.1.1, Some data may be lost because of the presence of consecutive packet dropouts in transmission and its number is denoted by $\delta_k = \frac{t_{(k+1)}-t_k}{h} - 1$ and there are two kinds of delays induced by the network arise, the delay from the sensors to the controller is denoted as τ_{sc_k} and the one from the controller to the actuators as τ_{ca_k}, $\forall k \in \mathbb{N}$. Hence, the whole feedback loop network-induced delay can be denoted as $\tau_k = \tau_{sc_k} + \tau_{ca_k}$, and the updating instants of the ZOH as $t_k + \tau_k$, for each $k \in \mathbb{N}$, which implies that every control signal is held by the ZOH and is only valid over the interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$.

Assumption 5.3 *The system* (5.1) *is use to give the mathematical model of the NCS only for a single packet transmission, in the case of a multi-packet transmission ,system* (5.1) *is always exploitable but we must place a buffer before both the controller and the actuator. Furthermore the number of consecutive data packet dropouts and the network-induced delay must be bounded and satisfy* $\delta_k \in [0, \bar{\delta}]$ *and* $\tau_k \in [\tau_{min}, \tau_{max}]$. *Consequently we can use* $\bar{\eta}$ *to give the value of the maximum allowable bound of packet dropout and maub*(τ_{max}) *for maximum allowable bound network-induced delays.*

Based on the previous assumptions, to stabilization the system (5.1) we consider, $\forall t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$, the state feedback control law given by:

$$u(t) = Kx(t_k). \tag{5.2}$$

where K is the state feedback matrix gain. Let us define, $\forall t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$:

$$\tau(t) = t - t_k. \tag{5.3}$$

The control law (5.2) can be rewritten as:

$$u(t) = Kx(t - \tau(t)),\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})\,. \tag{5.4}$$

Note that, from (5.3), we obviously get $\dot{\tau}(t) = 1$. Moreover, according to as (5.3), the input delay in(4.2) can be expressed as follows:

$$\begin{cases} 0 < \eta_m \leq \tau(t) \leq \eta_M, t \in [t_k + \tau_k, t_{(k+1)} + \tau_{k+1}]\,, \\ \dot{\tau}(t) = 1, \quad t \neq (t_k + \tau_k). \end{cases} \tag{5.5}$$

with $\eta_m = \inf_{k\in\mathbb{N}}(\tau_k) = \tau_{min}$ and $\eta_M = \sup_{k\in\mathbb{N}}(\tau_{k+1}) = \tau_{max}+(\bar{\delta}+1)h,\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})\,.$ Substituting the networked controller (5.4) into (5.1), the considered NCSs closed-loop dynamics can be express as the following systems with network-induced input time-varying delays:

$$\begin{cases} \dot{x}(t) = Ax(t) + BKx(t - \tau(t)), \\ x(t) = \phi(t), \forall t \in [-\tau_M, -\tau_m]. \end{cases} \tag{5.6}$$

where $\phi(t)$ is a vector valued function for initial state conditions.

Remark 5.1 *When no network-induced delays and packet dropout are non-considered, i.e.* $\tau_k = 0$ *and* $\delta_k = 0$, $\forall k \in \mathbb{N}$, *then the closed-loop system with input time-varying delay* (5.6) *reduces to the particular case of a sampled-data control system where the input delay satisfies* $0 \leq \tau(t) \leq h$ *and* $\dot{\tau}(t) = 1$, $\forall t \in [t_k, t_{k+1})$.

The goal of this paper is to propose new relaxed stability analysis and controller design conditions that ensure the asymptotically stability of (5.6), where the maximal available upper bound of $\tau(t)$, denoted $maub(\eta_M)$, for various values of η_m, will be considered as the conservatism index for comparison purpose with previous related results.

5.3 Main results

The main contribution of this paper is establish a new relaxed delay-dependent LMI-based stability conditions such that the NCSs (5.6) is asymptotically stable .

5.3.1 Stability analysis for NCSs.

In this section, a novel delay-dependent stability of NCSs with new augmented LKFs are proposed to obtain less conservative stability conditions via a single and double Wirtinger's inequalities and an improved reciprocal convexity. The main result of this section is proposed by the following theorem.

Theorem 5.1 *Let $i \in \mathcal{I}_2$ and $j \in \mathcal{I}_3$. For given scalars $\eta_m > 0$ and $\eta_M > 0$ such that $\eta_m \leq \tau(t) \leq \eta_M$ and feedback controller gain matrix K, the NCSs model with interval network-induced delay and packet losses is Asymptotically Stable if there exist symmetric positive definite matrices P, U_i, S_i, W_i, Z_i and R_j (in $\mathbb{R}^{n\times n}$), matrices variables $\mathcal{M}$ and $\mathcal{N}$ (in $\mathbb{R}^{2n\times n}$) and real matrix $\mathcal{T}$ (in $\mathbb{R}^{11n\times n}$) such that the following LMIs holds for $q = 1$ and 2:*

$$\begin{bmatrix} R_2 + W_1 & 0 & M_{11} & M_{12} \\ 0 & 3(R_2 + W_1) & M_{21} & M_{22} \\ M_{11}^T & M_{21}^T & R_2 + W_2 & 0 \\ M_{12}^T & M_{22}^T & 0 & 3(R_2 + W_2) \end{bmatrix} > 0. \tag{5.7}$$

$$\begin{bmatrix} R_3 + U_1 & 0 & N_{11} & N_{12} \\ 0 & 3(R_3 + U_1) & N_{21} & N_{22} \\ N_{11}^T & N_{21}^T & R_2 + U_2 & 0 \\ N_{12}^T & N_{22}^T & 0 & 3(R_3 + U_2) \end{bmatrix} > 0. \tag{5.8}$$

$$\sum_{i=2}^{8} \Phi_i + \mathcal{H}_e(\Pi_q^{1T}\mathcal{P}\Pi_q^2) + \mathcal{H}_e(\mathcal{T}\mathcal{G}) < 0, \quad (q \in \mathcal{I}_2). \tag{5.9}$$

where:

$$\mathcal{G} = \begin{bmatrix} A & 0 & BK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \tag{5.10}$$

$$\begin{aligned} \Phi_1 = {} & e_1 Z_1 e_1^T - e_2 Z_1 e_2^T - e_4 Z_2 e_4^T + e_{13}\left(\eta_m^2 R_1 + \eta_M^2 R_2 + (\eta_M - \eta_m)^2 R_3\right) e_{13}^T \\ & - (e_1 - e_2) R_1 (e_1 - e_2)^T - 3(e_1 + e_2 - 2e_5) R_1 (e_1 + e_2 - 2e_5)^T , \end{aligned} \tag{5.11}$$

$$\Phi_2 = \frac{\eta_m^2}{2} e_{13}(S_1 + S_2) e_{13}^T - \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}^T \begin{bmatrix} \mathcal{S}_1 & 0 \\ * & \mathcal{S}_2 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix}, \tag{5.12}$$

$$
\begin{aligned}
\Phi_3 =& \frac{\eta_M^2}{2} e_{13} \left(W_1 + W_2 \right) e_{13}^T \\
& - \begin{bmatrix} \chi_3 \\ \chi_4 \end{bmatrix}^T \begin{bmatrix} \mathcal{W}_1 & 0 \\ * & \mathcal{W}_1 \end{bmatrix} \begin{bmatrix} \chi_3 \\ \chi_4 \end{bmatrix} - \begin{bmatrix} \chi_5 \\ \chi_6 \end{bmatrix}^T \begin{bmatrix} \mathcal{W}_2 & 0 \\ * & \mathcal{W}_2 \end{bmatrix} \begin{bmatrix} \chi_5 \\ \chi_6 \end{bmatrix},
\end{aligned} \tag{5.13}
$$

$$
\begin{aligned}
\Phi_4 =& \frac{(\eta_M - \eta_m)^2}{2} e_{13} \left(U_1 + U_2 \right) e_{13}^T \\
& - \begin{bmatrix} \chi_7 \\ \chi_8 \end{bmatrix}^T \begin{bmatrix} \mathcal{U}_1 & 0 \\ * & \mathcal{U}_1 \end{bmatrix} \begin{bmatrix} \chi_7 \\ \chi_8 \end{bmatrix} - \begin{bmatrix} \chi_9 \\ \chi_{10} \end{bmatrix}^T \begin{bmatrix} \mathcal{U}_2 & 0 \\ * & \mathcal{U}_2 \end{bmatrix} \begin{bmatrix} \chi_9 \\ \chi_{10} \end{bmatrix},
\end{aligned} \tag{5.14}
$$

$$
\Phi_5 = - \begin{bmatrix} \chi_{11} \\ \chi_{12} \end{bmatrix}^T \begin{bmatrix} \mathcal{R}_2 & \mathcal{M} \\ * & \mathcal{R}_2 \end{bmatrix} \begin{bmatrix} \chi_{11} \\ \chi_{12} \end{bmatrix} - \begin{bmatrix} \chi_{13} \\ \chi_{14} \end{bmatrix}^T \begin{bmatrix} \mathcal{R}_3 & \mathcal{N} \\ * & \mathcal{R}_3 \end{bmatrix} \begin{bmatrix} \chi_{13} \\ \chi_{14} \end{bmatrix}, \tag{5.15}
$$

$$
\Pi_1^1 = \begin{bmatrix} e_1^T \\ \eta_m e_5^T \\ \eta_m e_6^T + (\eta_M - \eta_m) e_8^T \\ \eta_m e_7^T \\ \eta_m^2 e_{10}^T + (\eta_M - \eta_m)^2 e_{12}^T \\ \eta_m (\eta_M - \eta_m) e_6^T + (\eta_M - \eta_m)^2 e_{12}^T \end{bmatrix}, \Pi_2^1 = \begin{bmatrix} e_{13}^T \\ e_1^T - e_2^T \\ e_2^T - e_4^T \\ e_1^T - e_5^T \\ \eta_M e_1^T - \eta_m e_6^T - (\eta_M - \eta_m) e_8^T \\ (\eta_M - \eta_m) e_2^T - (\eta_M - \eta_m) e_8^T \end{bmatrix},
$$

$$
\Pi_1^2 = \begin{bmatrix} e_1^T \\ \eta_m e_5^T \\ \eta_M e_6^T + (\eta_M - \eta_m) e_7 \\ \eta_m e_7^T \\ \eta_M^2 e_{10}^T \\ (\eta_M - \eta_m)^2 e_{11}^T \end{bmatrix}, \qquad \Pi_2^2 = \begin{bmatrix} e_{13}^T \\ e_1^T - e_2^T \\ e_2^T - e_4^T \\ e_1^T - e_5^T \\ \eta_M e_1^T - \eta_M e_6^T \\ (\eta_M - \eta_m) e_2^T - (\eta_M - \eta_m) e_7^T \end{bmatrix},
$$

$$
\chi_1 = \begin{bmatrix} [e_2 - e_5]^T \\ [e_2 - 4e_5 + 6e_9]^T \end{bmatrix}, \chi_2 = \begin{bmatrix} [e_2 - e_5]^T \\ [e_2 - 4e_5 + 6e_9]^T \end{bmatrix}, \chi_3 = \begin{bmatrix} [e_1 - e_6]^T \\ [e_1 - 4e_6 + 6e_{10}]^T \end{bmatrix},
$$

$$
\chi_4 = \begin{bmatrix} [e_3 - e_8]^T \\ [e_3 + 2e_8 - 6e_{12}]^T \end{bmatrix}, \chi_5 = \begin{bmatrix} [e_3 - e_6]^T \\ [e_3 - 4e_6 + 6e_{10}]^T \end{bmatrix}, \chi_6 = \begin{bmatrix} [e_4 - e_8]^T \\ [e_4 + 2e_8 - 6e_{12}]^T \end{bmatrix},
$$

$$
\chi_7 = \begin{bmatrix} [e_2 - e_7]^T \\ [e_2 - 4e_7 + 6e_{11}] \end{bmatrix}^T, \chi_8 = \begin{bmatrix} [e_3 - e_8]^T \\ [e_3 + 2e_8 - 6e_{12}]^T \end{bmatrix}, \chi_9 = \begin{bmatrix} [e_3 - e_7]^T \\ [e_3 - 4e_7 + 6e_{11}]^T \end{bmatrix},
$$

$$
\chi_{10} = \begin{bmatrix} [e_4 - e_8]^T \\ [e_4 - 4e_8 + 6e_{12}]^T \end{bmatrix}, \chi_{11} = \begin{bmatrix} [e_1 - e_3]^T \\ [e_1 + e_3 - 2e_6]^T \end{bmatrix}, \chi_{12} = \begin{bmatrix} [e_3 - e_4]^T \\ [e_3 + e_4 - 2e_8]^T \end{bmatrix},
$$

$$
\chi_{13} = \begin{bmatrix} [e_2 - e_3]^T \\ [e_2 + e_3 - 2e_7]^T \end{bmatrix}, \chi_{14} = \begin{bmatrix} [e_3 - e_4]^T \\ [e_3 + e_4 - 2e_8]^T \end{bmatrix},
$$

$$\mathcal{R}_i = \begin{bmatrix} R_i & 0 \\ * & 3R_i \end{bmatrix}, \mathcal{S}_i = \begin{bmatrix} 2S_i & 0 \\ * & 4S_i \end{bmatrix}, \mathcal{U}_i = \begin{bmatrix} 2U_i & 0 \\ * & 4U_i \end{bmatrix}, \mathcal{W}_i = \begin{bmatrix} 2W_i & 0 \\ * & 4W_i \end{bmatrix}, (i \in \mathcal{I}_4).$$

Proof 5.1 *Consider a Lyapunov-Krasovskii functional of the following form :*

$$V(t) = \sum_{i=1}^{5} V_i(t). \tag{5.16}$$

where:

$$V_1(t) = \theta^T(t)\mathcal{P}\theta(t). \tag{5.17}$$

$$\theta(t) = \begin{bmatrix} \Xi_1 & \Xi_2 & \int_{-\eta_M}^{-\eta_m}\int_{t+\beta}^{t} x^T(s)dsd\beta \end{bmatrix}^T,$$

$$\Xi_1 = \begin{bmatrix} x^T(t) & \int_{t-\eta_m}^{t} x^T(s)ds & \int_{t-\eta_M}^{t} x^T(s)ds & \int_{t-\eta_M}^{t-\eta_m} x^T(s)ds \end{bmatrix}^T,$$

$$\Xi_2 = \begin{bmatrix} \frac{1}{\eta_m}\int_{-\eta_m}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta & \int_{-\eta_M}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta \end{bmatrix}^T,$$

$$\begin{aligned} V_2(t) = &\int_{t-\eta_m}^{t} x^T(s)Z_1x(s)ds + \int_{t-\eta_M}^{t-\tau(t)} x^T(s)Z_2x(s)ds \\ &+ \eta_m\int_{-\eta_m}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\beta + \eta_M\int_{-\eta_M}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\beta \\ &+ (\eta_M-\eta_m)\int_{-\eta_M}^{-\eta_m}\int_{t+\beta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\beta, \end{aligned} \tag{5.18}$$

$$V_3(t) = \int_{-\eta_m}^{0}\int_{\lambda}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)S_1\dot{x}(s)\,ds\,d\beta\,d\lambda + \int_{-\eta_m}^{0}\int_{-\eta_m}^{\lambda}\int_{t+\beta}^{t} \dot{x}^T(s)S_2\dot{x}(s)\,ds\,d\beta\,d\lambda, \tag{5.19}$$

$$V_4(t) = \int_{-\eta_M}^{0}\int_{\lambda}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)W_1\dot{x}(s)\,ds\,d\beta\,d\lambda + \int_{-\eta_M}^{0}\int_{-\eta_M}^{\lambda}\int_{t+\beta}^{t} \dot{x}^T(s)W_2\dot{x}(s)\,ds\,d\beta\,d\lambda, \tag{5.20}$$

$$V_5(t) = \int_{-\eta_M}^{-\eta_m}\int_{\lambda}^{-\eta_m}\int_{t+\beta}^{t-\eta_m} \dot{x}^T(s)U_1\dot{x}(s)\,ds\,d\beta\,d\lambda + \int_{-\eta_M}^{-\eta_m}\int_{-\eta_M}^{\lambda}\int_{t+\beta}^{t} \dot{x}^T(s)U_2\dot{x}(s)\,ds\,d\beta\,d\lambda. \tag{5.21}$$

The LKFs candidate (5.16) *is positive if P, $R_i (i \in \mathcal{I}_3)$, Z_j, S_j, W_j and U_j ($j \in \mathcal{I}_2$) are all positive definite matrices. In this case, the networked control system* (5.6) *with network-induce delay* (5.5) *is asymptotically stable if:*

$$\dot{V}(t) = \sum_{i=1}^{5} \dot{V}_i(t) < 0. \tag{5.22}$$

Let us define:

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-\eta_m) & x^T(t-\tau(t)) & x^T(t-\eta_M) & \mathcal{V}_1^T & \dots & \mathcal{V}_8^T & \dot{x}^T(t)\ . \end{bmatrix}^T \tag{5.23}$$

where

$$\mathcal{V}_1 = \frac{1}{\eta_m}\int_{t-\eta_m}^{t} x(s)ds,\ \mathcal{V}_2 = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} x(s)ds,$$
$$\mathcal{V}_3 = \frac{1}{(\tau(t)-\eta_m)}\int_{t-\tau(t)}^{t-\eta_m} x(s)ds, \mathcal{V}_4 = \frac{1}{(\eta_M-\tau(t))}\int_{t-\eta_M}^{t-\tau(t)} x(s)ds,$$
$$\mathcal{V}_5 = \frac{1}{\eta_m^2}\int_{-\eta_m}^{0}\int_{t+\beta}^{t} x(s)dsd\beta, \mathcal{V}_6 = \frac{1}{\tau(t)^2}\int_{-\tau(t)}^{0}\int_{t+\beta}^{t} x(s)dsd\beta,$$
$$\mathcal{V}_7 = \frac{1}{(\tau(t)-\eta_m)^2}\int_{-\tau(t)}^{-\eta_m}\int_{t+\beta}^{t-\eta_m} x(s)dsd\beta, \mathcal{V}_8 = \frac{1}{(\eta_M-\tau(t))^2}\int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)} x(s)dsd\beta.$$

The derivative of first term in (5.16) *is given as follow :*

$$\dot{V}_1(t) = \mathcal{H}_e(\theta^T(t)\mathcal{P}\dot{\theta}(t)). \tag{5.24}$$

where:

$$\dot{\theta}(t) = \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-\eta_m) \\ x(t) - x(t-\eta_M) \\ x(t-\eta_m) - x(t-\eta_M) \\ x(t) - \mathcal{V}_1 \\ \eta_M x(t) - \tau(t)\mathcal{V}_2 - (\eta_M - \tau(t))\mathcal{V}_4 \\ (\eta_M - \eta_m)x(t-\eta_m) - (\tau(t)-\eta_m)\mathcal{V}_3 - (\eta_M - \tau(t))\mathcal{V}_4 \end{bmatrix}. \tag{5.25}$$

and we can re-write $\theta(t)$ *in* (5.24) *as follow:*

$$\theta(t) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{bmatrix}^T, \tag{5.26}$$

$$\theta_1 = \begin{bmatrix} x^T(t) & \int_{t-\eta_m}^{t} x^T(s)ds & \int_{t-\tau(t)}^{t} x^T(s)ds + \int_{t-\eta_M}^{t-\tau(t)} x^T(s)ds \end{bmatrix}^T,$$
$$\theta_2 = \begin{bmatrix} \int_{t-\tau(t)}^{t-\eta_m} x^T(s)ds + \int_{t-\eta_M}^{t-\tau(t)} x^T(s)ds & \frac{1}{\eta_m}\int_{\eta_m}^{0}\int_{t-\eta_m}^{t} x^T(s)ds \end{bmatrix}^T,$$
$$\theta_3 = \int_{-\tau(t)}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta + \int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)} x^T(s)dsd\beta + (\eta_M-\tau(t))\int_{t-\tau(t)}^{t} x^T(s)ds,$$
$$\theta_4 = \int_{-\tau(t)}^{-\eta_m}\int_{t+\beta}^{t-\eta_m} x^T(s)dsd\beta + \int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)} x^T(s)dsd\beta + (\eta_M-\tau(t))\int_{t-\tau(t)}^{t-\eta_m} x^T(s)ds.$$

$$\begin{aligned}\theta(t) = \Big[\ & x(t) \quad \eta_1\mathcal{V}_1 \quad \tau(t)\mathcal{V}_2 + (\eta_2-\tau(t))\mathcal{V}_4 \quad (\tau(t)-\eta_1)\mathcal{V}_3 + (\eta_2-\tau(t))\mathcal{V}_4 \quad \eta_1\mathcal{V}_5 \\ & \tau(t)^2\mathcal{V}_6 + (\eta_2-\tau(t))^2\mathcal{V}_8 + \tau(t)(\eta_2-\tau(t))\mathcal{V}_2 \\ & (\tau(t)-\eta_1)^2\mathcal{V}_7 + (\eta_2-\tau(t))^2\mathcal{V}_8 + (\tau(t)-\eta_1)(\eta_2-\tau(t))\mathcal{V}_3\Big],\end{aligned} \tag{5.27}$$

Therefore, from (5.25) *and* (5.26) *and substituting* $\beta = \frac{\tau(t)-\eta_m}{\eta_M-\eta_m}$ *the following equations are* *true:*

$$\dot{V}_1(t) \leq \zeta^T(t)\mathcal{H}_e\left(\beta\Pi_1^1\mathcal{P}\Pi_2^1 + (1-\beta)\Pi_1^2\mathcal{P}\Pi_2^2\right)\zeta(t). \tag{5.28}$$

with Π_1^1, Π_1^2, Π_2^1 *and* Π_2^2 *detailed in Theorem 5.1.*

Likewise, let consider the time derivative of second term (4.26) *in* (5.16)*:*

$$\begin{aligned}\dot{V}_2(t) &= \dot{x}^T(t)\left(\eta_m^2R_1 + \eta_M^2R_2 + (\eta_M-\eta_m)^2R_3\right)\dot{x}(t) + x^T(t)Z_1x(t)\\ &- x^T(t-\eta_m)Z_1x(t-\eta_m) + (1-\dot{\tau}(t))x^T(t-\tau(t))Z_2x(t-\tau(t))\\ &- x^T(t-\eta_M)Z_2x(t-\eta_M) - \eta_m\int_{t-\eta_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds\\ &- \eta_M\int_{t-\eta_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds - (\eta_M-\eta_m)\int_{t-\eta_M}^{t-\eta_m}\dot{x}^T(s)R_3\dot{x}(s)ds.\end{aligned} \tag{5.29}$$

Applying lemma 3.3.4 to first integral terms in (5.29) *yields:*

$$\begin{aligned}&-\eta_m\int_{t-\eta_m}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \leq\\ &-\zeta^T(t)\left(\begin{bmatrix}[e_1-e_2]^T\\ [e_1+e_2-2e_5]^T\end{bmatrix}^T\begin{bmatrix}R_1 & 0\\ * & 3R_1\end{bmatrix}\begin{bmatrix}[e_1-e_2]^T\\ [e_1+e_2-2e_5]^T\end{bmatrix}\right)\zeta(t),\end{aligned} \tag{5.30}$$

The inequality (5.29) *can be written as:*

$$\begin{aligned}\dot{V}_2(t) \leq \zeta^T(t)\Phi_1\zeta(t) &- \eta_M\int_{t-\eta_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds\\ &- (\eta_M-\eta_m)\int_{t-\eta_M}^{t-\eta_m}\dot{x}^T(s)R_3\dot{x}(s)ds.\end{aligned} \tag{5.31}$$

with Φ_1 *is given in* (5.11).

Continuing by the time derivative of the third term in (5.16) *:*

$$\begin{aligned}\dot{V}_3(t) = \frac{\eta_m^2}{2}\dot{x}^T(t)(S_1+S_2)\dot{x}(t) &- \int_{-\eta_m}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)S_1\dot{x}(s)dsd\beta\\ &- \int_{-\eta_m}^{0}\int_{t-\eta_m}^{t+\beta}\dot{x}^T(s)S_2\dot{x}(s)dsd\beta.\end{aligned} \tag{5.32}$$

From lemma 3.3.4, we can write: and

$$\begin{aligned}&-\int_{-\eta_m}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)S_1\dot{x}(s)dsd\beta \leq\\ &-\zeta^T(t)\left(\begin{bmatrix}[e_1-e_5]^T\\ [e_1-4e_5+6e_9]^T\end{bmatrix}^T\begin{bmatrix}2S_1 & 0\\ * & 4S_1\end{bmatrix}\begin{bmatrix}[e_1-e_5]^T\\ [e_1-4e_5+6e_9]^T\end{bmatrix}\right)\zeta(t),\end{aligned} \tag{5.33}$$

and

$$-\int_{-\eta_m}^{0}\int_{t-\eta_m}^{t+\beta}\dot{x}^T(s)S_2\dot{x}^T(s)dsd\beta \leq -\zeta^T(t)\left(\begin{bmatrix}[e_2-e_5]^T\\ [e_2-4e_5+6e_9]^T\end{bmatrix}^T\begin{bmatrix}2S_2 & 0\\ * & 4S_2\end{bmatrix}\begin{bmatrix}[e_2-e_5]^T\\ [e_2-4e_5+6e_9]^T\end{bmatrix}\right)\zeta(t), \tag{5.34}$$

leading to:

$$\dot{V}_3(t) \leq \zeta^T(t)\Phi_2\zeta(t). \tag{5.35}$$

with Φ_2 *is given in* (5.12).

Next, let consider the time derivative of penultimate term in (5.16) *:*

$$\begin{aligned}\dot{V}_4(t) = {} & \frac{\eta_M^2}{2}\dot{x}^T(t)\left(W_1+W_2\right)\dot{x}(t)\\ & -\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)W_1\dot{x}(s)dsd\beta-\int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}\dot{x}^T(s)W_1\dot{x}(s)dsd\beta\\ & -(\eta_M-\tau(t))\int_{t-\tau(t)}^{t}\dot{x}^T(s)W_1\dot{x}(s)ds\\ & -\int_{-\tau(t)}^{0}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T(s)W_2\dot{x}(s)dsd\beta-\int_{-\eta_M}^{-\tau(t)}\int_{t-\eta_M}^{t+\beta}\dot{x}^T(s)W_2\dot{x}(s)dsd\beta\\ & -\tau(t)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)W_2\dot{x}(s)ds.\end{aligned} \tag{5.36}$$

From lemma 3.3.4 we can write:

$$-\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)W_1\dot{x}(s)dsd\beta \leq -\zeta^T(t)\left(\begin{bmatrix}[e_1-e_6]^T\\ [e_1-4e_6+6e_{10}]^T\end{bmatrix}^T\begin{bmatrix}2W_1 & 0\\ * & 4W_1\end{bmatrix}\begin{bmatrix}[e_1-e_6]^T\\ [e_1-4e_6+6e_{10}]^T\end{bmatrix}\right)\zeta(t), \tag{5.37}$$

$$-\int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}\dot{x}^T(s)W_1\dot{x}(s)dsd\beta \leq -\zeta^T(t)\left(\begin{bmatrix}[e_3-e_8]^T\\ [e_3+2e_8-6e_{12}]^T\end{bmatrix}^T\begin{bmatrix}2W_1 & 0\\ * & 4W_1\end{bmatrix}\begin{bmatrix}[e_3-e_8]^T\\ [e_3+2e_8-6e_{12}]^T\end{bmatrix}\right)\zeta(t), \tag{5.38}$$

$$-\int_{-\tau(t)}^{0}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T(s)W_2\dot{x}(s)dsd\beta \leq -\zeta^T(t)\left(\begin{bmatrix}[e_3-e_6]^T\\ [e_3-4e_6+6e_{10}]^T\end{bmatrix}^T\begin{bmatrix}2W_2 & 0\\ * & 4W_2\end{bmatrix}\begin{bmatrix}[e_3-e_6]^T\\ [e_3-4e_6+6e_{10}]^T\end{bmatrix}\right)\zeta(t), \tag{5.39}$$

and:

$$
\begin{aligned}
&-\int_{-\eta_M}^{-\tau(t)}\int_{t-\eta_M}^{t+\beta}\dot{x}^T(s)W_2\dot{x}(s)dsd\beta \leq \\
&-\zeta^T(t)\left(\begin{bmatrix}[e_4-e_8]^T\\ [e_4+2e_8-6e_{12}]^T\end{bmatrix}^T\begin{bmatrix}2W_2 & 0\\ * & 4W_2\end{bmatrix}\begin{bmatrix}[e_4-e_8]^T\\ [e_4+2e_8-6e_{12}]^T\end{bmatrix}\right)\zeta(t),
\end{aligned} \tag{5.40}
$$

from (5.37)-(5.40) *leading to:*

$$
\begin{aligned}
\dot{V}_4(t) \leq \zeta^T(t)\Phi_3\zeta(t) - (\eta_M-\tau(t))\int_{t-\tau(t)}^{t}\dot{x}^T(s)W_1\dot{x}(s)dsd\beta\\
-\tau(t)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)W_2\dot{x}(s)dsd\beta.
\end{aligned} \tag{5.41}
$$

with Φ_3 *given in* (5.13).
Finally, the time derivative of the last term in (5.16) *:*

$$
\begin{aligned}
\dot{V}_5(t) = &\frac{(\eta_M-\eta_m)^2}{2}\dot{x}^T(t)\left(U_1+U_2\right)\dot{x}(t)\\
&-\int_{-\tau(t)}^{-\eta_m}\int_{t+\beta}^{t-\eta_m}\dot{x}^T(s)U_1\dot{x}(s)dsd\beta - \int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}\dot{x}^T(s)U_1\dot{x}(s)ds\\
&-(\eta_M-\tau(t))\int_{t-\tau(t)}^{-\eta_m}\dot{x}^T(s)U_1\dot{x}(s)ds\\
&-\int_{-\tau(t)}^{-\eta_m}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T(s)U_2\dot{x}(s)dsd\beta - \int_{-\eta_M}^{-\tau(t)}\int_{t-\eta_M}^{t+\beta}\dot{x}^T(s)U_2\dot{x}(s)dsd\beta\\
&-(\tau(t)-\eta_m)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)U_2\dot{x}(s)ds.
\end{aligned} \tag{5.42}
$$

From lemma 3.3.4 we can write:

$$
\begin{aligned}
&-\int_{-\tau(t)}^{-\eta_m}\int_{t+\beta}^{t-\eta_m}\dot{x}^T(s)U_1\dot{x}(s)dsd\beta \leq \\
&-\zeta^T(t)\left(\begin{bmatrix}[e_2-e_7]^T\\ [e_2-4e_7+6e_{11}]^T\end{bmatrix}^T\begin{bmatrix}2U_1 & 0\\ * & 4U_1\end{bmatrix}\begin{bmatrix}[e_2-e_7]^T\\ [e_2-4e_7+6e_{11}]^T\end{bmatrix}\right)\zeta(t),
\end{aligned} \tag{5.43}
$$

$$
\begin{aligned}
&-\int_{-\eta_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}\dot{x}^T(s)U_1\dot{x}(s)dsd\beta \leq \\
&-\zeta^T(t)\left(\begin{bmatrix}[e_3-e_8]^T\\ [e_3+2e_8-6e_{12}]^T\end{bmatrix}^T\begin{bmatrix}2U_1 & 0\\ * & 4U_1\end{bmatrix}\begin{bmatrix}[e_3-e_8]^T\\ [e_3+2e_8-6e_{12}]^T\end{bmatrix}\right)\zeta(t),
\end{aligned} \tag{5.44}
$$

$$
\begin{aligned}
&-\int_{-\tau(t)}^{-\eta_m}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T(s)U_2\dot{x}(s)dsd\beta \leq \\
&-\zeta^T(t)\left(\begin{bmatrix}[e_3-e_7]^T\\ [e_3-4e_7+6e_{11}]^T\end{bmatrix}^T\begin{bmatrix}2U_2 & 0\\ * & 4U_2\end{bmatrix}\begin{bmatrix}[e_3-e_7]^T\\ [e_3-4e_7+6e_{11}]^T\end{bmatrix}\right)\zeta(t),
\end{aligned}
\tag{5.45}
$$

and

$$
\begin{aligned}
&-\int_{-\tau(t)}^{-\eta_m}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T(s)U_2\dot{x}(s)dsd\beta \leq \\
&-\zeta^T(t)\left(\begin{bmatrix}[e_4-e_8]^T\\ [e_4-4e_8+6e_{12}]^T\end{bmatrix}^T\begin{bmatrix}2U_2 & 0\\ * & 4U_2\end{bmatrix}\begin{bmatrix}[e_4-e_8]^T\\ [e_4-4e_8+6e_{12}]^T\end{bmatrix}\right)\zeta(t).
\end{aligned}
\tag{5.46}
$$

From (5.43)-(5.45) *it yields:*

$$
\begin{aligned}
\dot{V}_5(t) \leq \zeta^T(t)\Phi_4\zeta(t) &- (\eta_M-\tau(t))\int_{t-\tau(t)}^{-\eta_m}\dot{x}^T(s)U_1\dot{x}(s)ds \\
&- (\tau(t)-\eta_m)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)U_2\dot{x}(s)ds.
\end{aligned}
\tag{5.47}
$$

with Φ_4 *is given in* (5.14).

From (5.24)-(5.47)*, the equation* (5.22) *can be writing as follow:*

$$
\begin{aligned}
\dot{V}(t) \leq{}& \zeta^T(t)\left(\sum_{i=2}^{4}\Phi_i + \mathcal{H}_e\left(\beta\Pi_1^1\mathcal{P}\Pi_2 + (1-\beta)\Pi_1^2\mathcal{P}\Pi_2\right)\right)\zeta(t) \\
&- \eta_M\int_{t-\eta_M}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds - (\eta_M-\tau(t))\int_{t-\tau(t)}^{t}\dot{x}^T(s)W_1\dot{x}(s)ds \\
&- \tau(t)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)W_2\dot{x}(s)ds - (\eta_M-\eta_m)\int_{t-\eta_M}^{t-\eta_m}\dot{x}^T(s)R_3\dot{x}(s)ds \\
&- (\tau(t)-\eta_m)\int_{t-\eta_M}^{t-\tau(t)}\dot{x}^T(s)U_2\dot{x}(s)ds - (\eta_M-\tau(t))\int_{t-\tau(t)}^{-\eta_m}\dot{x}^T(s)U_1\dot{x}(s)ds.
\end{aligned}
\tag{5.48}
$$

So, the first three integral terms of (5.48) *can be bounded via lemma 3.3.4 as follows:*

$$\begin{aligned}
&-\eta_M \int_{t-\tau(t)}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \eta_M \int_{t-\eta_M}^{t-\tau(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&-(\eta_M-\tau(t))\int_{t-\tau(t)}^{t} \dot{x}^T(s)W_1\dot{x}(s)ds - \tau(t)\int_{t-\eta_M}^{t-\tau(t)} \dot{x}^T(s)W_2\dot{x}(s)ds \leq \\
&-\zeta^T(t)\left(\frac{1}{\alpha_1}\begin{bmatrix}[e_1-e_3]^T\\ [e_1+e_3-2e_6]^T\end{bmatrix}^T \begin{bmatrix}R_2 & 0\\ * & 3R_2\end{bmatrix}\begin{bmatrix}[e_1-e_3]^T\\ [e_1+e_3-2e_6]^T\end{bmatrix}\right.\\
&\quad+\frac{1}{\beta_1}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}^T \begin{bmatrix}R_2 & 0\\ * & 3R_2\end{bmatrix}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}\\
&\quad+\frac{\beta_1}{\alpha_1}\begin{bmatrix}[e_1-e_3]^T\\ [e_1+e_3-2e_6]^T\end{bmatrix}^T \begin{bmatrix}W_1 & 0\\ * & 3W_1\end{bmatrix}\begin{bmatrix}[e_1-e_3]^T\\ [e_1+e_3-2e_6]^T\end{bmatrix}\\
&\quad\left.+\frac{\alpha_1}{\beta_1}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}^T \begin{bmatrix}W_2 & 0\\ * & 3W_2\end{bmatrix}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}\right)\zeta(t).
\end{aligned} \tag{5.49}$$

with $\alpha_1 = \frac{\tau(t)}{\eta_M}$, $\beta_1 = \frac{\eta_M-\tau(t)}{\eta_M}$, $\frac{\beta_1}{\alpha_1} = \frac{\eta_M-\tau(t)}{\tau(t)}$ *and* $\frac{\alpha_1}{\beta_1} = \frac{\tau(t)}{\eta_M-\tau(t)}$.

In the same way, the last three integral terms of (5.48) *can be bounded via lemma 3.3.4 as follows:*

$$\begin{aligned}
&-(\eta_M-\eta_m)\int_{t-\tau(t)}^{t-\eta_m} \dot{x}^T(s)R_3\dot{x}(s)ds - (\eta_M-\eta_m)\int_{t-\eta_M}^{t-\tau(t)} \dot{x}^T(s)R_3\dot{x}(s)ds \\
&-(\eta_M-\tau(t))\int_{t-\tau(t)}^{t-\eta_m} \dot{x}^T(s)U_1\dot{x}(s)ds - (\tau(t)-\eta_m)\int_{t-\eta_M}^{t-\tau(t)} \dot{x}^T(s)U_2\dot{x}(s)ds \leq \\
&-\zeta^T(t)\left(\frac{1}{\alpha_2}\begin{bmatrix}[e_2-e_3]^T\\ [e_2+e_3-2e_7]^T\end{bmatrix}^T \begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix}\begin{bmatrix}[e_2-e_3]^T\\ [e_2+e_3-2e_7]^T\end{bmatrix}\right.\\
&\quad+\frac{1}{\beta_2}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}^T \begin{bmatrix}R_3 & 0\\ * & 3R_3\end{bmatrix}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}\\
&\quad+\frac{\beta_2}{\alpha_2}\begin{bmatrix}[e_2-e_3]^T\\ [e_2+e_3-2e_7]^T\end{bmatrix}^T \begin{bmatrix}U_1 & 0\\ * & 3U_1\end{bmatrix}\begin{bmatrix}[e_2-e_3]^T\\ [e_2+e_3-2e_7]^T\end{bmatrix}\\
&\quad\left.+\frac{\alpha_2}{\beta_2}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}^T \begin{bmatrix}U_2 & 0\\ * & 3U_2\end{bmatrix}\begin{bmatrix}[e_3-e_4]^T\\ [e_3+e_4-2e_8]^T\end{bmatrix}\right)\zeta(t).
\end{aligned} \tag{5.50}$$

with $\alpha_2 = \frac{\tau(t)-\eta_m}{\eta_M-\eta_m}$, $\beta_2 = \frac{\eta_M-\tau(t)}{\eta_M-\eta_m}$, $\frac{\beta_2}{\alpha_2} = \frac{\eta_M-\tau(t)}{\tau(t)-\eta_m}$ *and* $\frac{\alpha_2}{\beta_2} = \frac{\tau(t)-\eta_m}{\eta_M-\tau(t)}$.

Note that $\alpha_1+\beta_1 = 1$ *and* $\alpha_2+\beta_2 = 1$. *Thus, applying lemma 3.3.5 to* (5.49) *and* (5.50) *respectively, if there exist a matrix variables* $\mathcal{M} = \begin{bmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}$ *and* $\mathcal{N} = \begin{bmatrix} N_{11} & N_{12}\\ N_{21} & N_{22}\end{bmatrix}$

with appropriate dimension satisfying (5.7) *and* (5.8) *respectively, we can rewrite the inequality* (5.48) *as follow:*

$$\begin{aligned}
\dot{V}(t) \leq& \zeta^T(t)\Bigg(\sum_{i=1}^{4}\Phi_i + \mathcal{H}_e\left(\beta\Pi_1^1\mathcal{P}\Pi_2^1 + (1-\beta)\Pi_1^2\mathcal{P}\Pi_2^2\right)\\
&- \begin{bmatrix} [e_1-e_3]^T \\ [e_1+e_3-2e_6]^T \\ [e_3-e_4]^T \\ [e_3+e_4-2e_8]^T \end{bmatrix}^T \begin{bmatrix} R_2 & 0 & M_{11} & M_{12} \\ * & 3R_2 & M_{21} & M_{22} \\ * & * & R_2 & 0 \\ * & * & * & 3R_2 \end{bmatrix} \begin{bmatrix} [e_1-e_3]^T \\ [e_1+e_3-2e_6]^T \\ [e_3-e_4]^T \\ [e_3+e_4-2e_8]^T \end{bmatrix}\\
&- \begin{bmatrix} [e_2-e_3]^T \\ [e_2+e_3-2e_7]^T \\ [e_3-e_4]^T \\ [e_3+e_4-2e_8]^T \end{bmatrix}^T \begin{bmatrix} R_3 & 0 & N_{11} & N_{12} \\ * & 3R_3 & N_{21} & N_{22} \\ * & * & R_3 & 0 \\ * & * & * & 3R_3 \end{bmatrix} \begin{bmatrix} [e_2-e_3]^T \\ [e_2+e_3-2e_7]^T \\ [e_3-e_4]^T \\ [e_3+e_4-2e_8]^T \end{bmatrix}\Bigg)\zeta(t)\\
\leq& \zeta^T(t)\left(\sum_{i=1}^{5}\Phi_i + \mathcal{H}_e\left(\beta\Pi_1^1\mathcal{P}\Pi_2^1 + (1-\beta)\Pi_1^2\mathcal{P}\Pi_2^2\right)\right)\zeta(t).
\end{aligned} \tag{5.51}$$

with Φ_5 *is given in* (5.15).
Then, let us rewrite the NCSs model with the network-induced delay (5.6) *with* $\mathcal{G}$ *defined in* (5.10)*:*

$$\mathcal{G}\zeta(t) = 0. \tag{5.52}$$

Applying apply lemma 3.3.2, the NCSs model with network-induce delay (5.6) *is Asymptotically stable if there exists* $\mathcal{T} \in R^{13n\times n}$ *such that:*

$$\zeta^T(t)\mathcal{H}_e(\mathcal{T}\mathcal{G})\zeta(t) = 0. \tag{5.53}$$

with $\mathcal{G}$ *is defined in* (5.10).
Now, let consider the whole derivative of the LKFs, from (5.24)-(5.50)

$$\dot{V}(t) \leq \zeta^T(t)\left(\sum_{i=2}^{6}\Phi_i(t) + \mathcal{H}_e\left(\beta\Pi_1^1\mathcal{P}\Pi_2^1 + (1-\beta)\Pi_1^2\mathcal{P}\Pi_2^2\right) + \mathcal{H}_e(\mathcal{T}\mathcal{G})\right)\zeta(t). \tag{5.54}$$

Utilizing convex combination approach, so $\Pi_1^1\mathcal{P}\Pi_2^1 < 0$ *and* $\Pi_1^2\mathcal{P}\Pi_2^2 < 0$. *Therefore, if* (5.7), (5.8) *and* (5.9) *hold, then system* (5.6) *is asymptotically stable. This completes the proof.*

Remark 5.2 *As it is well known, the choice of suitable LKFs is the first key factor to reduce conservatism. for that, the double integral terms* $\int_{-\eta_M}^{-\eta_m}\int_{t+\beta}^{t} x(s)dsd\beta$, $\int_{-\eta_M}^{0}\int_{t+\beta}^{t} x(s)dsd\beta$ *and* $\frac{1}{\eta_m}\int_{-\eta_m}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta$ *are introduced in* $\theta(t)$ *to involve more cross terms among* $x(t)$, $x(t-\eta_m)$, $x(t-\tau(t))$, $x(t-\eta_M)$, $\frac{1}{\eta_M-\tau(t)}\int_{t-\eta_M}^{t-\tau(t)} x(s)ds$ *and* $\frac{1}{\tau(t)-\eta_m}\int_{t-\tau(t)}^{t-\eta_m} x(s)ds$. *This provide more freedom to choose matrices in LKFs achieving the maximum stability margin. The* *simultaneous presence of triple-integral terms in* (5.20) *along the interval* $[0, \eta_M]$ *could contain more information about the time delay and consequently provide more relaxation of the conservatism result.*

Remark 5.3 *Among the different methods reducing the conservatism, the free-weighting matrices method [168] is the most promising, it consists in the expense of many additional matrix variables. In our case, the matrices variable M and N in Theorem 5.1 are indispensable to derive the stability criterion. In addition the Wirtinger inequality is used to estimate the cross terms in the double and triple-integral forms, this give less conservative and more computationally stability criterion. Also, the application of the reciprocally convex approach combined with Wirtinger inequality can handle the cross terms in* (5.49) *and* (5.50) *without enlargement (only M and N are introduced).*

Remark 5.4 *In summary, the originality of the proposed method rely on both the choice of LKFs and the way to estimate the integral inequalities of its derivatives using Wirtinger-based inequalities and improved reciprocal convexity.*

5.3.2 Obtaining a robust state feedback control law.

In this section, based on Theorem 5.1 presented above, we will show how we can obtain a robust feedback control law using the state feedback controller (5.4) (stabilization problem), for an NCSs (5.6). The main results are presented below.

Theorem 5.2 *Let $i \in \mathcal{I}_2$ and $l \in \mathcal{I}_3$. For given scalars $\eta_m > 0$ and $\eta_M > 0$ such that $\eta_m \leq \tau(t) \leq \eta_M$, the NCSs model with interval network-induced delay* (5.1) *is Asymptotic Stabilize by the the feedback controller* (5.4) *if there exist symmetric positive definite matrices X, $\bar{P}$, $\bar{U}_i$, $\bar{S}_i$, $\bar{W}_i$, $\bar{Z}_i$ and $\bar{R}_l$ (in $\mathbb{R}^{n\times n}$), matrices variables $\bar{\mathcal{M}}$ and $\bar{\mathcal{N}}$ (in $\mathbb{R}^{2n\times n}$) and real matrix F (in $\mathbb{R}^{m\times n}$), such that the following LMIs holds:*

$$\begin{bmatrix} \bar{R}_2+\bar{W}_1 & 0 & \bar{M}_{11} & \bar{M}_{12} \\ 0 & 3\left(\bar{R}_2+\bar{W}_1\right) & \bar{M}_{21} & \bar{M}_{22} \\ \bar{M}_{11}^T & \bar{M}_{21}^T & \bar{R}_2+\bar{W}_1 & 0 \\ \bar{M}_{12}^T & \bar{M}_{22}^T & 0 & 3\left(\bar{R}_2+\bar{W}_1\right) \end{bmatrix} > 0. \tag{5.55}$$

$$\begin{bmatrix} \bar{R}_3+\bar{U}_1 & 0 & \bar{N}_{11} & \bar{N}_{12} \\ 0 & 3\left(\bar{R}_3+\bar{U}_1\right) & \bar{N}_{21} & \bar{N}_{22} \\ \bar{N}_{11}^T & \bar{N}_{21}^T & R_2+\bar{W}_1 & 0 \\ \bar{N}_{12}^T & \bar{N}_{22}^T & 0 & 3\left(\bar{R}_3+\bar{U}_1\right) \end{bmatrix} > 0. \tag{5.56}$$

$$\sum_{i=1}^{5} \bar{\Phi}_i + \mathcal{H}_e(\Pi_q^{1T}\bar{\mathcal{P}}\Pi_q^2) + \mathcal{H}_e(\tilde{\mathcal{X}}) < 0 \quad (q \in \mathcal{I}_2). \tag{5.57}$$

where:

$$\tilde{X} = \begin{bmatrix} AX & 0 & BF & 0_{(1\times 9)} & -X \\ 0 & 0 & 0 & 0_{(1\times 9)} & 0 \\ \varepsilon_1 AX & 0 & \varepsilon_1 BF & 0_{(1\times 9)} & -\varepsilon_1 X \\ 0_{(9\times 1)} & 0_{(9\times 1)} & 0_{(9\times 1)} & 0_{(9\times 9)} & 0_{(9\times 1)} \\ \varepsilon_2 AX & 0 & \varepsilon_2 BF & 0_{(1\times 9)} & -\varepsilon_2 X \end{bmatrix},$$

$$\begin{aligned}\Phi_1 &= e_1\bar{Z}_1e_1^T - e_2\bar{Z}_1e_2^T - e_4\bar{Z}_2e_4^T + e_{13}\left(\eta_m^2\bar{R}_1 + \eta_M^2\bar{R}_2 + (\eta_M-\eta_m)^2R_3\right)e_{13}^T \\ &\quad - (e_1-e_2)\bar{R}_1(e_1-e_2)^T - 3(e_1+e_2-2e_5)\bar{R}_1(e_1+e_2-2e_5)^T,\end{aligned} \tag{5.58}$$

$$\bar{\Phi}_2 = \frac{\eta_m^2}{2}e_{13}\left(\bar{S}_1+\bar{S}_2\right)e_{13}^T - \begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{S}}_1 & 0\\ * & \bar{\mathcal{S}}_2\end{bmatrix}\begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix}, \tag{5.59}$$

$$\begin{aligned}\bar{\Phi}_3 =& \frac{\eta_M^2}{2}e_{13}\left(\bar{W}_1+\bar{W}_2\right)e_{13}^T \\ &- \begin{bmatrix}\chi_3\\ \chi_4\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{W}}_1 & 0\\ * & \bar{\mathcal{W}}_1\end{bmatrix}\begin{bmatrix}\chi_3\\ \chi_4\end{bmatrix} - \begin{bmatrix}\chi_5\\ \chi_6\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{W}}_2 & 0\\ * & \bar{\mathcal{W}}_2\end{bmatrix}\begin{bmatrix}\chi_5\\ \chi_6\end{bmatrix},\end{aligned} \tag{5.60}$$

$$\begin{aligned}\Phi_4 =& \frac{(\eta_M-\eta_m)^2}{2}e_{13}\left(\bar{U}_1+\bar{U}_2\right)e_{13}^T \\ &- \begin{bmatrix}\chi_7\\ \chi_8\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{U}}_1 & 0\\ * & \bar{\mathcal{U}}_1\end{bmatrix}\begin{bmatrix}\chi_7\\ \chi_8\end{bmatrix} - \begin{bmatrix}\chi_9\\ \chi_{10}\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{U}}_2 & 0\\ * & \bar{\mathcal{U}}_2\end{bmatrix}\begin{bmatrix}\chi_9\\ \chi_{10}\end{bmatrix},\end{aligned} \tag{5.61}$$

$$\Phi_5 = -\begin{bmatrix}\chi_{11}\\ \chi_{12}\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{R}}_2 & \bar{\mathcal{M}}\\ * & \bar{\mathcal{R}}_2\end{bmatrix}\begin{bmatrix}\chi_{11}\\ \chi_{12}\end{bmatrix} - \begin{bmatrix}\chi_{13}\\ \chi_{14}\end{bmatrix}^T \begin{bmatrix}\bar{\mathcal{R}}_3 & \bar{\mathcal{N}}\\ * & \bar{\mathcal{R}}_3\end{bmatrix}\begin{bmatrix}\chi_{13}\\ \chi_{14}\end{bmatrix}, \tag{5.62}$$

$$\bar{\mathcal{R}}_i = \begin{bmatrix}\bar{R}_i & 0\\ * & 3\bar{R}_i\end{bmatrix}, \bar{\mathcal{S}}_i = \begin{bmatrix}2\bar{S}_i & 0\\ * & 4\bar{S}_i\end{bmatrix}, \bar{\mathcal{U}}_i = \begin{bmatrix}2\bar{U}_i & 0\\ * & 4\bar{U}_i\end{bmatrix}, \bar{\mathcal{W}}_i = \begin{bmatrix}2\bar{W}_i & 0\\ * & 4\bar{W}_i\end{bmatrix}, (i \in \mathcal{I}_4).$$

Proof 5.2 *let* $D_x^l = diag\begin{bmatrix} X & X_{(l\times 1)}\end{bmatrix}^T$. *Pre-and-post-multiplying* (5.7) *and* (5.8) *by* D_x^3 *and its transpose,* (5.54) *by* D_x^{10} *and its transpose, respectively, with the following variables changes* $F = KX$, $\bar{P} = X^TPX$, $\bar{Z}_i = X^TZ_iX$, $\bar{S}_i = X^TS_iX$, $\bar{R}_i = X^TR_iX$, $\bar{W}_i = X^TW_iX (i \in \mathcal{I}_2)$, $\bar{Q}_j = X^TQ_jX$, $\bar{\mathcal{M}} = X^T\mathcal{M}X$ *and* $\bar{\mathcal{N}} = X^T\mathcal{N}X (j \in \mathcal{I}_3)$, *we obtain the conditions expressed in Theorem 5.2. This completes the proof.*

5.4 Numerical examples

In this section, we provide academic numerical examples to illustrate the effectiveness and the conservatism reduction of the results proposed in Theorems 1 and 2 regarding to previous relevant results. The purpose is to compare the maximum allowable bounds of time-delay that guarantees the global asymptotic stability of the systems.

Example 5.1 *Let us consider the NCSs* (5.6), *with:*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, B \times K = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

This system has been often considered in previous articles for the conservatism comparison (see the references given in table 5.1) . The goal is to find the maximum allowable upper bound η_M (denoted as maub(η_M) of the network-induced delay and packet dropouts $\tau(t)$, which ensure that the considered NCSs is GAS).
Table 5.1 summarizes the obtained maub(τ_M) with fixed $h = 0.01$ and $\bar{\delta} = 10$ from various previous results, compared with ones obtained from Theorem 5.1. As one can notice,

Table 5.1: The obtained *maub*(η_M) with varying η_m of integrator system

.

Considered results $/\tau_m$	2	3	4	5
[103]	2.5660	3.3400	4.1690	5.0270
[88]	2.6700	3.3800	4.1600	-
[89]	2.6900	3.4100	4.2000	5.0300
[62]	2.6987	3.4186	4.2097	5.0440
[64] ($\lambda = 3$)	2.7990	3.4990	4.2960	5.1400
[113]	2.8000	3.5000	4.3000	5.1400
[3]	3.0545	3.6939	4.4026	5.1557
[25]	3.4671	4.1535	4.4731	-
[25]($\sigma = 10$)	3.5836	4.2009	4.9268	5.8774
[57] ($\lambda = 3, \sigma = 12$)	3.6741	4.3021	5.0379	5.9536
[57] ($\lambda = 3, \sigma = 15$)	3.7293	4.3847	5.1125	6.0497
Theorem 5.1	**4.1497**	**5.0601**	**6.0258**	**7.0019**

among all the considered results, the ones obtained from Theorem 5.1 provide the biggest maximal allowable values of η_M and the considered networked control system NCSs is GAS for all $\tau(t) \in [\eta_m, maub(\eta_M)]$. These numerical results confirm the reduction of the conservatism provided by the LMI stability conditions proposed in Theorem 5.1 regarding to the other considered results. The NCSs of example 5.1 state responses dynamics with network-induce delay (5.3) (with delay profile in Figure 5.2) are plotted in Figure 5.1 with the initial conditions $x(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$, $\forall t \in [-\eta_M, -\eta_m]$, and with $\eta_M = 3.0253$, $\eta_m = 0.5$. Figure 1 confirm the system stability.

Example 5.2 *Consider the following closed loop linear time-varying system described by (K is the controller gain matrix):*

$$A = \begin{bmatrix} 0.0 & 1.0 \\ -10.0 & -1.0 \end{bmatrix}, B \times K = \begin{bmatrix} 0.0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$

In this example the upper bounds of input delay (maub(η_M)) (with delay profile in Figure

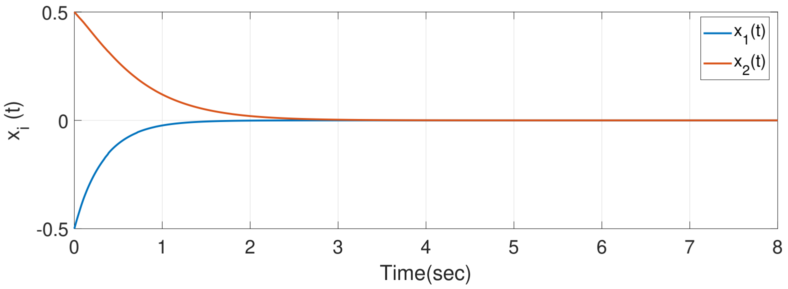

Figure 5.1: states trajectories of Integrator system.

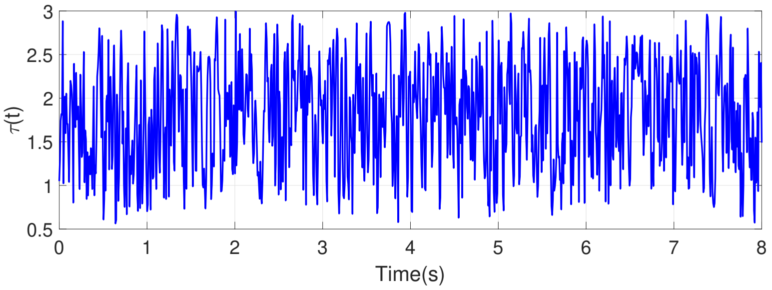

Figure 5.2: Delay profile in the simulation of Integrator system.

3) are calculated with various value of lower bounds of network-induced delay η_m by the proposed Theorem 5.1, the obtained results are compared with those of some recent papers results given in Table 5.2. The improvements brought by the proposed Theorem 5.1

Table 5.2: The obtained $maub(\eta_M)$ with varying η_m of motor system.

Considered results	$\eta_m = 0.3$	$\eta_m = 0.7$	$\eta_m = 1$	$\eta_m = 2$
[82]	1.28	1.64	1.94	2.94
[89]	1.33	1.73	2.03	3.03
[44]	1.64	2.02	2.31	3.31
[55](δ = 10)	1.93	2.22	2.44	3.37
Theorem 5.1	**2.04**	**2.37**	**2.61**	**3.48**

compared with other methods are clearly shown. For $2 \leq \tau(t) \leq 3.48$, *Figure 5.3 shows the state dynamics of the system* (4.2) *with the initial condition* $x(0) = \begin{bmatrix} 0.5 & -0.8 \end{bmatrix}^T$. *As one can see, the considered closed loop NCSs* (5.6) *is asymptotically stable.*

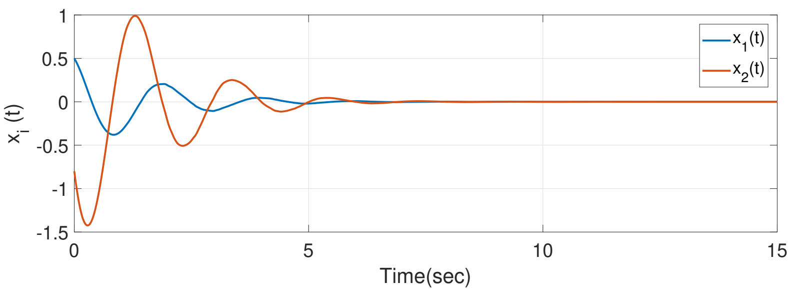

Figure 5.3: Delay profile in the simulation of Motor system.

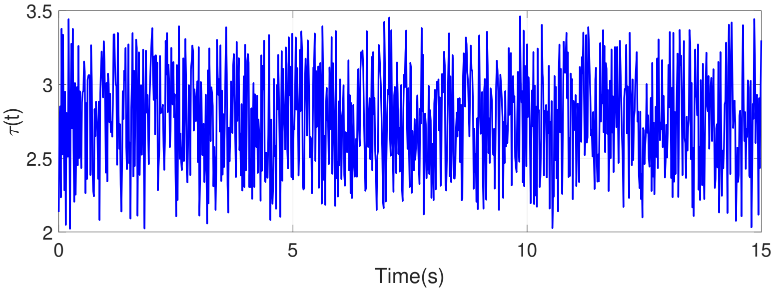

Figure 5.4: Delay profile in the simulation of Motor system.

Example 5.3 *(Controller design for quadruple-tank process)*
In this example, we apply the proposed method presented in section 5.3.2 to obtain a robust state feedback controller for a real application namely "quadruple-tank process system" controlled over network. The systems consists of four interconnected water tanks and two pumps (Fig.5.5). the goal is to control the water level in the two lower tanks using pump 1 and 2 (more details of this system can be found in [36] and [159]). The author in Ref. [36] proposed a state-space model of the quadruple-tank process and designed a state feedback controller as follows:

$$\dot{x}(t) = A_1 x(t) + A_2 x(t - \eta_m) + B_1 x(t - \eta_M) + B_2 x(t - \tau_3). \tag{5.63}$$

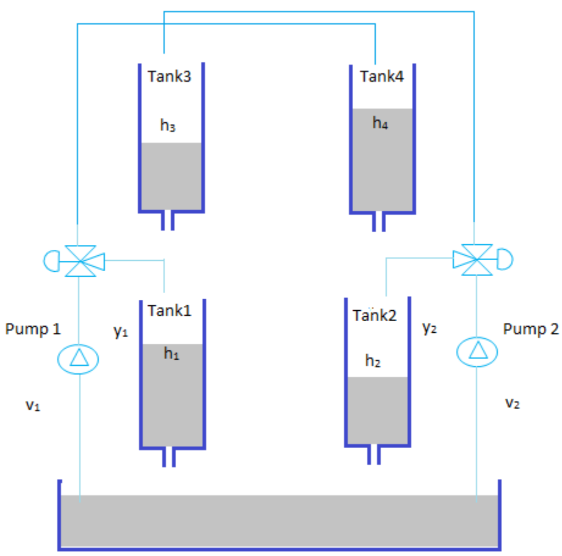

Figure 5.5: Schematic representation of the quadruple-tank process ([36]).

where

$$A_1 = \begin{bmatrix} -0.0021 & 0 & 0 & 0 \\ 0 & -0.0021 & 0 & 0 \\ 0 & 0 & -0.0424 & 0 \\ 0 & 0 & 0 & -0.0424 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 0.0424 & 0 \\ 0 & 0 & 0 & 0.0424 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1113\gamma_1 & 0 & 0 & 0 \\ 0 & 0.1042\gamma_2 & 0 & 0 \end{bmatrix}^T, B_2 = \begin{bmatrix} 0 & 0 & 0 & 0.1113(1-\gamma_1) \\ 0 & 0 & 0.1042(1-\gamma_2) & 0 \end{bmatrix}^T,$$

$$\gamma_1 = 0.333, \quad \gamma_2 = 0.307, \quad u = Kx(t).$$

For our method presented in subsection 5.3.2, for simplicity purpose, assume that $\tau_1 = 0$ *and* $\eta_M = \tau_3 = \tau(t)$*. Therefore, the control law is given by* $u(t) = Kx(t - \tau(t))$*. Let* $A = A_1 + A_2$, $B = B_1 + B_2$*, then, the quadruple-tank process model* (5.63) *can be rewritten to the form of system* (5.6)*. For given* $\eta_m = 0.01$ *and* $\varepsilon_1 = 0.05$ *and* $\varepsilon_2 = 0.03$*, the maximum allowable value network-induced delay are* $\eta_M = 0.9621$ *under the state feedback controller*

$$u(t) = \begin{bmatrix} 0.3587 & -0.8530 & 0.3775 & -0.8975 \\ -0.8769 & 0.4156 & -0.9227 & 0.4374 \end{bmatrix} x(t).$$

With the above control law, the state trajectories of system (5.6) *with the delay profile of Figure 5.7 is given in Figure 6, when the initial conditions are* $x(0) = \begin{bmatrix} -0.6 & 0.7 & -0.2 & 0.4 \end{bmatrix}^T$ *and* $0.01 \le \tau(t) \le 0.9621$*. It can be seen that the state trajectories of system* (5.6) *with the commands signal trajectories of Figure 5.8 tends to zero, which demonstrates that the proposed state feedback controller stabilize the considered NCSs* (5.6).

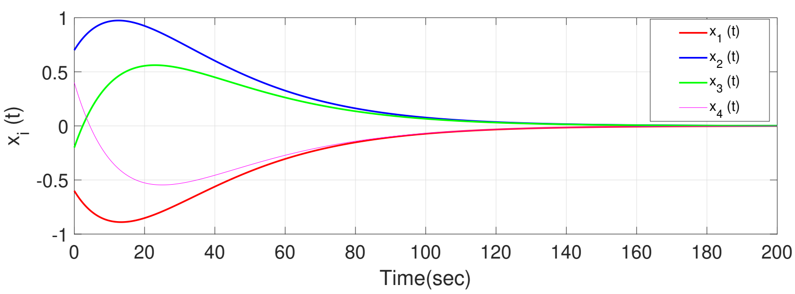

Figure 5.6: state trajectories of quadruple-tank process.

Example 5.4 *(batch reactor) Let consider the benchmark example of a batch reactor under networked state feedback controller suffering from network-induced delays, where*

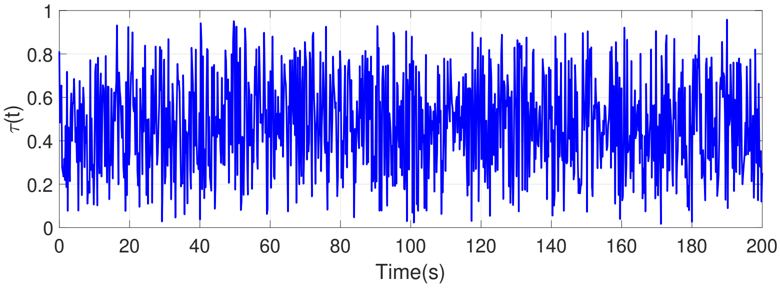

Figure 5.7: Delay profile in the simulation of example 5.3.

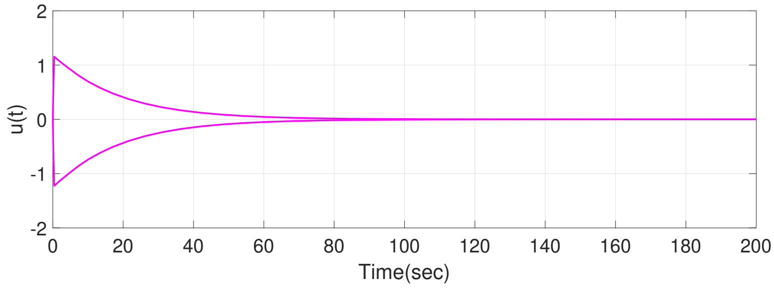

Figure 5.8: commands trajectories of quadruple-tank process 'u1(t) and u2(t)'.

:

$$A = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.2902 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}.$$

For various values of η_m given in Table 5.3, we apply Theorem 5.2 with $\varepsilon_1 = 10$ and $\varepsilon_2 = 10$ to find the maximum values of η_M that guarantee that our NCSs (5.6) *still GAS under various induced-network delays with profile presented in Figure 9 . For given*

Table 5.3: The obtaned $maub(\eta_M)$ with varying η_m of Batch reactor

.

Results / η_m	0	0.1	0.2	0.3	0.5
Theorem 5.2	**0.41**	**0.51**	**0.56**	**0.60**	**0.64**

$\eta_m = 0.5$ and $\varepsilon_{1,2} = 10$ the maximum allowable value $\eta_M = 0.64$ under the state feedback controller

$$u(t) = \begin{bmatrix} 0.0738 & -0.0211 & 0.0562 & -0.0734 \\ 1.1220 & 0.2648 & 0.8187 & -0.3670 \end{bmatrix} x(t).$$

To validate the effectiveness of our methods, the state trajectories of system (5.6) *is plotted and shown in Figure 5.9 for the initial condition $x(0) = \begin{bmatrix} -0.1 & 0.5 & -0.3 & 0.2 \end{bmatrix}^T$ and $0.1 \leq \tau(t) \leq 0.51$. It can be seen It can clearly be seen that the calculated state feedback controller with the trajectories presented in Figure 5.9, can stabilize the considered NCSs* (5.6) *under the considered network-induced delays.*

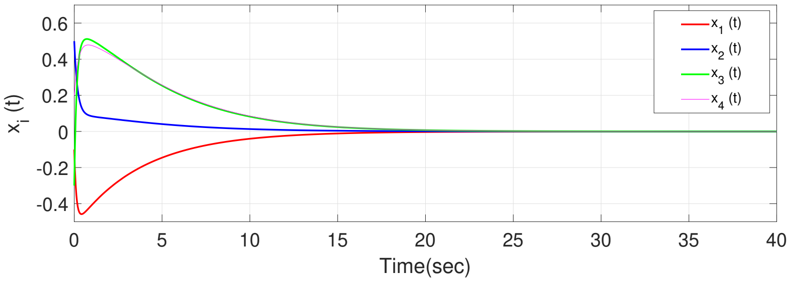

Figure 5.9: state trajectories of Batch reactor.

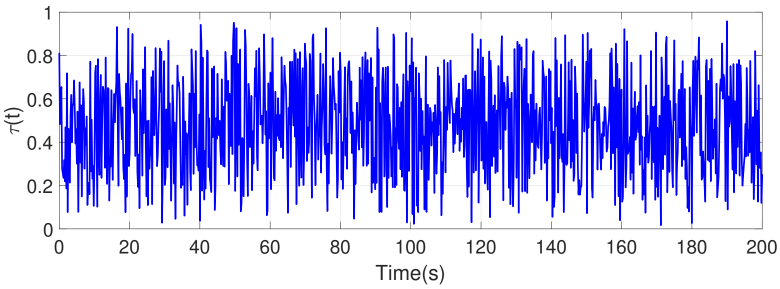

Figure 5.10: Delay profile in the simulation of Batch reactor.

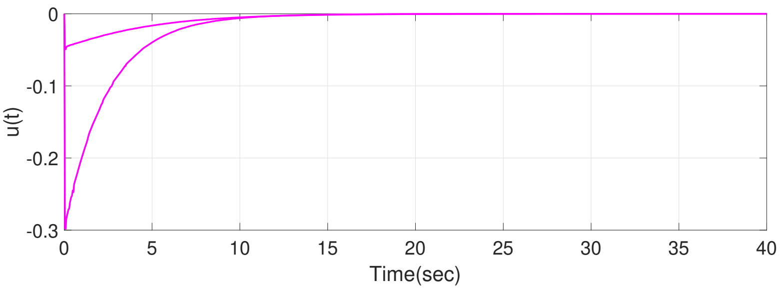

Figure 5.11: commands trajectories of Batch reactor 'u1(t) and u2(t)'.

5.5 Conclusion

A new method of stability and stabilization improvement of Networked Control Systems (NCSs), subject to network-induced delay is presented in this chapter. The method is based on the combination of Wirtinger inequality with improved reciprocal convexity. This allow us to address the problem of stability analysis and controller design for NCSs in the form of LMIs. The resolution of such optimisation problem permits a relaxation of the accepted network-induced delays. The proposed approach is validated with four numerical examples and the obtained results are compared with those of other existing methods, and their superiority in terms of conservativeness are proved.

Chapter 6

Conclusions and Future Work

This thesis presents some contributions in the stability analysis and robust state feedback controller design for NCSs subject to some network-induced disadvantages such as network-induced delay and packet dropouts, as it is not possible to avoid these imperfections, they were incorporated into the controller design. In Chapter 1, we presented an introduction in the field of Networked control systems including a brief History and recent trends in the modeling and stability analysis of NCSs, Chapter 2 was dedicated to the presentation of the principal definitions of NCSs and these stability analysis/robust controller design challenges. In addition, different architectures and topologies for NCSs were presented and actual and real implementation of NCSs in an industrial area were discussed, then we mentioned different control techniques involved the study and design controllers for NCSs, and we concluded the chapter by presenting the different protocols used in the real implementation of NCSs.

In Chapter 3, we presented fundamental concepts of the stability analysis using the Lyapunov theorem and different definitions and notation related to them, then we moved to the presentation of the robust control techniques where we focused on the $\mathcal{H}_\infty$ technique to avoid perturbations and modeling uncertainties. In Chapter 4, we presented a first contribution for the stability analysis and controller synthesis of NCSs where a stochastic NCSs model with stochastic time-varying network-induced is used, the non-uniform and distribution of network-induced delays which simulates the real delay in the real network are fully introduced in the modeling and stability of NCSs. Then, the Lyapunov-Krasovskii functional (LKF) is formulated using probabilistic information's of both lower and upper bounds of the time-varying network-induced delay and Wirtinger-based integral inequalities are used to estimate the accuracy of the resulting time derivatives and also to reduce conservatism of the stability conditions. Afterward, stability condition based on $\mathcal{H}_\infty$ disturbance attenuation level is expressed in terms of a set of linear matrix inequalities (LMIs), and Finsler's lemma is used to relax it by adding slack decision variables and decoupling the systems matrices from those of Lyapunov-Krasovskii. This procedure makes the state feedback controller design as simple as a variables change. Finally, a maximum

allowable upper bound of the network induced delay and state feedback controller gains are calculated by resolving the above relaxed LMIs' convex optimization problem.
In the last chapter, we presented a second contribution concerning the stability analysis and controller design for NCSs modeled as a sampled-data system with network-induced delay. Less conservative stability conditions in the form of linear matrix inequality (LMIs) are derived by constructing new augmented Lyapunov-Krasovskii functional (LKFs) with double and triple integral terms with a novel augmented vector. Wirtinger-based inequalities and extended reciprocally convex approach are also used to estimate the time derivative of LKFs. The obtained stability condition gives maximum allowable bound (MAUB) of network-induced delay that guarantees the stability of an NCSs compared with other existing methods. A state feedback controller is also derived from the proposed stability condition.
Finally, The contributions presented in chap 4 and 5 were validated by practical numerical examples; the results show that the negative effects of the unpredictable network-induced delays are compensated and the stability of NCSs guaranteeing high disturbance attenuation level. These results were published in many international conference paper's and also in an international peer-reviewed journal.

6.1 Future Work

The work accomplished in this thesis has explored several NCSs stability analysis and robust controller design, it achieved significant results in this domain where the researches still at these initial stage. We believed that the improvements addressed in this thesis should be more improved in future researches beginning from our obtained results, here below some future research investigations :

- The Stability condition derived in Chapter 4 and 5 depends on the Network-induced delay and Packet dropout, while the NCSs suffer from others several network-induced imperfections and constraints (quantization error, variable sampling, bandwidth limitation,...etc.). It is obvious that the inclusion of all the imperfections will be most suitable for a possible real implementation of the obtained control laws. Therefore, we will focus in our future works on the Trade-off between all the network-induced imperfections to design an NCSs that can be implemented in industries.

- The periodic transmission of closed-loop data through the network causes different problems such as packet collision, buffer overflow, and network congestion which lead to packet losses and degrade the network quality of services. this means that it is necessary to design new data transmission strategies to avoid the loss in packets by reducing the amount of data transmitted through a new event-triggered control design strategy based on aperiodic transmission and threshold between the latest and current transmission to decide exactly the time required to transmit, or by inserting the buffer overflow challenge into the design of the network control system.

- Most of the real physical systems are inherently nonlinear, for that we will focus in the future on the analysis of the nonlinear NCSs subject to network-induced imperfections by designing fuzzy controllers with Takagi–Sugeno (T-S) fuzzy NCSs model subject to network-induced imperfections.

References

[1] Ali-Akbar Ahmadi and Farzad R Salmasi. Observer-based reliable control for lipschitz nonlinear networked control systems with quadratic protocol. *International Journal of Control, Automation and Systems*, 13(3):753–763, 2015.

[2] S Massoud Amin and Bruce F Wollenberg. Toward a smart grid: power delivery for the 21st century. *IEEE power and energy magazine*, 3(5):34–41, 2005.

[3] Jiyao An, Zhiyong Li, and Xiaomei Wang. A novel approach to delay-fractional-dependent stability criterion for linear systems with interval delay. *ISA transactions*, 53(2):210–219, 2014.

[4] Yassine Ariba and Frédéric Gouaisbaut. Construction of lyapunov-krasovskii functional for time-varying delay systems. In *2008 47th IEEE Conference on Decision and Control*, pages 3995–4000. IEEE, 2008.

[5] Jianjun Bai, Renquan Lu, Hongye Su, and Anke Xue. Modeling and h_∞ control of wireless networked control system with both delay and packet loss. *Journal of the Franklin Institute*, 352(10):3915–3928, 2015.

[6] Yan Bai, Zhichen Li, and Congzhi Huang. New h_∞ control approaches for interval time-delay systems with disturbances and their applications. *ISA transactions*, 65:174–185, 2016.

[7] Lubomir Bakule. Decentralized control: An overview. *Annual reviews in control*, 32(1):87–98, 2008.

[8] Lei Bao, Mikael Skoglund, and Karl Henrik Johansson. Encoder˜ decoder design for event-triggered feedback control over bandlimited channels. In *2006 American Control Conference*, pages 4183–4188. IEEE, 2006.

[9] Aude Bartholomeus-Goubet. *Sur la stabilité et la stabilisation des sytèmes retardés: critères dépendant des retards*. PhD thesis, Lille 1, 1996.

[10] Saverio Bolognani and Sandro Zampieri. A gossip-like distributed optimization algorithm for reactive power flow control. *IFAC Proceedings Volumes*, 44(1):5700–5705, 2011.

[11] Sathyam Bonala, Bidyadhar Subudhi, and Sandip Ghosh. On delay robustness improvement using digital smith predictor for networked control systems. *European Journal of Control*, 34:59–65, 2017.

[12] Fayçal Bourahala and Kevin Guelton. A finsler-based result for the stability analysis of takagi-sugeno fuzzy models with interval time-varying delays. In *2017 11th Asian Control Conference (ASCC)*, pages 1743–1748. IEEE, 2017.

[13] Fayçal Bourahala, Kevin Guelton, Farid Khaber, and Noureddine Manamanni. Improvements on pdc controller design for takagi-sugeno fuzzy systems with state time-varying delays. *IFAC-PapersOnLine*, 49(5):200–205, 2016.

[14] Stephen Boyd, Laurent El Ghaoui, Eric Feron, and Venkataramanan Balakrishnan. *Linear matrix inequalities in system and control theory*, volume 15. Siam, 1994.

[15] Guoliang Chen, Jianwei Xia, Guangming Zhuang, and Junsheng Zhao. Improved delay-dependent stabilization for a class of networked control systems with nonlinear perturbations and two delay components. *Applied Mathematics and Computation*, 316:1–17, 2018.

[16] Jie Chen, Su Meng, and Jian Sun. Stability analysis of networked control systems with aperiodic sampling and time-varying delay. *IEEE transactions on cybernetics*, 47(8):2312–2320, 2016.

[17] Liang Chen, Jianyan Sun, and Chunxiang Xu. Regularized extreme learning machine–based intelligent adaptive control for uncertain nonlinear systems in networked control systems. *Personal and Ubiquitous Computing*, 23(3-4):617–625, 2019.

[18] Burak Demirel, Corentin Briat, and Mikael Johansson. Deterministic and stochastic approaches to supervisory control design for networked systems with time-varying communication delays. *Nonlinear Analysis: Hybrid Systems*, 10:94–110, 2013.

[19] Lei Ding, Qing-Long Han, and Ge Guo. Network-based leader-following consensus for distributed multi-agent systems. *Automatica*, 49(7):2281–2286, 2013.

[20] John Doyle, Keith Glover, Pramod Khargonekar, and Bruce Francis. State-space solutions to standard h_2 and h_∞ control problems. In *1988 American Control Conference*, pages 1691–1696. IEEE, 1988.

[21] Baozhu Du, James Lam, and Zhan Shu. Stabilization for state/input delay systems via static and integral output feedback. *Automatica*, 46(12):2000–2007, 2010.

[22] Arezou Elahi and Alireza Alfi. Finite-time h_∞ control of uncertain networked control systems with randomly varying communication delays. *ISA transactions*, 69:65–88, 2017.

[23] Fabio Fagnani and Sandro Zampieri. Stability analysis and synthesis for scalar linear systems with a quantized feedback. *IEEE Transactions on automatic control*, 48(9):1569–1584, 2003.

[24] Arash Farnam and Reza Mahboobi Esfanjani. Improved stabilization method for networked control systems with variable transmission delays and packet dropout. *ISA transactions*, 53(6):1746–1753, 2014.

[25] Arash Farnam and Reza Mahboobi Esfanjani. Improved linear matrix inequality approach to stability analysis of linear systems with interval time-varying delays. *Journal of Computational and Applied Mathematics*, 294:49–56, 2016.

[26] Yuan Ge, Qigong Chen, Ming Jiang, and Yiqing Huang. Modeling of random delays in networked control systems. *Journal of Control Science and Engineering*, 2013:8, 2013.

[27] Keqin Gu, Jie Chen, and Vladimir L Kharitonov. *Stability of time-delay systems*. Springer Science & Business Media, 2003.

[28] Rachana Ashok Gupta and Mo-Yuen Chow. Networked control system: Overview and research trends. *IEEE transactions on industrial electronics*, 57(7):2527–2535, 2009.

[29] Sandra Hirche, Chih-Chung Chen, and Martin Buss. Performance oriented control over networks–switching controllers and switched time delay–. In *Proceedings of the 45th IEEE Conference on Decision and Control*, pages 4999–5005. IEEE, 2006.

[30] Zhipei Hu and Feiqi Deng. Robust h_∞ control for networked systems with transmission delays and successive packet dropouts under stochastic sampling. *International Journal of Robust and Nonlinear Control*, 27(1):84–107, 2017.

[31] Dan Huang and Sing Kiong Nguang. Dynamic output feedback control for uncertain networked control systems with random network-induced delays. *International Journal of Control, Automation and Systems*, 7(5):841, 2009.

[32] Dan Huang and Sing Kiong Nguang. *Robust control for uncertain networked control systems with random delays*, volume 386. Springer Science & Business Media, 2009.

[33] Changki Jeong, PooGyeon Park, and Sung Hyun Kim. Improved approach to robust stability and h_∞ performance analysis for systems with an interval time-varying delay. *Applied Mathematics and Computation*, 218(21):10533–10541, 2012.

[34] Li Jie, Zhou Ying, and Fan Chunxia. Stability analysis for variable sampling networked control systems with data packet dropout and partly unknown transition probabilities. In *2019 Chinese Control And Decision Conference (CCDC)*, pages 2367–2372. IEEE, 2019.

[35] Wei Jin, Bin Tang, Jiali Qin, and Yun Zhang. Improvement on fuzzy-model-based stabilization of nonlinear networked control systems. In *Proceedings of the 33rd Chinese Control Conference*, pages 5766–5773. IEEE, 2014.

[36] Karl Henrik Johansson. The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on control systems technology*, 8(3):456–465, 2000.

[37] Hamid Reza Karimi. Robust h-infinity filter design for uncertain linear systems over network with network-induced delays and output quantization. 2009.

[38] Hassan K Khalil and Jessy W Grizzle. *Nonlinear systems*, volume 3. Prentice hall Upper Saddle River, NJ, 2002.

[39] Jin-Hoon Kim. Further improvement of jensen inequality and application to stability of time-delayed systems. *Automatica*, 64:121–125, 2016.

[40] SA Koubias and GD Papadopoulos. Modern fieldbus communication architectures for real-time industrial applications. *Computers in industry*, 26(3):243–252, 1995.

[41] Oh-Min Kwon, Myeong-Jin Park, Sang-Moon Lee, Ju H Park, and Eun-Jong Cha. Stability for neural networks with time-varying delays via some new approaches. *IEEE transactions on neural networks and learning systems*, 24(2):181–193, 2012.

[42] Oh-Min Kwon, Myeong-Jin Park, Ju H Park, Sang-Moon Lee, and Eun-Jong Cha. Analysis on robust $h\infty$ performance and stability for linear systems with interval time-varying state delays via some new augmented lyapunov–krasovskii functional. *Applied Mathematics and Computation*, 224:108–122, 2013.

[43] Won Il Lee, Changki Jeong, and PooGyeon Park. Further improvement of delay-dependent stability criteria for linear systems with time-varying delays. In *2012 12th International Conference on Control, Automation and Systems*, pages 1300–1304. IEEE, 2012.

[44] Won Il Lee, Seok Young Lee, and Poogyeon Park. Improved criteria on robust stability and h_∞ performance for linear systems with interval time-varying delays via new triple integral functionals. *Applied Mathematics and Computation*, 243:570–577, 2014.

[45] Won Il Lee and Poogyeon Park. Second-order reciprocally convex approach to stability of systems with interval time-varying delays. *Applied Mathematics and Computation*, 229:245–253, 2014.

[46] Huan Lei, Shouming Zhong, and Peiming Wang. Exponential stability analysis for switched systems with distributed time-varying delays. In *2012 international conference on computer science and service system*, pages 2189–2193. IEEE, 2012.

[47] Bing Li and Jun-feng WU. A new delay-dependent stability criteria for networked control systems. *Journal of Theoretical and Applied Information Technology*, 43(1):127–132, 2012.

[48] Jiarui Li, Yugang Niu, and Yuanyuan Zou. Sliding mode control for networked control system under fading channels. In *2019 IEEE Conference on Control Technology and Applications (CCTA)*, pages 561–566. IEEE, 2019.

[49] Jing Li and Bin Zuo. Modeling and stability analysis of ncs with markov delay and dropout. In *Recent Advances in Computer Science and Information Engineering*, pages 307–313. Springer, 2012.

[50] Liming Li and Tao Li. Stability analysis of sample data systems with input missing: A hybrid control approach. *ISA transactions*, 90:116–122, 2019.

[51] Meng Li, Yong Chen, Anjian Zhou, Wen He, and Xu Li. Adaptive tracking control for networked control systems of intelligent vehicle. *Information Sciences*, 503:493–507, 2019.

[52] Min Li, Feng Shu, Duyu Liu, and Shouming Zhong. Robust $h\infty$ control of ts fuzzy systems with input time-varying delays: A delay partitioning method. *Applied Mathematics and Computation*, 321:209–222, 2018.

[53] Tao Li, Zhipeng Li, Li Zhang, and Shumin Fei. Improved approaches on adaptive event-triggered output feedback control of networked control systems. *Journal of the Franklin Institute*, 355(5):2515–2535, 2018.

[54] Xiuying Li and Shuli Sun. Robust h_∞ control for networked systems with random packet dropouts and time delays. *Procedia Engineering*, 29:4192–4197, 2012.

[55] Zhichen Li, Yan Bai, Congzhi Huang, and Yunfei Cai. Novel delay-partitioning stabilization approach for networked control system via wirtinger-based inequalities. *ISA transactions*, 61:75–86, 2016.

[56] Zhichen Li, Yan Bai, Congzhi Huang, and Huaicheng Yan. A generalized double integral inequalities approach to stability analysis for time-delay systems. *Journal of the Franklin Institute*, 354(8):3455–3471, 2017.

[57] Zhichen Li, Yan Bai, and Tianqi Li. Improved stability and stabilization design for networked control systems using new quadruple-integral functionals. *ISA transactions*, 63:170–181, 2016.

[58] F-L Lian, James R Moyne, and Dawn M Tilbury. Network protocols for networked control systems. In *Handbook of Networked and Embedded Control Systems*, pages 651–675. Springer, 2005.

[59] Leipo Liu and Xiaona Song. Static output tracking control of nonlinear systems with one-sided lipschitz condition. *Mathematical Problems in Engineering*, 2014, 2014.

[60] Pin-Lin Liu. Further results on delay-range-dependent stability with additive time-varying delay systems. *ISA transactions*, 53(2):258–266, 2014.

[61] Xinghua Liu, Xinghuo Yu, Guoqi Ma, and Hongsheng Xi. On sliding mode control for networked control systems with semi-markovian switching and random sensor delays. *Information Sciences*, 337:44–58, 2016.

[62] Yun Liu, Li-Sheng Hu, and Peng Shi. A novel approach on stabilization for linear systems with time-varying input delay. *Applied Mathematics and Computation*, 218(10):5937–5947, 2012.

[63] Yuzhi Liu and Muguo Li. An improved delay-dependent stability criterion of networked control systems. *Journal of the Franklin Institute*, 351(3):1540–1552, 2014.

[64] Yuzhi Liu and Muguo Li. Improved robust stabilization method for linear systems with interval time-varying input delays by using wirtinger inequality. *ISA transactions*, 56:111–122, 2015.

[65] Merid Lješnjanin, Dragan Nešić, and Daniel E Quevedo. Robust stability of a class of networked control systems. *Automatica*, 73:117–124, 2016.

[66] Lei Lu, Jinxing Lin, and Kanglei Ren. h_∞ output tracking control for networked control systems with network-induced delays. In *2018 Chinese Control And Decision Conference (CCDC)*, pages 1416–1421. IEEE, 2018.

[67] Jing Ma, Junchen Li, Xiang Gao, and Zengping Wang. Research on time-delay stability upper bound of power system wide-area damping controllers based on improved free-weighting matrices and generalized eigenvalue problem. *International Journal of Electrical Power & Energy Systems*, 64:476–482, 2015.

[68] Magdi S Mahmoud, Nezar M Alyazidi, and Abdul-Wahed A Saif. Lqg control design over lossy communication links. *International Journal of Systems Science*, 45(11):2309–2326, 2014.

[69] Magdi S Mahmoud, SZ Selim, P Shi, and MH Baig. New results on networked control systems with non-stationary packet dropouts. *IET Control Theory & Applications*, 6(15):2442–2452, 2012.

[70] Magdi S Mahmoud, SZ Selim, and Peng Shi. Global exponential stability criteria for neural networks with probabilistic delays. *IET control theory & applications*, 4(11):2405–2415, 2010.

[71] Cai Meng, Tianmiao Wang, Wusheng Chou, Sheng Luan, Yuru Zhang, and Zengmin Tian. Remote surgery case: robot-assisted teleneurosurgery. In *IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA'04. 2004*, volume 1, pages 819–823. IEEE, 2004.

[72] ROUAMEL Mohamed and GHERBI Sofiane. Using smith predictor compensator to avoid delay in a networked wind system. In *International Conference on Advanced Engineering of Petrochemical Industry (ICAEPI 2017), Skikda-Algeria*, page 12, 2017.

[73] ROUAMEL Mohamed, GHERBI Sofiane, and BOURAHALA Faycal. Further stability improvements of networked control system with delay distribution approach. In *Third International Conference on Technological Advanced in Electical Engineering (ICTAEE'18), Skikda-Algeria*, page 12, 2018.

[74] ROUAMEL Mohamed, GHERBI Sofiane, and BOURAHALA Faycal. A new stability improvements of networked control system with delay distribution approach:less conservative result. In *Second International Conference on Electrical Engineering (ICEEB'18), Biskra-Algeria*, page 12, 2018.

[75] James R Moyne, Nader Najafi, Daniel Judd, and Allen Stock. Analysis of sensor/actuator bus interoperability standard alternatives for semiconductor manufacturing. In *Sensors Expo Conf. Proc*, 1994.

[76] Richard M Murray, Karl J Astrom, Stephen P Boyd, Roger W Brockett, and Gunter Stein. Future directions in control in an information-rich world. *IEEE Control Systems Magazine*, 23(2):20–33, 2003.

[77] Oreste Riccardo Natale, Olivier Sename, and Carlos Canudas-de Wit. Inverted pendulum stabilization through the ethernet network, performance analysis. In *Proceedings of the 2004 American Control Conference*, volume 6, pages 4909–4914. IEEE, 2004.

[78] Yugang Niu, Tinggang Jia, Xingyu Wang, and Fuwen Yang. Output-feedback control design for ncss subject to quantization and dropout. *Information Sciences*, 179(21):3804–3813, 2009.

[79] Yingnan Pan and Guang-Hong Yang. Event-based output tracking control for fuzzy networked control systems with network-induced delays. *Applied Mathematics and Computation*, 346:513–530, 2019.

[80] Zhonghua Pang, Guoping Liu, Donghua Zhou, and Dehui Sun. Data-based predictive control for networked nonlinear systems with packet dropout and measurement noise. *Journal of Systems Science and Complexity*, 30(5):1072–1083, 2017.

[81] Antonis Papachristodoulou and Stephen Prajna. On the construction of lyapunov functions using the sum of squares decomposition. In *Proceedings of the 41st IEEE Conference on Decision and Control, 2002.*, volume 3, pages 3482–3487. IEEE, 2002.

[82] PooGyeon Park, Jeong Wan Ko, and Changki Jeong. Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, 47(1):235–238, 2011.

[83] PooGyeon Park, Won Il Lee, and Seok Young Lee. Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems. *Journal of the Franklin Institute*, 352(4):1378–1396, 2015.

[84] Pablo A Parrilo and Sanjay Lall. Semidefinite programming relaxations and algebraic optimization in control. *European Journal of Control*, 9(2-3):307–321, 2003.

[85] Chen Peng and Yu-Chu Tian. Networked h_∞ control of linear systems with state quantization. *Information Sciences*, 177(24):5763–5774, 2007.

[86] Chen Peng, Min Wu, Xiangpeng Xie, and Yu-Long Wang. Event-triggered predictive control for networked nonlinear systems with imperfect premise matching. *IEEE Transactions on Fuzzy Systems*, 26(5):2797–2806, 2018.

[87] Chen Peng, Dong Yue, Engang Tian, and Zhou Gu. A delay distribution based stability analysis and synthesis approach for networked control systems. *Journal of the Franklin Institute*, 346(4):349–365, 2009.

[88] Wei Qian, Tao Li, Shen Cong, and Shumin Fei. Stability analysis for interval time-varying delay systems based on time-varying bound integral method. *Journal of the Franklin Institute*, 351(10):4892–4903, 2014.

[89] Wei Qian and Juan Liu. New stability analysis for systems with interval time-varying delay. *Journal of the Franklin Institute*, 350(4):890–897, 2013.

[90] Wei Qian, Weiwei Xing, Lei Wang, and Bingfeng Li. New optimal analysis method to stability and h_∞ performance of varying delayed systems. *ISA transactions*, 2019.

[91] Si-Wei Qiao, Xiao-Lin Xu, Jie Wu, Ling-Yan Wang, and Xi-Sheng Zhan. Stability analysis of networked control systems with multi-parameter constraints. In *2019 Chinese Control Conference (CCC)*, pages 5345–5349. IEEE, 2019.

[92] Li Qiu, Fengqi Yao, Gang Xu, Shanbin Li, and Bugong Xu. Output feedback guaranteed cost control for networked control systems with random packet dropouts and time delays in forward and feedback communication links. *IEEE Transactions on Automation Science and Engineering*, 13(1):284–295, 2014.

[93] Daniel E Quevedo, Eduardo I Silva, and Graham C Goodwin. Control over unreliable networks affected by packet erasures and variable transmission delays. *IEEE Journal on Selected Areas in Communications*, 26(4):672–685, 2008.

[94] Kadangode K Ramakrishnan and Henry Yang. The ethernet capture effect: Analysis and solution. In *Proceedings of 19th Conference on Local Computer Networks*, pages 228–240. IEEE, 1994.

[95] Hongru Ren, Renquan Lu, Junlin Xiong, and Yong Xu. Optimal estimation for discrete-time linear system with communication constraints and measurement quantization. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2018.

[96] Young-Hoon Roh and Jun-Ho Oh. Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation. *Automatica*, 35(11):1861–1865, 1999.

[97] Mohamed Rouamel, Sofiane Gherbi, and Fayçal Bourahala. Robust stability and stabilization of networked control systems with stochastic time-varying network-induced delays. *Transactions of the Institute of Measurement and Control*, 42(10):1782–1796, 2020.

[98] R Saravanakumar, M Syed Ali, He Huang, Jinde Cao, and Young Hoon Joo. Robust h_∞ state-feedback control for nonlinear uncertain systems with mixed time-varying

delays. *International Journal of Control, Automation and Systems*, 16(1):225–233, 2018.

[99] Shankar Sastry. Lyapunov stability theory. In *Nonlinear Systems*, pages 182–234. Springer, 1999.

[100] Mouquan Shen, Shen Yan, and Guangming Zhang. A new approach to event-triggered static output feedback control of networked control systems. *ISA transactions*, 65:468–474, 2016.

[101] Ting Shi, Peng Shi, and Shuoyu Wang. Robust sampled-data model predictive control for networked systems with time-varying delay. *International Journal of Robust and Nonlinear Control*, 29(6):1758–1768, 2019.

[102] RE Skelton, T Iwasaki, and KM Grigoriadis. *A unified algebraic approach to linear control design*. Taylor and Francis, London, 1998.

[103] Jian Sun, GP Liu, Jie Chen, and D Rees. Improved stability criteria for linear systems with time-varying delay. *IET Control Theory & Applications*, 4(4):683–689, 2010.

[104] Jiandong Sun and Jingping Jiang. Delay and data packet dropout separately related stability and state feedback stabilisation of networked control systems. *IET Control Theory & Applications*, 7(3):333–342, 2013.

[105] Cheng Tan, Lin Li, and Huanshui Zhang. Stabilization of networked control systems with both network-induced delay and packet dropout. *Automatica*, 59:194–199, 2015.

[106] Andrew S Tanenbaum. Computer networks, prentice-hall international. *Upper Saddle River, NJ*, 2, 1996.

[107] Bin Tang, Shiguo Peng, and Yun Zhang. Fuzzy-model-based robust $h\infty$ design of nonlinear packetized networked control systems. *IEEE Trans. Fuzzy Systems*, 24(3):544–557, 2016.

[108] Bin Tang, Jun Wang, and Yun Zhang. A delay-distribution approach to stabilization of networked control systems. *IEEE Transactions on Control of Network Systems*, 2(4):382–392, 2015.

[109] Ze Tang, Ju H Park, and Tae H Lee. Dynamic output-feedback-based h_∞ design for networked control systems with multipath packet dropouts. *Applied Mathematics and Computation*, 275:121–133, 2016.

[110] Yodyium Tipsuwan and Mo-Yuen Chow. Gain scheduler middleware: a methodology to enable existing controllers for networked control and teleoperation-part i: networked control. *IEEE transactions on Industrial Electronics*, 51(6):1218–1227, 2004.

[111] Domagoj Tolić. Stabilizing transmission intervals and delays in nonlinear networked control systems through hybrid-system-with-memory modeling and lyapunov–krasovskii arguments. *Nonlinear Analysis: Hybrid Systems*, 36:100834, 2020.

[112] Li Tongtao, Li Lixiong, and Fei Minrui. Exponential stability of a classic networked control systems with variable and bounded delay based on impulsive control theory. In *2009 Second International Conference on Intelligent Computation Technology and Automation*, volume 1, pages 785–789. IEEE, 2009.

[113] Hieu Trinh et al. Refined jensen-based inequality approach to stability analysis of time-delay systems. *IET Control Theory & Applications*, 9(14):2188–2194, 2015.

[114] SJLM van Loon, MCF Donkers, Nathan van de Wouw, and WPMH Heemels. Stability analysis of networked control systems with periodic protocols and uniform quantizers. *IFAC Proceedings Volumes*, 45(9):186–191, 2012.

[115] Jing Wang, Mengshen Chen, Hao Shen, Ju H Park, and Zheng-Guang Wu. A markov jump model approach to reliable event-triggered retarded dynamic output feedback h_∞ control for networked systems. *Nonlinear Analysis: Hybrid Systems*, 26:137–150, 2017.

[116] Jun-Wei Wang, Huai-Ning Wu, Lei Guo, and Yue-Sheng Luo. Robust h_∞ fuzzy control for uncertain nonlinear markovian jump systems with time-varying delay. *Fuzzy Sets and Systems*, 212:41–61, 2013.

[117] Tian-Bao Wang, Cheng-Dong Wu, Yu-Long Wang, and Yun-Zhou Zhang. Dynamic output feedback h_∞ control for continuous-time networked control systems. *IET Control Theory & Applications*, 7(7):1005–1013, 2013.

[118] Yanfeng Wang, Ping He, Heng Li, Xiaoyue Sun, Haoyang Mi, Wei Wei, Xingzhong Xiong, and Yangmin Li. h_∞ control of networked control system with data packet dropout via observer-based controller. *IEEE Access*, 2020.

[119] Yanqian Wang, Gongfei Song, Junjie Zhao, Jing Sun, and Guangming Zhuang. Reliable mixed h_∞ and passive control for networked control systems under adaptive event-triggered scheme with actuator faults and randomly occurring nonlinear perturbations. *ISA transactions*, 89:45–57, 2019.

[120] Yu-Long Wang and Qing-Long Han. Modelling and controller design for discrete-time networked control systems with limited channels and data drift. *Information Sciences*, 269:332–348, 2014.

[121] Zidong Wang, Fuwen Yang, Daniel WC Ho, and Xiaohui Liu. Robust h_∞ control for networked systems with random packet losses. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 37(4):916–924, 2007.

[122] Di Wu, Xi-Ming Sun, Yun-Bo Zhao, and Wei Wang. Stability analysis of nonlinear switched networked control systems with periodical packet dropouts. *Circuits, Systems, and Signal Processing*, 32(4):1931–1947, 2013.

[123] Dongxiao Wu, Jun Wu, and Sheng Chen. Robust h_∞ control for networked control systems with uncertainties and multiple-packet transmission. *IET Control Theory & Applications*, 4(5):701–709, 2010.

[124] Jing Wu and Tongwen Chen. Design of networked control systems with packet dropouts. *IEEE Transactions on Automatic control*, 52(7):1314–1319, 2007.

[125] Ying Wu and Yanpeng Wu. An enhanced smith predictor based robust control over networked control system with random time delays. In *2016 2nd International Conference on Control Science and Systems Engineering (ICCSSE)*, pages 50–54. IEEE, 2016.

[126] Q.-L. Han X.-M. Zhang. Networked control systems: A survey of trends and techniques. *IEEE/CAA JOURNAL OF AUTOMATICA SINICA*, Early Access:1–17, 2019.

[127] Zhiyu Xi. Dynamic sliding mode controller design for networked control systems with random packet loss and event driven quantisation. *IET Control Theory & Applications*, 12(17):2433–2440, 2018.

[128] Zhiyu Xi. Mean-square stability of networked control systems with event driven state quantisation and packet loss. *IET Control Theory & Applications*, 12(7):980–984, 2018.

[129] Yuanqing Xia, Wen Xie, Bo Liu, and Xiaoyun Wang. Data-driven predictive control for networked control systems. *Information Sciences*, 235:45–54, 2013.

[130] Jinxia Xie, Yang Song, Xiaomin Tu, and Minrui Fei. Stabilization for networked control systems with packet dropout based on average dwell time method. In *Life System Modeling and Intelligent Computing*, pages 8–13. Springer, 2010.

[131] Shixun Xiong, Mou Chen, and Qianxian Wu. Predictive control for networked switch flight system with packet dropout. *Applied Mathematics and Computation*, 354:444–459, 2019.

[132] Hao Xu, Avimanyu Sahoo, and Sarangapani Jagannathan. Stochastic adaptive event-triggered control and network scheduling protocol co-design for distributed networked systems. *IET Control Theory & Applications*, 8(18):2253–2265, 2014.

[133] Binqiang Xue, Haisheng Yu, and Mengling Wang. Robust h_∞ output feedback control of networked control systems with discrete distributed delays subject to packet dropout and quantization. *IEEE Access*, 7:30313–30320, 2019.

[134] Bu Xuhui, Hou Zhongsheng, Jin Shangtai, and Chi Ronghu. An iterative learning control design approach for networked control systems with data dropouts. *International Journal of Robust and Nonlinear Control*, 26(1):91–109, 2016.

[135] Jingjing Yan. Event-driven control for ncss with logarithmic quantization and packet losses. *Journal of Control Science and Engineering*, 2017, 2017.

[136] Jingjing Yan and Yuanqing Xia. Decentralized quantized control for ncss under periodic protocol. *Information Sciences*, 408:234–251, 2017.

[137] Fuwen Yang, Wu Wang, Yugang Niu, and Yongmin Li. Observer-based h_∞ control for networked systems with consecutive packet delays and losses. *International Journal of Control, Automation and Systems*, 8(4):769–775, 2010.

[138] Hongjiu Yang, Yang Xu, and Jinhui Zhang. Event-driven control for networked control systems with quantization and markov packet losses. *IEEE transactions on cybernetics*, 47(8):2235–2243, 2016.

[139] Rongni Yang, Guo-Ping Liu, Peng Shi, Clive Thomas, and Michael V Basin. Predictive output feedback control for networked control systems. *IEEE Transactions on Industrial Electronics*, 61(1):512–520, 2013.

[140] Tao Yang. *Impulsive control theory*, volume 272. Springer Science & Business Media, 2001.

[141] Tao Yang. *Impulsive systems and control: Theory and applications*. Nova Science Publishers, Inc., 2001.

[142] Xunyuan Yin, Lixian Zhang, Yanzheng Zhu, Changhong Wang, and Zhaojian Li. Robust control of networked systems with variable communication capabilities and application to a semi-active suspension system. *IEEE/ASME Transactions on Mechatronics*, 21(4):2097–2107, 2016.

[143] Mei Yu, Long Wang, Tianguang Chu, and Fei Hao. Stabilization of networked control systems with data packet dropout and transmission delays: continuous-time case. *European Journal of Control*, 11(1):40–49, 2005.

[144] Sun Yu and Xiaofeng Wang. Quantization scheme design in distributed event-triggered networked control systems. *IFAC Proceedings Volumes*, 44(1):13257–13262, 2011.

[145] Dong Yue, Qing-Long Han, and James Lam. Network-based robust $h\infty$ control of systems with uncertainty. *Automatica*, 41(6):999–1007, 2005.

[146] Dong Yue, Engang Tian, Yijun Zhang, and Chen Peng. Delay-distribution-dependent stability and stabilization of t–s fuzzy systems with probabilistic interval delay. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 39(2):503–516, 2008.

[147] George Zames. Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses. *IEEE Transactions on automatic control*, 26(2):301–320, 1981.

[148] George Zames and Bruce Francis. Feedback, minimax sensitivity, and optimal robustness. *IEEE transactions on Automatic Control*, 28(5):585–601, 1983.

[149] Hong-Bing Zeng, Yong He, Min Wu, and Jinhua She. Free-matrix-based integral inequality for stability analysis of systems with time-varying delay. *IEEE Transactions on Automatic Control*, 60(10):2768–2772, 2015.

[150] Dawei Zhang, Zhiyong Zhou, and Xinchun Jia. Network-based pi control for output tracking of continuous-time systems with time-varying sampling and network-induced delays. *Journal of the Franklin Institute*, 355(12):4794–4808, 2018.

[151] Hao Zhang, Gang Feng, Huaicheng Yan, and Qijun Chen. Sampled-data control of nonlinear networked systems with time-delay and quantization. *International Journal of Robust and Nonlinear Control*, 26(5):919–933, 2016.

[152] Hongmei Zhang, Jinde Cao, and Lianglin Xiong. Novel synchronization conditions for time-varying delayed lur'e system with parametric uncertainty. *Applied Mathematics and Computation*, 350:224–236, 2019.

[153] Huaipin Zhang, Dong Yue, Xiuxia Yin, and Ji Chen. Adaptive model-based event-triggered control of networked control system with external disturbance. *IET control theory & applications*, 10(15):1956–1962, 2016.

[154] Jinhui Zhang, James Lam, and Yuanqing Xia. Output feedback delay compensation control for networked control systems with random delays. *Information Sciences*, 265:154–166, 2014.

[155] Jinhui Zhang and Yuanqing Xia. Design of h_∞ fuzzy controllers for nonlinear systems with random data dropouts. *Optimal Control Applications and Methods*, 32(3):328–349, 2011.

[156] Jun Zhang and Da-Yong Luo. A new stability condition for networked control system with time-varying delay based on delay uneven-partitioning approach. In *2015 Chinese Automation Congress (CAC)*, pages 145–150. IEEE, 2015.

[157] Liansheng Zhang, Liu He, and Yongduan Song. New results on stability analysis of delayed systems derived from extended wirtinger's integral inequality. *Neurocomputing*, 283:98–106, 2018.

[158] Luo Zhang, Qingxian Wu, and Shixun Xiong. Robust stabilization of uncertain nonlinear networked control systems with packet dropouts and quantization. In *2017 32nd Youth Academic Annual Conference of Chinese Association of Automation (YAC)*, pages 525–530. IEEE, 2017.

[159] Ruimei Zhang, Deqiang Zeng, Xinzhi Liu, Shouming Zhong, and Qishui Zhong. Improved results on state feedback stabilization for a networked control system with additive time-varying delay components' controller. *ISA transactions*, 75:1–14, 2018.

[160] Xian-Ming Zhang and Qing-Long Han. Network-based $h\infty$ filtering using a logic jumping-like trigger. *Automatica*, 49(5):1428–1435, 2013.

[161] Xian-Ming Zhang and Qing-Long Han. Event-triggered dynamic output feedback control for networked control systems. *IET Control Theory & Applications*, 8(4):226–234, 2014.

[162] Zhenxing Zhang, Hongjing Liang, Chengwei Wu, and Choon Ki Ahn. Adaptive event-triggered output feedback fuzzy control for nonlinear networked systems with packet dropouts and actuator failure. *IEEE Transactions on fuzzy systems*, 27(9):1793–1806, 2019.

[163] Yun-Bo Zhao, Tao Huang, Yu Kang, and Xugang Xi. Stochastic stabilisation of wireless networked control systems with lossy multi-packet transmission. *IET Control Theory & Applications*, 13(4):594–601, 2018.

[164] Yun-Bo Zhao, Guo-Ping Liu, Yu Kang, and Li Yu. Packet-based networked control systems in continuous time. In *Packet-Based Control for Networked Control Systems*, pages 61–74. Springer, 2018.

[165] Shuo Zhen, Zhongsheng Hou, and Chenkun Yin. A novel data-driven predictive control for networked control systems with random packet dropouts. In *2017 6th Data Driven Control and Learning Systems (DDCLS)*, pages 335–340. IEEE, 2017.

[166] Kemin Zhou, John Comstock Doyle, Keith Glover, et al. *Robust and optimal control*, volume 40. Prentice hall New Jersey, 1996.

[167] X-L Zhu, Y Wang, and G-H Yang. New stability criteria for continuous-time systems with interval time-varying delay. *IET control theory & applications*, 4(6):1101–1107, 2010.

[168] X-L Zhu and G-H Yang. Jensen integral inequality approach to stability analysis of continuous-time systems with time-varying delay. *IET Control Theory & Applications*, 2(6):524–534, 2008.

[169] Xun-Lin Zhu and Guang-Hong Yang. State feedback controller design of networked control systems with multiple-packet transmission. *International Journal of Control*, 82(1):86–94, 2009.

Appendix A

LMI problems

Linear matrix inequalities are used to solve several automation problems (optimization problems in control theory, system identification and signal processing) which are generally difficult to solve analytically. More specifically, they are used for stability analysis and the synthesis of control laws for NCSs models. The advantage of LMI-based methods is that they can be solved using convex programming.
In this appendix, we will give a brief reminder on convex analysis and linear matrix inequalities as well as the techniques used to solve the LMI problems established during this thesis. For more details on the use of LMI in the field of control, the reader can refer to [14].

A.1 Convex set

A set is said to be convex if:

$$\forall \lambda \in [0,1], \quad \forall (x_1, x_2) \in C^2 \Leftrightarrow (\lambda x_1 + (1-\lambda) x_2) \in C. \tag{A.1}$$

- An important property of convex sets is that the intersection of two convex sets is a

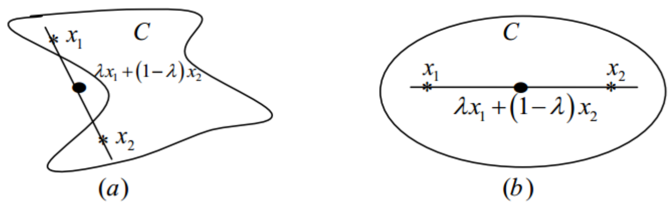

Figure A.1: Convex (a) and non-convex (b) sets.

convex set.

A.2 Convex function

Let be a function $f : C \to \mathbb{R}$ where the set C is convex, the function f is a convex function if and only if:

$$\forall (x, y) \in C^2 \ \forall \lambda \in [0, 1], \ then \ f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y). \tag{A.2}$$

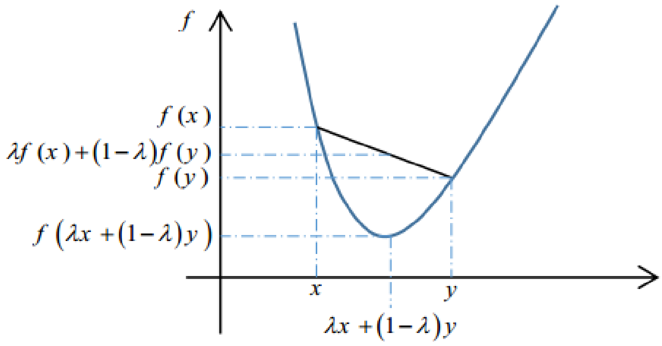

Figure A.2: convex function.

A.3 Linear Matrix Inequalities LMIs

An LMI is a matrix inequality defined in the following form:

$$F(x) = F(0) + \sum_{i=1}^{m} x_i F_i > 0. \tag{A.3}$$

where $x \in \mathbb{R}^{>}$ is the variable to be determined and symmetric matrices $F_i = F_i^T \in \mathbb{R}^{\ltimes \times \ltimes}$, $i = 0, \ldots, m$, are given.

- The inequality ">" in (A.3) means that it is defined positive, that is to say, $u^T F(x) u > 0$ for all $u \in \mathbb{R}$, $u \neq 0$, that is to say all the eigenvalues $\lambda(F(x))$ of $F(x)$ are negative.

- The LMI (A.3) is a convex constraint in, in other words the set $\{x | F(x) > 0\}$ is convex.

- The resolution of the LMI (A.3) amounts to determining the vector $x(t) = [x_1, x_2, \ldots, x_n]$, so $F(x) > 0$ that the convex constraint is verified. This then amounts to minimizing the eigenvalues $\lambda_{max}(F(x)) > 0$. This problem can be solved using the algorithm of the interior point $\lambda(F(x)) > 0$ so that the function is convex.

- It is possible to convert non-linear matrix inequalities (BMI) into LMIs of the form (A.3). The LMI formatting of a convex optimization problem therefore requires writing all the constraints of the problem in the form of affine LMI as a function of the optimization variables.

- We can have a non-strict linear matrix inequality given by: $F(x) \leq 0$.

A.4 Classic LMI problems

There are three types of optimization problems convex encountered in the form of LMI.

A.4.1 Feasibility problem:

It is a question of finding a vector x such that the convex constraint $F(x) > 0$ is satisfied. This problem is generally solved by looking for the vector x minimizing the scalar t such that:

$$-F(x) < t \times I. \tag{A.4}$$

If the minimum value of is negative then the problem is doable.

A.4.2 Eigen-value problem EVP:

It is a question of minimizing the greatest eigenvalue of a symmetric matrix under a constraint of type LMI:

$$\begin{cases} \text{minimize } \lambda, \\ \text{subject to } \begin{cases} \lambda I - A(x) > 0, \\ B(x) > 0. \end{cases} \end{cases} \tag{A.5}$$

where the matrices $A(x)$ and $B(x)$ are symmetrical and linear with respect to the variable x. In the literature, we can find another equivalent formulation of the EVP problem. For example, the problem of minimizing a linear function is posed in the form of an x EVP problem, as a variable as follows:

$$\begin{cases} \text{minimize } c^T x, \\ \text{subject to } F(x) > 0. \end{cases} \tag{A.6}$$

A.4.3 Generalized Eigen-Value Problem GEVP:

It is a question of minimizing the greatest generalized eigenvalue of a pair of matrices, linearly dependent on the x variable, under LMIs constraints. This problem is expressed as follows:

$$\begin{cases} \text{minimize } \lambda, \\ \text{subject to } \begin{cases} \lambda B(x) - A(x) > 0, \\ B(x) > 0, C(x) > 0. \end{cases} \end{cases} \tag{A.7}$$