ALGERIAN DEMOCRATIC AND PEOPLE'S REPUBLIC

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

20 AUGUST 1955 UNIVERSITY - SKIKDA

Faculty of Sciences

Department of Physics

Physics 1 and 2
Course Handout

1st Year Engineer

Sector: Agricultural sciences

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2024- 2025

Preface

This course handout has been carefully prepared to serve as a comprehensive guide for **first-year engineering students**. It encompasses fundamental topics that are essential for developing a solid foundation in engineering sciences. The material is structured into **four key chapters**, each addressing a crucial area of study:

1. **Mathematical Review** – This chapter provides the necessary mathematical tools required for engineering applications. It covers fundamental concepts such as algebra, calculus, and vector analysis, ensuring that students develop strong problem-solving skills.
2. **Fluid Mechanics** – Understanding the behavior of fluids is vital in various engineering fields. This chapter introduces the basic principles governing fluid motion, including hydrostatics, Bernoulli's equation, and applications in real-world engineering systems.
3. **Geometric Optics** – This chapter explores the principles of light propagation, including reflection, refraction, lens systems, and optical instruments. These concepts are fundamental in disciplines such as physics, engineering design, and telecommunications.
4. **Crystallography** – The final chapter delves into the atomic structure of solids, covering crystal systems, unit cells, Bravais lattices, and symmetry in crystals. This knowledge is crucial in materials science, solid-state physics, and engineering applications involving crystalline structures.

Each chapter includes **theoretical explanations, illustrative examples, and exercises** to enhance understanding and practical application. The content is designed to be accessible to students at the beginning of their engineering journey while providing a strong conceptual framework for more advanced studies.

It is our hope that this handout will serve as a valuable resource for students, helping them grasp fundamental scientific principles and their applications in engineering.

Dr. Kenza Kamli

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Chapter-I:

Mathematical Review

CHAPTER I : *Mathematical Review*

Physics is an exact science where laws are expressed by mathematical formulas. To describe these laws, physics uses the notions of physical quantities. Each of them must be well defined and we must know how to measure it. There are two types of quantities:

- Scalar: It is represented by a number followed by a unit. such as mass, time, length, etc.;
- Vector: which are characterized by a direction, a sense, a module and a point of application. For example, force, speed, etc.

In this chapter, we review some basic mathematical concepts that are highly useful and necessary for first-year students to understand the coherence between physical quantities and their dimensions, uncertainties, and related topics.

1. Physical quantities and dimensional analysis

A **physical quantity** is a property or characteristic of a physical system that can be measured or quantified and expressed using a numerical value and a unit of measurement. It is used to describe the physical world in a precise and standardized manner.

A. Components of a physical quantity

- **Numerical Value**: Indicates the magnitude or size of the quantity.
- **Unit of Measurement**: Specifies the standard used to measure the quantity.

For instance, in 10 m, "10" is the numerical value, and "m" (meter) is the unit.

B. Types of physical quantities

Base quantities: Fundamental quantities defined in terms of basic units, such as length, mass, time, electric current, temperature, amount of substance, and luminous intensity. Example: Length (m), Mass (kg).

Derived quantities: Quantities derived from the base quantities through mathematical relationships. Example: Velocity (m/s), Force (N).

C. Importance

Physical quantities allow scientists and engineers to:

- Measure and compare properties of matter and phenomena.
- Formulate laws of nature using mathematical relationships.
- Communicate precise and standardized information universally.

In essence, they are the building blocks of science and engineering, providing a common language for understanding and describing the physical world.

1.1. International System of Units (SI)

The set of definitions, measurement methods and units of fundamental quantities constitutes what is called a system of units. There are several systems of units but the most common is the International System (SI). The S.I. is made up of the units of the rationalized MKSA system (M: Meter, K: Kilogram, S: Second and A: Ampere) and includes additional definitions of the unit of temperature and the unit of luminous intensity. In this unit system, the base or fundamental units are those mentioned in Table 1 below.

Table 1: fundamental units.

Physical quantities	Length	Mass	Time	Electrical intensity	Temperature	Luminous intensity	Quantity of matter
Symbol	L	M	T	I	θ	J	µ
Unit	Meter (m)	Kilogram (Kg)	Second (S)	Ampere (A)	Kelvin (K)	Candela (cd)	Mole (mol)

1.2. Derived units or Secondary units

Secondary units are units of measurement that are derived from fundamental (primary) units, based on specific relationships between physical quantities. They are not independently defined but depend on combinations of the fundamental units of a measurement system, such as the International System of Units (SI).

Examples:

In Mechanics:

- Speed (m/s) is a secondary unit derived from the fundamental units of length (meter, m) and time (second, s).
- Force (Newton, N) is derived as kg.m/s^2, combining mass (kg), length (m), and time (s).

In Electricity:

Energy (Joule, J) is derived as N.m, which translates to kg.m^2/s^2.

Therefore, from the previously defined basic units, we can easily define units that result from them:

- *Surface*: square meter (m^2);
- *Volume*: cubic meter (m^3);
- *Speed*: meter per second (m/s);
- *Acceleration*: meter per second, per second (m/s^2);
- *Angular speed*: radian per second (rd/s);

- *Force*: Newton (N);
- *Moment*: Meter. Newton (m.N);
- *Energy, Work, Quantity of Heat*: Joule (J);
- *Power*: Watt (W).

1.3. An additional unit

The official unit for plane angles is the Radian (rad). It constitutes an additional unit to the seven units mentioned above.

- Solid angle: steradian (sr);

1.4. Multiples and sub-multiples

Multiples are the integer products of a given quantity or base unit. In mathematics or measurement, multiples represent values that are greater than or equal to the original quantity, obtained by multiplying it by whole numbers. For example: In the metric system, multiples of a meter include **kilometer (km)** (1 km = 1000 m) or **megameter (Mm)** (1 Mm = 10^6 m).

Sub-multiples are fractions of a given quantity or base unit, representing values smaller than the original quantity. They are obtained by dividing the base unit by powers of 10 or other standard fractions. For example: In the metric system, sub-multiples of a meter include **centimeter (cm)** (1 cm = 0.01 m) or **millimeter (mm)** (1 mm = 0.001 m).

In other words we can say that:

- Multiples: Values **greater than** the base unit (e.g., km, Mm).
- Sub-multiples: Values **smaller than** the base unit (e.g., cm, mm).

This concept is fundamental in measurement systems to express quantities efficiently across different scales.

Table 2: The multiples and sub-multiples.

Multiples							
Coefficient	10^{+1}	10^{+2}	10^{+3}	10^{+6}	10^{+9}	10^{+12}	10^{+15}
Prefix	deca	hecta	kilo	mega	giga	tera	peta
Symbol	da	h	k	M	G	T	P
Sub-multiples							
Coefficient	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}
Prefix	deci	centi	milli	micro	nano	pico	femto
Symbol	d	c	m	µ	n	p	f

2. Dimensional analysis (dimensional equations)

2.1. Definition

Dimensional analysis is a method in physics and engineering used to analyze and simplify physical relationships by focusing on the dimensions of the physical quantities involved. It examines how physical quantities (such as length, mass, time, etc.) relate to each other in terms of their fundamental units (e.g., meters, kilograms, seconds).

It is a powerful tool for:

- Checking the correctness of equations.
- Deriving relationships between physical quantities.
- Converting units.

2.2. Dimensional equations

Each physical quantity is expressed in terms of its fundamental dimensions. Dimensional equations are conventional notations that simply summarize the definition of quantities derived from the fundamental units: Length, Mass, Time, etc.., symbolized by the capital letters L, M and T… etc (***As given in table 1***).

Therefore, any derived quantity **G** can be expressed in terms of these quantities according to the expression:

$$[G] = [L^a . M^b . T^c . I^d . \theta^e . \mu^f . J^j]$$

With: a, b, c, d, e, f and j are real numbers. The expression of **[G]** is the dimensional equation of the quantity **G**.

Remark: To express the dimensional equation all the physical quantities must be written between brackets.

- **What is the use of this expression?**

1. The first interest of using dimensional analysis is to verify the **Dimensional Homogeneity**: An equation is dimensionally correct if the dimensions on both sides of the equation are the same.

Example: In F = m.a; $[F] = [M.L.T^{-2}]$, and $[m.a] = [M.L.T^{-2}]$, so the equation is dimensionally homogeneous.

2. The second role is the **Unit Conversion**: The dimensional analysis assists in converting one unit system to another.

3. Another important use is the **Deriving Formulas**: Because this method allows estimation of relationships between physical quantities based on their dimensions.

4. And the last point is **Simplifying Problems**: Helps reduce the number of variables in a problem through dimensionless quantities.

Note: Exponential, logarithmic, trigonometric functions, constants, and everything inside these functions have dimension 1.

$[x] = 1$, $[\alpha] = 1$, $[\sin \alpha] = 1$, $[e^{\alpha}] = 1$, $[\log x] = 1$, $[\pi] = 1$, $[8] = 1$…

Example: Suppose you want to check if the equation for the period of a pendulum, $T = 2\pi\sqrt{\frac{L}{g}}$, is dimensionally correct.

Answer:

We have the period (T) is given by the formula: $T = 2\pi\sqrt{\frac{L}{g}}$

Therefore, Dimension of the period T = [T]; and the second side of the equation:

$[2\pi] = 1$; L (length) = [L]; g (acceleration) = $[L.T^{-2}]$

Substitute dimensions into the equation:

$$[T] = 1\sqrt{\frac{[L]}{[L.T^{-2}]}} = \sqrt{[T^2]} = [T]$$

Since the dimensions are consistent, the equation is correct.

Dimensional analysis is an essential tool for ensuring the accuracy and validity of physical models and equations.

3. Error calculations

3.1. Definition

Error calculations involve determining and quantifying the uncertainty or deviation in measurements or results. Errors are inherent in any measurement process and arise due to limitations in instruments, methods, or external factors. Error calculations help in assessing the accuracy and reliability of experimental or observed data.

3.2. Types of errors

When measuring a physical quantity experimentally, the found value does not correspond to the exact value of the quantity; rather, it is close to the reality, herein, we have to measure the errors of calculation. Different types of errors exists such as:

A. Systematic errors

This type of errors are related to the consistent and repeatable errors caused by flaws in equipment, calibration, or methodology. For example, a miscalibrated scale consistently adds 0.5 kg to every measurement.

B. Random errors

Unpredictable variations caused by environmental factors or observer inconsistencies are called random errors. Like the fluctuations in temperature which affects the measurements slightly differently each time.

C. Gross errors

Significant errors caused by human mistakes, such as incorrect readings or data entry.

3.3. Common error metrics in calculations

3.3.1. Absolute, Relative and Percentage Error

- The absolute error **(δx)** is defined as the difference between the measured value and the true or accepted value: $\delta x = | x_{true} - x_{measured} |$.
- While, the relative error signifies the ratio of the absolute error to the true value, expressed as a fraction: $\textbf{Relative error} = \delta x / x_{true}$.
- The Percentage Error represents the relative error expressed as a percentage: $\textbf{Percentage error} = (\delta x / x_{true}) * 100$.

3.3.2. Steps in error calculations

- ***Identify the true value*:** Determine the accepted or theoretical value for comparison.
- ***Measure the observed value***: Collect the experimental or measured data.
- ***Calculate the absolute, relative, or percentage Errors***: Use the appropriate formula depending on the application.
- ***Aggregate errors (if multiple measurements)***: Combine errors using statistical methods such as averaging or standard deviation.

Meanwhile, the exact value is often inaccessible, the error is unknown we are obliged to use other concepts in order to estimate the errors made during the measurements. Herein, comes another notion called the ***Uncertainty***.

4. Uncertainty

4.1. Definition

Uncertainty quantifies the *range of possible values* within which the true value is likely to lie. It reflects the *confidence* or *limits* of a measurement based on the precision of the instrument and method used.

It is not a mistake but an *inherent property of the measurement process*, acknowledging the limitations of precision. It focuses on the *precision* and reliability of the measurement.

Uncertainty is expressed as a range (e.g., $x \pm \Delta x$) or confidence interval (e.g., 95%).

Example: If a length is measured as 10.0 ± 0.2 cm, the uncertainty is ± 0.2 cm.

4.2. Absolute uncertainty Δx

Absolute uncertainty is the measure of the margin of error in a measurement, expressed in the same unit as the measurement itself. It quantifies the amount of possible deviation from the measured value due to limitations in the measuring instrument or process.

Absolute uncertainty is written as: $\mathbf{\bar{x} \pm \Delta x}$

Where: $\bar{x}$ is the measured value or the best estimate and Δx is the absolute uncertainty.

The absolute uncertainty is expressed in the **same units** as the measured quantity.

4.3. Calculation of absolute uncertainty Δx

Let x be a quantity obtained experimentally. We assume that the best estimate of x (denoted as $\bar{\mathbf{x}}$) is a value located between two bounds: $\mathbf{x_{min} < \bar{x} < x_{max}}$.

The best estimate $\bar{\mathbf{x}}$ and the absolute uncertainty Δx are given by the relations:

$$\Delta \mathbf{x} = \frac{x_{max} - x_{min}}{2} \; ; \; \bar{\mathbf{x}} = \frac{x_{max} + x_{min}}{2}$$

Thus, we write: $\mathbf{x} = \bar{x} \pm \Delta x$

Example: You read a measurement between 5.7 cm and 5.8 cm on a graduated ruler, where the smallest division of the ruler is in mm.

What is $\bar{d}$ (the best estimate)? What is Δd ? How do we write the measurement?

Answer:

1. Best Estimate ($\bar{d}$):

The best estimate of the measurement is the midpoint of the range:

$$\bar{d} = \frac{5.7 + 5.8}{2} = 5.75\ cm.$$

2. Absolute Uncertainty (Δd):

$$\Delta d = \frac{5.8 - 5.7}{2} = 0.05\ cm$$

3. Writing the Measurement*:*

The measurement is expressed as: d = 5.75 ± 0.05 cm.

This means the true value lies within the range 5.70 cm to 5.80 cm.

Importance of Absolute Uncertainty

- It provides a concrete, easily interpretable range for the measured value.
- Helps convey the precision of the measurement and the limitations of the measuring instrument.
- Forms the foundation for calculating relative and percentage uncertainties, as well as propagating uncertainty in derived quantities.

4.4. Relative uncertainty ($\Delta x/|\bar{x}|$)

A. Relative uncertainty is the ratio of the absolute uncertainty (Δx) to the best-esteemed measured value ($|\bar{x}|$). It is a ***dimensionless quantity*** that expresses how significant the uncertainty is compared to the size of the measurement. It provides a sense of the measurement's precision.

The relative uncertainty is given by the formula: $\textbf{Relative uncertainty} = {}^{\Delta \mathbf{x}}/_{|\bar{\mathbf{x}}|}$

B. Relative uncertainty can be expressed as:

A **fraction**: $\Delta x/|\bar{x}|$, or as a **percentage (%)**: Relative Uncertainty×100.

Example: For a measurement of d = 5.75 ± 0.05 cm, calculate the relative uncertainty.

Answer:

- Absolute uncertainty (Δd): 0.05 cm,
- Measured value ($\bar{d}$): 5.75 cm.

Relative uncertainty ($\Delta d / \bar{d}$):

Relative Uncertainty = $\Delta d / \bar{d}$ = 0.05 / 5.75 ≈ 0.0087

To express it as a percentage:

Relative Uncertainty (%) = 0.0087×100 ≈ 0.87%.

4.5. Calculation of uncertainties for indirect measurements

In cases where physical quantities are not measured directly but are calculated from other measured values, the uncertainty in the calculated result is determined through **error propagation**. This process involves combining the uncertainties of the measured quantities using mathematical rules.

4.5.1. Addition and Subtraction

When quantities are added or subtracted, their **absolute uncertainties** are added:

$$\Delta R = \Delta A + \Delta B + \Delta C\ldots$$

Where:

- $R = A+B+C$, $R = A-B-C$ or $R = A+B-C$
- ΔA, ΔB and ΔC: Absolute uncertainties of A, B and C.

Example: Two lengths are measured as A = 12.5 ± 0.2 cm and B = 8.3 ± 0.1 cm. Find the total length and its uncertainty.

Solution:

- *Total length*: R = A+B = 12.5 + 8.3 = 20.8 cm.
- *Absolute uncertainty*: ΔR = ΔA + ΔB = 0.2 + 0.1 = 0.3 cm.

$$R = 20.8 \pm 0.3 \text{ cm}$$

4.5.2. Multiplication and Division

When quantities are multiplied or divided, their **relative uncertainties** can be calculated by using one of these two method:

A. Total Differential Method

The total differential method is a mathematical approach for calculating uncertainties in a function f (x, y, z...), where the function depends on multiple measured variables x, y, z... Each variable has an associated uncertainty. This method involves propagating uncertainties through the function using partial derivatives.

- **Calculation method**

Example, if **G = *f* (x, y)** then:

$$dG = \left(\frac{\partial G}{\partial x}\right) dx + \left(\frac{\partial G}{\partial y}\right) dy$$

1. Calculate the partial derivatives;
2. Replace d (the symbol of derivation) by Δ and change the negative signs (-) to a positive signs (+).

3. Finally, we get the ***absolute*** uncertainty, which can be switched to the relative uncertainty by dividing the obtained result by G.

This method has many advantages such as:

- Handles complex functions systematically.
- Provides accurate results when uncertainties are small and independent.
- Can be applied to any mathematical function.

B. Logarithmic Differential Method

The logarithmic differential method is a specialized technique for propagating uncertainties in cases where the functional relationship between variables involves multiplication, division, or powers. It simplifies the calculations by converting the function into a logarithmic form and using relative uncertainties.

- **Calculation method**

1. Express the Function Logarithmically: If G = f (x, y, z...,), take the natural logarithm of both sides: Ln G = ln f(x, y, z)

2. Differentiate Logarithmically: Apply the chain rule to the natural logarithm:

$$\frac{dG}{G} = \left(\frac{\partial G}{\partial x}\right)\frac{dx}{x} + \left(\frac{\partial G}{\partial y}\right)\frac{dy}{y}$$

3. Replace d (the symbol of derivation) by Δ and change the negative signs (-) to a positive signs (+).
4. Finally, we get the ***relative*** uncertainty, which can be switched to the absolute uncertainty by multiplying the obtained result by G.

Exercises

Exercise 1:

Verify whether the following equation is dimensionally consistent: F = ma + b

where **F** is force, **m** is mass, **a** is acceleration, and **b** is a constant with dimensions of force.

Exercise 2:

1. Write the dimensional equation of the following physical quantities:

Kinetic energy $\mathbf{K_E}$

Energy **E**

Electrical charge **Q**

Electrical resistance **R**

The power **P**

2. Express in the International System of Units (IS) the following derived units: The Newton (N), Joule (J), Watt (W), Pascal (Pa), and the Coulomb (C).
3. Check the homogeneity of the different expressions of energy: The work of a force (W), kinetic energy (K_E), Potential energy of gravity (E_p), Energy stored by a capacitor (1/2 QV).

Exercise 3:

Find by dimensional analysis the expression of the distance **Y** traveled by a body of mass **m** that falls by free fall without initial velocity for a time **t**.

Knowing that this distance depends on the weight of the body **P**, the gravitational acceleration **g** and the time **t**.

Exercise 4:

Calculate by the total differential method and the logarithmic differential method the relative uncertainty of the following physical quantities:

$$X = \frac{1}{2}yt^2, R = \frac{R_1.R_2}{R_1+R_2}$$

Exercise 5 :

We want to calculate the density **ρ** of a liquid of mass **m** contained in a cylinder of radius **r** and height **h**. The experimental measurements of **m**, **r** and **h** and their relative uncertainty are as follows: m = (620 ± 2) g; r = (4.3 ± 0.1) cm; h = (25.6 ± 0.1) cm.

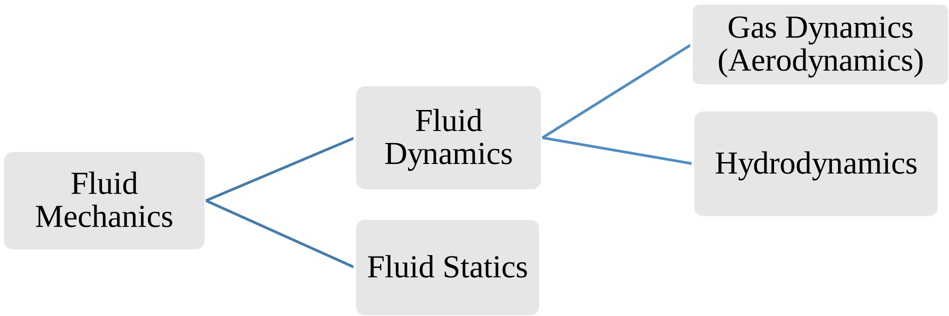
1. Calculate the density **ρ** of the liquid.
2. Calculate Relative Uncertainty and Absolute Uncertainty of **ρ**.

Chapter-II:

Fluid Mechanics

CHAPTER II : *Fluid Mechanics*

Fluid mechanics is a branch of engineering that focuses on the study of fluids at rest (i.e., fluid statics) as well as the study of fluids in motion or the study of fluid flow (i.e., fluid kinematics and dynamics).



The applications of fluid mechanics are diverse such as: Hydraulics (Use of liquids in systems (e.g., brakes, lifts)); Aerodynamics (Study of gases (air) around objects (e.g., airplanes, cars)); Medicine (Understanding blood flow (fluid dynamics)); Climate Science (Studying ocean currents and atmospheric gases), air conditioning, heating, hydroelectricity, urban aerodynamics, vibration analysis, aeronautics, chemical and food industries, meteorology, etc.

1. Definition of fluids

A **fluid** is a substance that continuously deforms under the application of a shear stress, no matter how small the stress may be. Fluids include both **liquids** and **gases**, as they share the property of being able to flow and take the shape of their container.

- **Liquids**: Have a definite volume but no fixed shape; they take the shape of their container up to the level of their free surface.
- **Gases**: Have neither a definite volume nor shape; they expand to fill the entire volume of their container.

Table I.1: Differences Between Liquids and Gases

Property	Liquids	Gases
Volume	Fixed	Variable
Shape	Takes the shape of its container	Expands to fill the container
Compressibility	Low (nearly incompressible)	High
Molecular Spacing	Close	Far apart
Density	Relatively high and constant	Low and variable

The fluid is an isotropic material:

- **Continuous**: its properties vary in a continuous manner;
- **Deformable** (it has no fixed shape): the molecules can easily slide over one another, and this mobility causes the fluid to take the shape of the container holding it;
- **Capable of flowing**: any fluid can flow more or less easily from one container to another or through a pipe. Frictional forces opposing the sliding of fluid particles against each other may arise, as every real fluid has viscosity;
- **Isotropy**: at any point within the fluid, all its properties are the same in every direction in space.

The fluid particle: it is a portion of fluid that, at a given instant (t), is characterized by properties such as pressure, temperature, density, etc.

Fluids can be classified as follows:

A. *Perfect Fluid*: A perfect fluid is a fluid in which the tangential forces due to internal friction are zero, meaning there is no friction (no viscosity, $\mu=0$). The contact forces are perpendicular to the surface elements on which they act.

B. *Real Fluid*: In a real fluid, the tangential forces due to internal friction, which oppose the relative sliding of fluid layers, are taken into account. This phenomenon, known as viscous friction, occurs during fluid motion. The statics of real fluids coincides with the statics of perfect fluids.

C. *Incompressible Fluid*: A fluid is considered incompressible when the volume occupied by a given mass does not change with external pressure (ρ = constant). Liquids, such as water and oil, can generally be regarded as incompressible fluids.

D. *Compressible Fluid*: A fluid is considered compressible when the volume occupied by a given mass changes with external pressure ($\rho \neq$ constant). Gases are compressible fluids.

2. Characteristics of fluids

The main characteristics of fluids are:

- **Ability to flow**: Fluids have the ability to flow due to weak intermolecular forces compared to solids. This property allows them to adapt to the shape of the container they occupy.
- **Continuum hypothesis**: Fluids are treated as a continuum for most engineering and scientific applications, meaning their properties (e.g., density, velocity, pressure) are assumed to vary smoothly and continuously, even though they are made up of discrete molecules.

- **Molecular arrangement**: The molecules in fluids are less tightly packed than in solids, allowing greater freedom of movement.
  - **Liquids**: Molecules are close together but can slide past one another.
  - **Gases**: Molecules are widely spaced and move independently in random directions.
- **Compressibility**:
  - **Liquids**: Generally incompressible (their volume changes negligibly under pressure).
  - **Gases**: Highly compressible (their volume changes significantly with pressure and temperature).
- **Surface tension (for Liquids)**: Caused by cohesive forces between molecules at the surface of the liquid. Surface tension allows small objects (like insects or needles) to float on the surface of water without sinking.
- **Buoyancy**: Fluids exert an upward force on objects submerged in them, known as the buoyant force (Archimedes' Principle).
- **Thermal expansion**: Fluids expand when heated and contract when cooled. Gases exhibit significant thermal expansion compared to liquids.

The *Absolute Temperature Scale*: The Kelvin scale (**K**): In the International System (SI), the absolute temperature is given by the relationship: **T(K) = T(°C) + 273.15**

The Celsius scale (**°C**), which is the *Relative Temperature Scale*: In the International System (SI), temperature is divided into 100 increments.

- **Diffusion and mixing**: Molecules in a fluid can move and mix due to random motion, enabling diffusion of substances throughout the fluid.
- **Adhesion and cohesion**:

Cohesion: Attraction between like molecules (e.g., water molecules).

Adhesion: Attraction between fluid molecules and a solid surface (responsible for capillary action).

- **Compressibility and speed of sound**: The speed at which sound propagates through a fluid depends on its compressibility and density. Gases generally have slower sound propagation speeds compared to liquids due to their higher compressibility.
- **Interaction with solids**: Fluids exert shear forces and normal forces when in motion relative to solid surfaces, leading to frictional effects such as drag.
- **Flow behavior**:
- **Newtonian fluids**: Have a constant viscosity (e.g., water, air).

- **Non-Newtonian fluids**: Viscosity depends on the shear rate (e.g., ketchup, cornstarch solutions).

- **Viscosity**: Viscosity is a measure of a fluid's resistance to flow. High-viscosity fluids (e.g., honey) flow slower than low-viscosity fluids (e.g., water or air).

- **Density (ρ)**: Density is the mass per unit volume of the fluid.

$$\boldsymbol{\rho} = \frac{\mathbf{m}}{\mathbf{V}}\text{, expressed in: Kg.m}^{-3}.$$

Liquids have a relatively constant density. Gases have variable density that depends on pressure and temperature (described by the Ideal Gas Law).

For gases, the density depends on temperature and pressure. For an ideal gas, the density can be calculated using its equation of state.

$$PV = mRT \rightarrow P = \rho RT \rightarrow \rho = P/(RT)$$

Where: P is the absolute pressure in Pa, T the absolute temperature in K, R the specific gas constant in $J.kg^{-1}·K^{-1}$.

- **Pressure**: Fluids exert pressure equally in all directions at a point within the fluid (Pascal’s Principle). The pressure increases with depth in a liquid due to the weight of the fluid above.

- **Specific Volume**: The specific volume (**ν**) is the inverse of the density:

$$\boldsymbol{\nu} = 1/\rho = V/m\text{, expressed in: m}^3\text{/Kg}$$

- **Weight Density**: The weight density or specific weight ($\boldsymbol{\varpi}$) is the weight per unit volume:

$$\boldsymbol{\varpi} = \boldsymbol{\rho}\,\boldsymbol{g} = (m.g)/V$$

It is expressed in $N.m^{-3}$, where: **m** is the mass in kg, **g** the gravity acceleration in $m.s^{-2}$, and **V** is the volume in m^3.

- **Relative Density (Specific Gravity)**: The relative density (**δ**) of a substance is the ratio of its density to the density of a reference substance under the same conditions of temperature and pressure:

$$\delta = \frac{\rho_{substance}}{\rho_{reference}}\text{, dimensionless (no units)}$$

- ❖ For **liquids**, the reference is **water**, with a density of approximately 1000 $kg.m^{-3}$: $\delta = \frac{\rho_{liquid}}{\rho_{water}}$.
- ❖ For **gases**, the reference is **air**, with its density depending on the standard temperature and pressure conditions: $\delta = \frac{\rho_{gas}}{\rho_{air}}$.

If δ > 1, the substance is denser than the reference material. If δ < 1, the substance is less dense than the reference material.

2.1. Viscosity of a fluid

Viscosity is a property of a fluid resulting from the cohesion and interaction between its molecules, which present resistance to deformation. This property comes into play whenever the fluid is in motion.

2.1.1. Dynamic viscosity (μ)

Dynamic or absolute viscosity (μ) is a measure of a fluid's resistance to flow when an external force is applied. It describes the internal friction between layers of a fluid in motion. A fluid with high dynamic viscosity flows more slowly because it resists deformation, while a fluid with low viscosity flows more easily.

A. Mathematical representation

To derive a relation for viscosity, consider a volume of fluid placed between two parallel, infinite horizontal plates separated by a distance h.

- The lower plate is fixed, while the upper plate is mobile.
- To maintain the upper plate at a constant velocity V, a constant tangential force F must be applied.

This demonstrates a viscous interaction between the plate and the fluid, which manifests as drag on the upper plate and shear force in the fluid.

- The fluid in contact with the upper plate adheres to it and moves at velocity U, while the fluid in contact with the fixed plate has zero velocity (no-slip condition).

The **shear stress** is expressed as:

$$\boldsymbol{\tau = F/S}$$

Where:

τ: Shear stress ($N.m^{-2}$), F: Force (N), S: Surface area of the plate (m^2).

For laminar flow (fluid moving in parallel layers), the fluid velocity changes linearly between 0 and V, with the velocity profile and gradient given by: $\frac{\Delta \mathbf{v}}{\Delta \mathbf{y}} = \frac{\mathbf{V}}{\mathbf{h}}$.

Newton's law states that the force F is proportional to both the surface area S and the velocity gradient: F = τ.S = $\mu . S\frac{\Delta \mathrm{v}}{\Delta y}$.

Thus, the shear stress can also be expressed as:

$$\tau = \mu \frac{\Delta \mathrm{v}}{\Delta y}$$

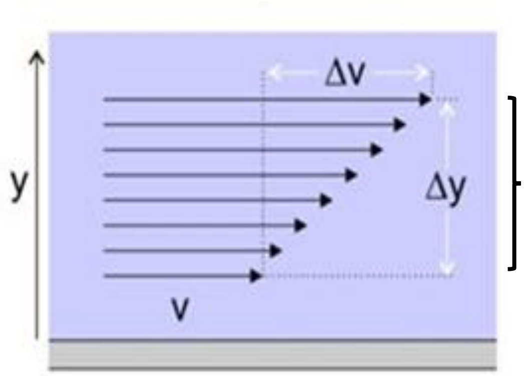


Figure II.1: *Velocity profile.*

Where: $\Delta v/\Delta y$ is the velocity gradient or rate of shear deformation..

Therefore, in summery *Absolute* (*dynamic*) viscosity is a measure of how a fluid resists the deformation of shear stress due to its inter-molecular friction.

The SI unit for absolute viscosity is **Ns/m²** (or 1Kg/ms). Also in common use is **Poise** (**P**) although this is often quoted as cP (centiPoise).

$$1\ Ns/m^2 = 1\ Kg/ms = 10\ P = 1000\ cP$$

B. *Physical interpretation*

Dynamic viscosity quantifies the force required to move one layer of fluid relative to another. For example:

- Water has a low viscosity (flows easily).
- Honey has a high viscosity (resists flow).

C. *Factors affecting dynamic viscosity*

- **Temperature**:

Liquids: Viscosity decreases as temperature increases because intermolecular forces weaken.
Gases: Viscosity increases as temperature increases due to increased molecular collisions.

- **Pressure**:

For liquids, viscosity increases slightly with pressure according to: $\mu=\mu_0.a^{(P-P0)}$; where P_0 is atmospheric pressure, and a depends on the fluid type (a=1.003 for mineral oils).

For gases, pressure has minimal effect under normal conditions.

- **Nature of the fluid**: Viscosity depends on the molecular structure and interactions of the fluid.

2.1.2. Kinematic viscosity (ν)

Kinematic viscosity (ν) is the ratio of a fluid's dynamic viscosity (μ) to its density (ρ). It represents the fluid's ability to flow under the influence of gravity, without considering external forces.

Mathematically, it is expressed as:

$$\nu = \frac{\mu}{\rho}$$

Where: ν is the kinematic viscosity ($m^2.s^{-1}$); μ is the dynamic viscosity, and ρ is the density of the fluid.

In addition, in common use is the **Stoke** (St) although this is often quoted as cSt (centiStoke).

$$1\ m/s^2 = 10000\ St = 10\ cSt$$

A. Physical interpretation

Kinematic viscosity describes how easily a fluid flows under the influence of gravity. A fluid with a high kinematic viscosity flows more slowly because it has higher resistance to motion relative to its density.

For example: Water has low kinematic viscosity, meaning it flows freely. Honey has higher kinematic viscosity, meaning it flows slowly.

B. Factors affecting kinematic viscosity

- Dynamic Viscosity (**μ**): If μ increases, ν also increases.
- Density (**ρ**): If ρ increases (e.g., at higher pressure), ν decreases, as ρ is in the denominator.
- Temperature:

For **liquids**: As temperature increases, ν decreases (lower μ).

For **gases**: As temperature increases, ν increases (higher μ).

C. Applications of kinematic viscosity

- **Fluid flow analysis**:

Used in fluid dynamics to calculate the Reynolds number (Re), which determines whether flow is laminar or turbulent:

$$R_e = \frac{V.d}{\nu}$$

This is a dimensionless number determined from the pipe diameter (d), the flow velocity (V) and the viscosity (ν) of the fluid.

The Reynolds Number is effectively the ratio of the forces of mass flow and shear stress due to the fluid's viscosity. Pipe flow can be considered to be laminar if the Reynolds number is less than 2300 and fully turbulent if it is greater than 4000. Flow characteristics are unpredictable if the value is between these two values.

Pump data supplied by manufacturers generally refers to performance with water and corrections may be necessary if you intend to pump other fluids.

- **Finding the Newtonian or Non-Newtonian fluid**

Kinematic viscosity is a standard property of a Newtonian fluid whereas dynamic viscosity refers to how a fluid (particularly a non-Newtonian fluid) resists flow under an external force (such as a pump).

What is a Newtonian fluid?

If the viscosity of a liquid remains constant under shear stress such as agitation or pumping at a constant temperature (that is, it obeys Newton's Law of Viscosity) it is called a Newtonian Fluid. Water and most oils are Newtonian liquids.

What is a non-Newtonian fluid?

Any liquid that does not obey Newton's Law of Viscosity is called a non-Newtonian fluid. This includes several different types of fluids that show an apparent increase or decrease in viscosity with shear rate. These changes can also be time dependent or time independent.

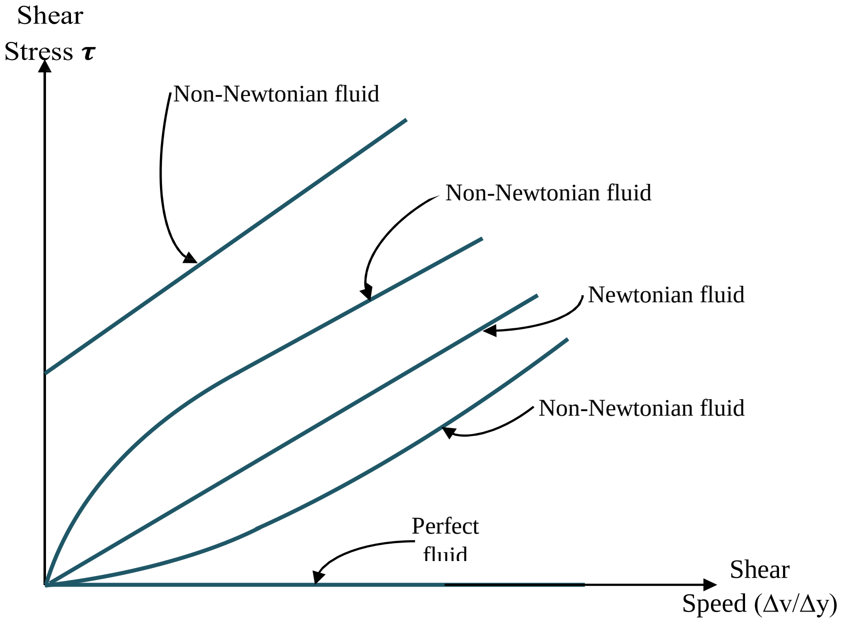


Figure II.2: Variation *of Shear Stress as a Function of Shear Velocity for Newtonian and Non-Newtonian Fluids.*

- **Engineering design**: Designing pipelines, pumps, and heat exchangers to optimize fluid flow.

- **Weather and oceanography**: Understanding the motion of air and water in the atmosphere and oceans.
- **Industrial applications**: Used in lubricants, fuels, and other liquids to ensure efficient machinery operation.

2.2. Surface Tension for fluids

Surface tension is the property of a fluid's surface that causes it to behave like a stretched elastic membrane. It results from the cohesive forces between fluid molecules at the interface of two different phases (e.g., liquid and gas).

Surface tension arises because molecules at the surface experience unbalanced intermolecular forces, pulling them inward toward the bulk of the liquid. This creates a "skin-like" effect at the fluid's surface.

2.2.1. Expression for Surface Tension (σ)

Surface tension (σ) is defined as the force per unit length along the interface or the energy required to create a unit area of surface:

$$\sigma = \frac{F}{L} \text{ or } \sigma = \frac{\delta W}{\delta S},$$

Where:

σ: Surface tension ($N.m^{-1}$),

F: Force acting along the interface (N),

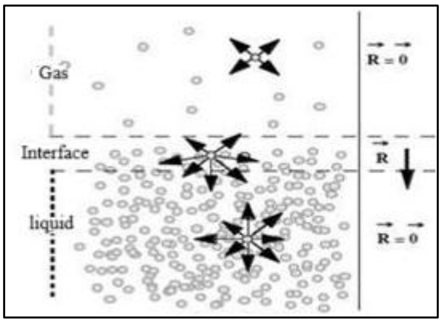
L: Length of the interface (m),

δW: Work done to increase the surface area (J),

δS: Increase in surface area (m^2).

2.2.2. Physical Mechanism

Cohesion and Surface Tension: Molecules within the bulk of a liquid are surrounded by other molecules and experience balanced intermolecular forces.



Molecules at the surface, however, experience a net inward force due to the absence of neighboring molecules above them. This inward pull creates tension at the surface.

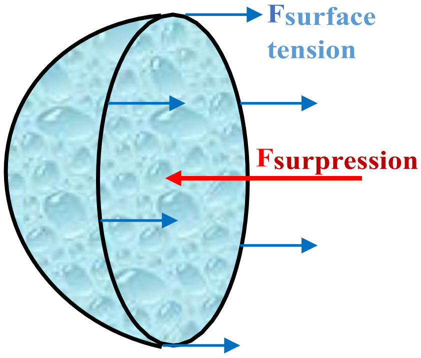
Minimization of Surface Area: To minimize energy, liquids tend to form shapes with the smallest surface area for a given volume (e.g., spherical droplets).

2.2.3. Applications of surface tension

- Formation of Droplets: Rain droplets form spherical shapes due to surface tension.
- Capillary Action: Surface tension plays a role in capillary action, where a liquid rises or falls in a narrow tube due to adhesion and cohesion.
- Insects Walking on Water: Small insects can walk on water due to the high surface tension of water supporting their weight.
- Soap Bubbles: The elasticity of soap films is a result of surface tension.
- Medical Applications: In lungs, surfactants reduce the surface tension of alveolar fluid, preventing lung collapse.
- Industrial Processes: Surface tension is crucial in processes like painting, inkjet printing, and cleaning.

2.2.4. Expression of surface tension for curved surfaces

Surface tension causes pressure differences across curved surfaces like droplets or bubbles, described by the **Young-Laplace Equation**:



A. For a droplet:

$$\Delta P = 2\sigma/R,$$

Where: ΔP is the pressure difference, σ is the surface tension, and R is the droplet radius.

B. For a bubble (two interfaces):

$$\Delta P = 4\sigma/R .$$

Surface tension is a fundamental property of fluids that governs many natural and industrial phenomena, making it an essential topic in fluid mechanics.

2.3. Wettability and contact angle

The contact angle θ determines whether a liquid wets a solid surface based on its value.

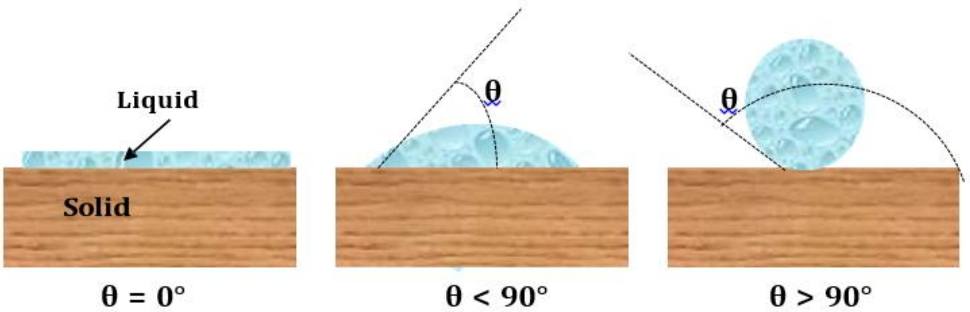


Figure II.3: *Different liquid shapes on a solid flat plate forming different contact angle cases.*

θ=0°: Perfect wetting.

θ < 90°: Partial wetting.

θ > 90°: Non-wetting.

The contact angle θ depends on the liquid or the solid that supports or contains it, and the gas that surrounds both. Three parameters must therefore be taken into account:

• The surface tension σ_{SL} between the solid and the liquid;

• The surface tension σ_{LG} between the liquid and the gas;

• The surface tension σ_{SG} between the solid and the gas.

The droplet balance results in:

$$\boldsymbol{\sigma_{SG} = \sigma_{SL} + \sigma_{LG}\cos\theta \rightarrow \cos\theta = \frac{\sigma_{SG}-\sigma_{SL}}{\sigma_{LG}}}$$

2.4. Capillary effect – Jurin's Law

The capillary effect, or capillarity, is the phenomenon where a liquid rises or falls in a narrow tube (capillary) due to the interplay of **cohesive forces** (between the liquid molecules) and **adhesive forces** (between the liquid and the solid surface of the tube).

- **If adhesive forces dominate (e.g., water in a glass tube)**: The liquid rises in the tube, forming a concave meniscus.
- **If cohesive forces dominate (e.g., mercury in a glass tube)**: The liquid is depressed in the tube, forming a convex meniscus.

2.4.1. Jurin's Law

Jurin's Law provides the mathematical relationship between the height **h** of a liquid column in a capillary tube and the properties of the liquid and tube

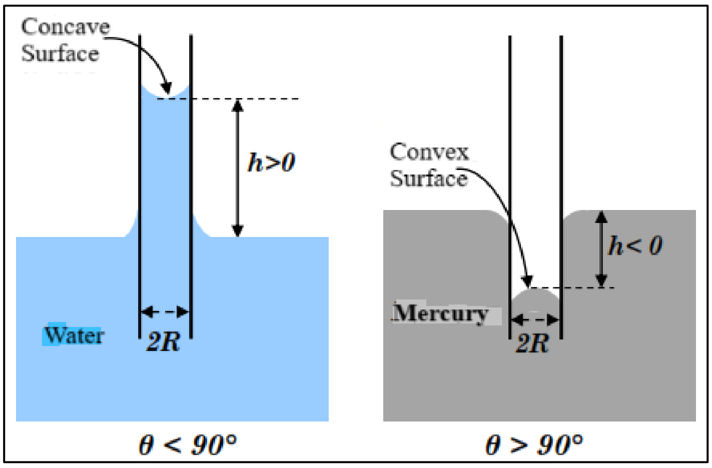


Figure II.4: *(a) The capillary ascent of water and (b) the capillary depression of mercury in a small diameter glass tube.*

The height **h** of a liquid in a capillary tube is given by:

$$h = \frac{2\sigma\cos\theta}{R\rho g}$$

Here: σ is the surface tension of the liquid, ρ the density of the liquid, R the radius of the tube, and θ is the contact angle.

For θ > 90°, h is negative, indicating depression.

2.4.2. Applications of the capillary effect

- Biology: Transport of water and nutrients in plants through xylem vessels. Movement of fluids in porous tissues and membranes.
- Industrial Processes: Ink flow in inkjet printers. Capillary action in oil wicks and lamp wicks.
- Geology: Water movement in soil and porous rocks.
- Medicine: Blood sampling using capillary tubes.
- Daily Life: Absorption of liquid by paper towels. The rise of liquid in thin straws.

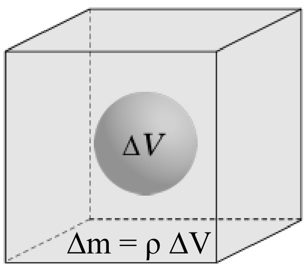
3. Hydrostatics

3.1. Definition

Hydrostatics, also known as fluid statics, is the branch of fluid mechanics that deals with fluids at rest. It studies the properties of stationary fluids, the forces they exert on surfaces, and the pressure distribution within the fluid. Unlike fluid dynamics, hydrostatics assumes there is no relative motion between fluid particles, meaning the fluid is in a state of equilibrium.

3.2. Pressure in a fluid at rest

- ***Pressure forces*****:** Consider a fluid volume element of mass **Δm** and density **ρ** in mechanical equilibrium with the rest of the surrounding fluid:



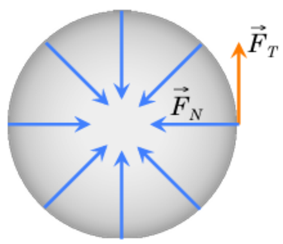
This volume element is subject to:

• <u>volume forces</u>: due to the existence of fields to which ΔV can be subjected and which apply to all the atoms contained in ΔV: Gravity, and Acceleration.

• <u>surface forces</u>: which can be broken down into:

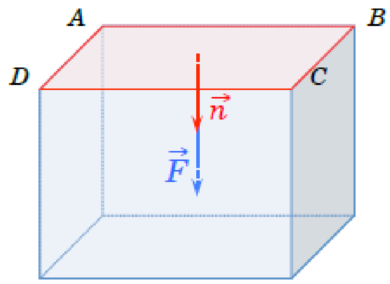
Tangential forces $\overrightarrow{F_T}$ related to the viscosity of the fluid (concern only moving fluids);

Normal forces $\overrightarrow{F_N}$ at the surface at any point on it and directed towards the interior of the volume element: *hydrostatic pressure forces*.



In this chapter, we will limit ourselves to the study of the hydrostatic pressure forces.

Consider a planar surface (quadrilateral ABCD of surface S) on which the pressure P is uniform:



The hydrostatic pressure force is defined by:

$$\vec{\mathbf{F}} = \boldsymbol{P.S}\,\vec{\mathbf{n}}$$

$\vec{n}$: Unit vector ⊥ to S.

In this case, the pressure is defined by:

$$\mathbf{P} = \frac{\mathbf{F}}{\mathbf{S}}$$

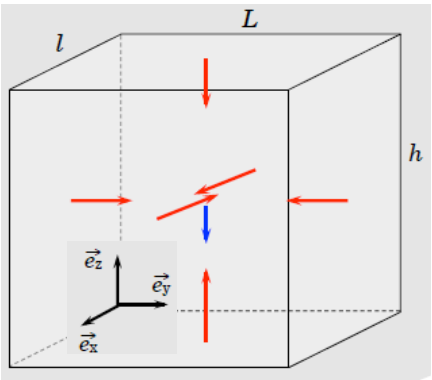
Therefore, the ***Pressure*** in a static fluid is the force exerted per unit area due to the weight of the fluid above.

Where: P: Pressure (Pa), F: Force (N), S: Area (m^2).

3.3. Fundamental relation of hydrostatics

3.3.1. Case of an incompressible homogeneous fluid

We consider a homogeneous and incompressible liquid whose volumic mass ρ is constant. We are more particularly interested in a parallelepiped volume element V (l × L × h):



This volume element is at rest, it is immobile in the frame (O, $\overrightarrow{e_x}$, $\overrightarrow{e_y}$, $\overrightarrow{e_z}$). By virtue of Newton's first law, this translates to the fact that the sum of the forces exerted on this volume element is zero:

$$\sum \overrightarrow{F_{ext}^{i}} = \vec{0}$$

The forces exerted on this volume are: Pressure forces and Volume forces.

The equilibrium condition is written:

$$\underbrace{\vec{F}(x) + \vec{F}(x+l)}_{\vec{e_x}} + \underbrace{\vec{F}(y) + \vec{F}(y+L)}_{\vec{e_y}} + \underbrace{\vec{F}(z) + \vec{F}(z+h)}_{\vec{e_z}} + \underbrace{\overrightarrow{P_{weight}}}_{\vec{e_z}} = \vec{0}$$

It therefore appears that:

- The horizontal pressure forces according to $\overrightarrow{e_x}$ and $\overrightarrow{e_y}$ compensate each other two by two:

$$\vec{F}(x) + \vec{F}(x+l) = \vec{0}$$

$$\vec{F}(y) + \vec{F}(y+L) = \vec{0}$$

- The sums of the vertical pressure forces along $\overrightarrow{e_z}$ are opposed to the weight of the volume considered.

$$\vec{F}(z) + \vec{F}(z+h) + \overrightarrow{P_{weight}} = \vec{0}$$

With: $\vec{F}(z) = P(z)\, l\, L\, \overrightarrow{e_z}$ (Up);

$\vec{F}(z+h) = -P(z+h)\, l\, L\, \overrightarrow{e_z}$ (down);

$\vec{P}_{weight} = \rho V \vec{g} = -\rho\, l\, L\, h\, g\, \overrightarrow{e_z}$ (down).

That to say: P(z) – P(z+h) – ρgh =0

P(z) = P(z + h) + ρ g h, with h > 0

Fundamental Equation of Hydrostatics

This equation shows that pressure in a static fluid is directly proportional to the depth and density of the fluid.

The hydrostatic pressure at a given depth is given by: $P = P_0 + \rho g h$

3.3.2. Case of a compressible fluid

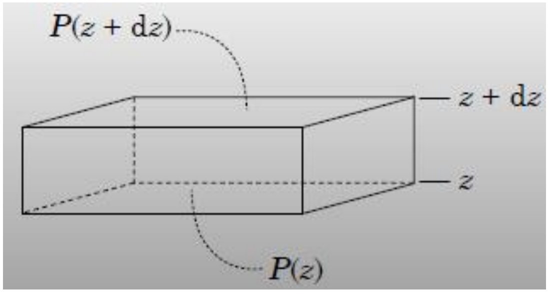
For compressible fluids such as gases, the density depends on the pressure. In the case of a volume of gas in a gravitational field (example: Earth's atmosphere), it is easy to understand that the layers of gas located at low altitude are compressed by the weight of the upper layers. The reasoning used previously for liquids is no longer valid because the pressure and density of compressible fluids (gases) can vary significantly with the height **z** at which we are placed. It is then appropriate to reason on a small difference in height **dz** for which we can legitimately assume that the variation in density is very small or even negligible. The fluid can then be considered homogeneous over the thickness concerned.

In this case, the fundamental relation of hydrostatics can be written:

P(z) = P(z + dz) + ρ(z) g dz

or even:

$$\frac{P(z+dz) - P(z)}{dz} = -\rho(z) g$$



By making the thickness of the volume considered dz → 0, we arrive at the following differential equation:

$$\lim_{dz \to 0} \frac{P(z+dz) - P(z)}{dz} = \frac{dP}{dz} = -\rho(z) g$$

The fundamental relationship of hydrostatics remains valid but the density ρ is no longer a constant.

3.4. Archimedes' Principle

Archimedes' principle deals with the forces applied to an object by fluids surrounding it. This applied force reduces the net weight of the object submerged in a fluid. In this article, let us familiarize ourselves with Archimedes' principle.

- **What is the Archimedes' Principle?**

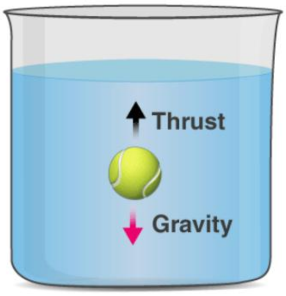
Archimedes' principle states that:

"*The upward buoyant force that is exerted on a body immersed in a fluid, whether partially or fully submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the center of mass of the displaced fluid***"**.

The value of thrust force is given by the Archimedes law which Archimedes of Syracuse of Greece discovered. When an object is partially or fully immersed in a liquid, the apparent loss of weight is equal to the weight of the liquid displaced by it.

- **Archimedes' Principle Explanation**

If you look at the figure, the weight due to gravity is opposed by the thrust provided by the fluid. The object inside the liquid only feels the total force acting on it as the weight. Because the actual gravitational force is decreased by the liquid's upthrust, the object feels as though its weight is reduced. The apparent weight is thus given by:



Apparent weight= Weight of object (in the air) – Thrust force (buoyancy)

Archimedes' principle tells us that the weight loss is equal to the weight of liquid the object displaces.

- **Archimedes' Principle Formula**

In simple form, the Archimedes law states that the buoyant force on an object is equal to the weight of the fluid displaced by the object. Mathematically written as:

$$F_b = \rho \times g \times V$$

Where F_b is the buoyant force, ρ is the density of the fluid, V is the submerged volume, and g is the acceleration due to gravity.

- The principle is used to measure the volume and density of irregularly shaped objects. It is also used in a large variety of scientific research subjects, including medical, engineering, entomology, engineering, and geology

- **Archimedes' Principle Derivation**

We know that the density is defined as: $\rho = m/V$

Therefore, the mass of the displaced liquid can be written as follows: $m = \rho \times V$

Now, the weight of the displaced liquid can be calculated as follows:

$$\text{Weight} = \text{Mass} \times \text{Acceleration due to gravity}$$

$$\text{Weight} = \text{Mass} \times g = \rho \times V \times g$$

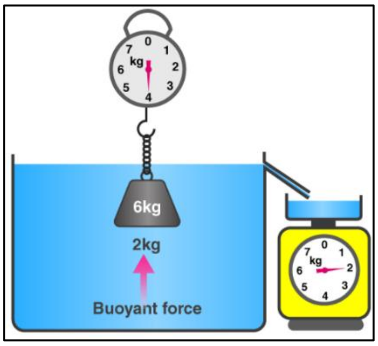
From Archimedes' principle, we know that the apparent loss of weight is equal to the weight of the water displaced therefore the thrust force is given by the following equation:

$$\text{Thrust Force} = \rho \times V \times g$$

Where ρ is the density of the liquid, V is the volume of liquid displaced and g is the acceleration due to gravity.

The ***thrust force*** is also called the ***buoyant force*** because it is responsible for objects floating. Thus, this equation is also called ***the law of buoyancy***.

- **Archimedes' Principle Experiment**



Take a container filled with water to the brim.

Now take any solid object you like and measure its weight using a spring balance. Note this down.

Keep the object attached to the spring balance and submerge it in the water. Just make sure the spring balance is not submerged.

Now, note down the weight shown by the spring balance. You will notice that it is less. Some water will be displaced into the bowl.

Collect this water and weigh it. You will find that the weight of the water will be exactly equal to the loss of weight of the object!

- **Summary of Hydrostatics**

Hydrostatics is fundamental for understanding fluid behavior at rest.

Key principles like Pascal's principle, Archimedes' principle, and hydrostatic pressure provide tools for solving problems involving fluids in equilibrium.

Applications extend across engineering, marine science, meteorology, and medicine.

4. Hydrodynamics

4.1. Definition

Hydrodynamics is a branch of fluid mechanics that deals with the motion of fluids (liquids and gases) and the forces acting on them. It focuses on the study of fluid flow patterns, behavior under different conditions, and the interaction of fluids with solid boundaries or other fluids. Hydrodynamics is essential for understanding and designing systems involving fluid motion, such as pipelines, ships, and turbines.

4.2. Types of Fluid Flow

As already stated in the upper sections the fluids can be classified according to the type of the flow by using *Reynolds number* (*Re*) to ***laminar, turbulent or transitional flow***.

- **Laminar Flow**: Smooth, orderly motion of fluid particles in parallel layers with no disruption between them. The velocity of the fluid at any point remains constant over time. Example: Flow of oil through a thin tube.
- **Turbulent Flow**: Irregular, chaotic flow characterized by eddies and vortices. Velocity fluctuates over time and space. Example: Flow of water in rivers or air over an airplane wing.
- **Transitional Flow**: A mix of laminar and turbulent flow that occurs at intermediate flow rates.

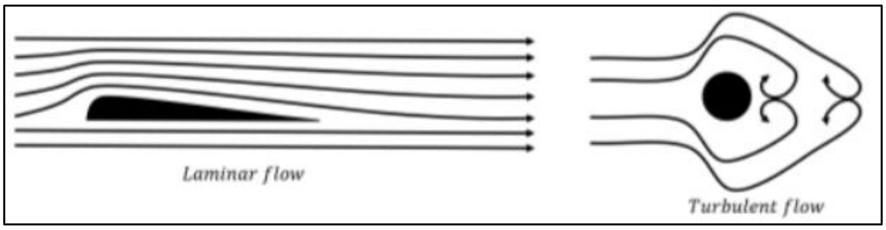


Figure II.5 : *Laminar (left) and turbulent (right) flow of a fluid around an object*.

4.3. Continuity of flow (Equation of continuity)

Consider the laminar flow of a fluid through a pipe whose cross-sectional area narrows from A_1 to A_2 in the direction of flow, as illustrated in Figure II.6.

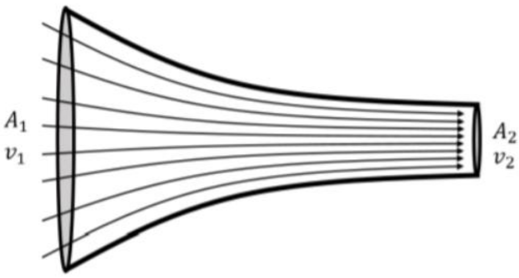


Figure II.6 : *Laminar flow of a fluid in a narrowing pipe*.

The particles that make up the fluid have a speed $\boldsymbol{v_1}$ at the wide end of the pipe and speed $\boldsymbol{v_2}$ at the narrow end. The **equation of continuity** is based on the premise that the fluid that enters the pipe must exit the pipe, as there is nowhere else for the fluid to go. That is, if during a period of time, $\boldsymbol{\Delta t}$, a mass, $\boldsymbol{\Delta m}$, of fluid enters the wide end of the pipe, then during that same period of time, the same mass of fluid must exit the narrow end of the pipe.

During a period of time, $\boldsymbol{\Delta t}$, the fluid at the wide end of the pipe will travel a distance $l_1 = v_1\Delta t$. Thus, a volume of fluid, ΔV_1, will enter the wide end of the pipe:

$$\Delta V_1 = A_1 l_1 = A_1 v_1 \Delta t$$

Similarly, during that period of time, a volume ΔV_2 will exit the narrow end of the pipe:

$$\Delta V_2 = A_2 l_2 = A_2 v_2 \Delta t$$

If the fluid is compressible, its density can change. Let $\boldsymbol{\rho_1}$ be the density of the fluid at the wide end of the pipe and $\boldsymbol{\rho_2}$ be the density of the fluid at the narrow end. The mass of fluid, $\boldsymbol{\Delta m}$, entering the wide end of the pipe is given by:

$$\Delta m = \rho_1 \Delta V_1 = \rho_1 A_1 v_1 \Delta t.$$

The mass of fluid exiting the narrow end of the pipe is given by:

$$\Delta m = \rho_2 \Delta V_2 = \rho_2 A_2 v_2 \Delta t$$

The mass of fluid entering the wide end of the pipe must equal the mass exiting the narrow end of the pipe: $\boldsymbol{\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t}$

Leading to the **equation of continuity**:

$$\boldsymbol{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

The quantity $\boldsymbol{\rho A v}$ has dimensions of mass per time, and corresponds to the mass of fluid passing through a cross section A per unit time.

If the ***fluid is incompressible***, as are most liquids, then the density is the same on both sides of the pipe, and the equation simplifies to:

$$\boldsymbol{A_1 v_1 = A_2 v_2} \text{ (Incompressible fluid)}$$

For a liquid, we can define the "***volumetric flow***", Q, as:

$$\boldsymbol{Q = Av}$$

where A is the cross-sectional area of the surface through which a fluid with speed, v, flows. Q has the dimension of volume per time, and corresponds to the volume of fluid passing through the cross section A per unit time.

For an incompressible fluid, the equation of continuity is thus equivalent to stating that the volumetric flow, Q, of the fluid is a constant.

Example: When water flows out of your faucet, you observe that the stream of water gets narrower as the water moves down, as shown in the figure. Why is this?

Answer:

- The atmospheric pressure increases as the waters moves downwards, so the stream of water is more and more compressed.
- As the water accelerates due to gravity, the cross-sectional area of the flowing water must reduce in order to preserve a constant flow rate.

4.4. Bernoulli's Principle

In this section, we examine how the pressure and speed of a fluid change as a the fluid flows. We will restrict ourselves to discussing the **laminar** flow of an **incompressible** fluid with no friction. Bernoulli was the first to quantitatively describe the flow of incompressible fluids, with the “Bernoulli’s Principle”.

Consider the laminar flow of an incompressible fluid through a pipe that changes height, from y_1 to y_2, as well as cross-sectional area, from A_1 to A_2, as shown in Figure II.7. The figure shows an element of fluid, in blue, as it moves through the pipe. The top panel corresponds to the location of the fluid element at time t = 0, whereas the bottom panel shows the location of the element of fluid at time t = Δt.

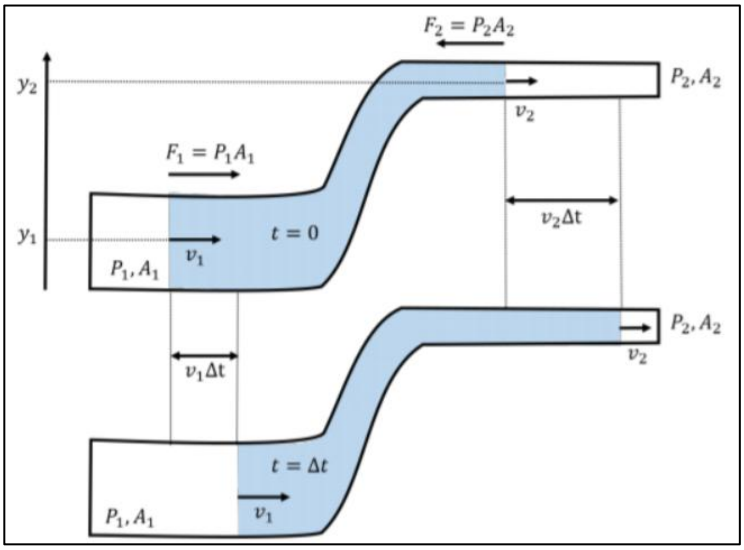


Figure II.7: *Laminar flow of an incompressible fluid through a pipe that changes cross-sectional area and height in the direction of flow. An element of fluid, in blue, is shown at time t=0 (top panel), and then, at a later time, t=Δt (bottom panel).*

To model how the fluid moves through this pipe, we can use energy and the Work-Energy Theorem. We start by considering the amount of work done on the element of fluid as it moves from the position in the top panel to the position in the bottom panel.

The fluid that is to the left of the element of fluid exerts a pressure, P_1, on the fluid element that leads to a net force, $\overrightarrow{F_1}$, towards the right. Similarly, the fluid to the right of the element of fluid exerts a net force $\overrightarrow{F_2}$ in the opposite direction, due to the pressure P_2 on that side of the fluid element.

In a period of time, Δt, the left part of the fluid element will move a distance $l_1 = v_1 \Delta t l_1$, while the right part of the fluid element will move a distance $l_2 = v_2 \Delta t l_2$. We can calculate the work done by each force, defining positive work to be in the direction of motion:

$$W_1 = F_1 l_1 = (P1A1)(v_1 \Delta t)$$

$$W_2 = F_2 l_2 = (P2A2)(v_2 \Delta t)$$

Gravity will also do (negative) work on the fluid as it changes height. In a period of time, Δt, a mass of fluid, Δm, will move from position $y = y_1$ to position $y = y_2$. The mass of fluid that changes height is given by the part of the fluid that moves a distance, l_1, on the right side of the pipe:

$$\Delta m = V_1 \rho = A_1 l_1 \rho = A_1 v_1 \Delta t \rho$$

Because of the equation of continuity, this is also equal to the mass of fluid that moves a distance, l_2, on the left side of the pipe:

$$\Delta m = V_2 \rho = A_2 l_2 \rho = A_2 v_2 \Delta t \rho$$

since $A_1 v_1 = A_2 v_2 A_1$. The force of gravity will thus do negative work on that mass element:

$$W_g = -\Delta m g\,(y_2 - y_1) = -(A_1 v_1 \Delta t \rho) g (y_2 - y_1)$$

The net work done on the element of fluid over the time Δt is thus:

$$W_{net} = W_1 + W_2 + W_g = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t - A_1 v_1 \Delta t \rho g\,(y_2 - y_1)$$

Note that, because of the equation of continuity, $A_1 v_1 = A_2 v_2$, we can factor out a $A_1 v_1$ from each term:

$$W_{net} = A_1 v_1 \Delta t\,(P_1 - P_2 - \rho g (y_2 - y_1))$$

The net work done on the fluid must equal the change in kinetic energy, ΔK, of the mass element, Δm, from one end of the pipe to the other:

$$\Delta K = \tfrac{1}{2}\,\Delta m\, v_2^2 - \tfrac{1}{2}\,\Delta m\, v_1^2 = \tfrac{1}{2}\,(A_1.v_1.\Delta t.\rho)(v_2^2 - v_1^2)$$

Using the Work-Energy Theorem, we have: $W_{net} = \Delta K$

We can re-arrange this so that all the quantities for each side of the pipe are on the same side of the equation:

$$P_1 + \tfrac{1}{2}\,\rho\, v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\,\rho\, v_2^2 + \rho g y_2$$

Since the locations 1 and 2 that we chose are arbitrary, we can state that, for laminar incompressible flow, the following quantity evaluated at any position is a constant.

Bernoulli's principle states that in a streamline flow, the total energy (pressure, kinetic, and potential) remains constant. The Bernoulli equation is:

$$P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$$

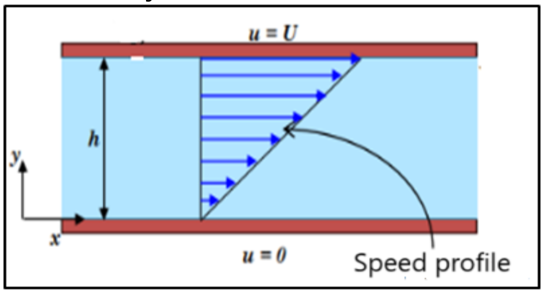
This statement “Bernoulli’s Equation” is equivalent to conservation of energy for the fluid.

Exercises

Exercise 1:

A Newtonian fluid with density $\delta = 0{,}92$ and kinematic viscosity $\upsilon = 4$ stokes flows over a flat plate in steady state.

The velocity profile (u) is shown in Figure 1 and follows the law: $u(y) = U \sin(\frac{\pi y}{2h})$.



Determine the shear stress on the plate as a function of U and h.

Exercise 2:

The space between two parallel square flat plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.50 mm. The upper plate which is moving at 2.50 meters per second requires a force of 98.10 N to maintain the speed.

Determine :

- The dynamic viscosity of oil in poise.
- The kinematic viscosity of oil in stokes.

Data: $\delta_{oil} = 0{,}95$; $g = 10$ m/s^2.

Exercise 3:

Determine the kinematic viscosity of water, if the friction force on the surface S = 0.06 m^2 is equal to $12 * 10^{-4}$ N and the velocity gradient is equal to 1s^{-1}.

Exercise 4:

A viscous Newtonian liquid with a dynamic viscosity coefficient μ =1 mPa.s, with a volumetric weight of 1000 kg.m^{-3}, flows into a 1cm diameter pipe.

What is the value of the flow rate in liters/min to change from laminar to turbulent regime?

Exercise 5:

1. Compare the surface tension energy of a droplet of a liquid with a diameter of 10 µm to the energy it gains when falling freely from a height of 10 m.
2. The same question for a droplet with a radius of 5 mm.

σ = 72 mJ.m^{-2}, ρ = 1 g.cm^{-3}, g = 10 ms^{-2}.

Exercise 6:

A liquid that perfectly wets glass, with a density of $\rho = 1.05 \times 10^3$ kg/m³, rises to an average height of h = 1.5 cm in a vertical glass capillary tube with an inner diameter of d = 1 mm.

1. Calculate the surface tension coefficient of the liquid.
2. What is the height reached in the same capillary tube when it is vertically immersed in mercury?

Given: $\sigma_{Hg} = 500\ 10^{-3}$ N/m, $\rho_{Hg} = 13600$ kg.m^{-3}, $\Theta = 135°$.

Exercise 7:

A glass capillary tube with a radius r = 2 mm is partially immersed in a vessel containing mercury. The mercury descends in the tube to a height difference h relative to the level of mercury in the vessel. The contact angle is $\theta = 130°$ and the surface tension is $\sigma = 0.49$ N/m.

1. Draw a detailed diagram showing r, θ, and h.
2. Determine the depression h of the mercury.

$\delta_{mercury} = 13.6$

Exercise 8:

Part 1:

1. Calculate the resulting force, if a steel ball of radius 6 cm is immersed in water.
2. Calculate the buoyant force, if a floating body is 95% submerged in water. The density of water is 1000 kg.m^{-3}.

Part 2:

Your garden hose has a diameter of D = 2cm. How fast must water flow out of the hose if you are to fill a 5L bucket in one minute?

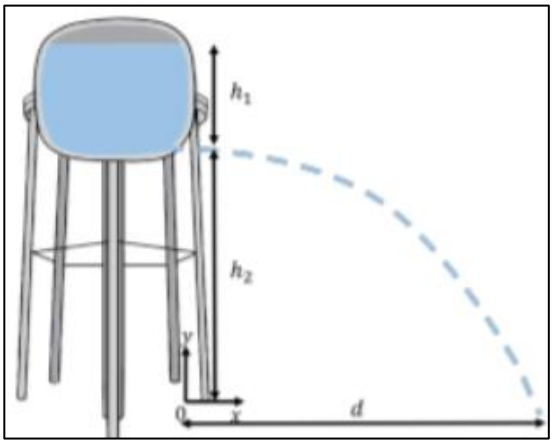
Exercise 9:

When a high speed train is traveling at constant speed, is there a net force on the windows from air pressure?

A. No, since the windows are stationary relative to the train, there is no net force on them from air pressure.
B. Yes, there is a net outwards force on the windows from air pressure.
C. Yes, there is a net inwards force on the windows from air pressure.

Exercise 10:

A water tower is constructed so that the bottom of the water tank is a height h_2 above the ground, as illustrated in the figure below.



The water in the tank is at a height h_1 from the bottom of the tank. A leak from a hole is found at the base of the tank (the water flows horizontally out of the hole).

1. What is the horizontal distance, d, from the bottom of the tower to where the water from the leak hits the ground?
2. Assume that the water level in the tank is constant and that atmospheric pressure does not change appreciably over the height of the tower.

Exercise 11:

You measure that water comes out of your kitchen faucet at a rate of 6 L/min. The faucet has a diameter of 2 cm.

1. At what rate will water flow out of your basement faucet, which has a diameter of 1 cm and is located a height, h = 3 m, below your kitchen faucet?
2. Assume that atmospheric pressure, P_0, does not change appreciably between your kitchen and basement.

Chapter-III:

Geometric Optics

CHAPTER III : *Geometric Optics*

Geometric optics, also known as ray optics, is the branch of optics that studies the behavior of light in terms of rays. It focuses on the laws of reflection and refraction and how light interacts with optical systems such as mirrors, lenses, and prisms. Geometric optics assumes that light travels in straight lines and neglects the wave nature of light, making it valid when the wavelength of light is much smaller than the dimensions of the objects it encounters.

1. Definitions

There are three ways in which light can travel from a source to another location:

- It can come directly from the source through empty space, such as from the Sun to Earth.
- Or light can travel through various media, such as air and glass, to the person.
- Light can also arrive after being reflected, such as by a mirror. In all of these cases, light is modeled as traveling in straight lines called rays.

Light may change direction when it encounters objects (such as a mirror) or in passing from one material to another (such as in passing from air to glass), but it then continues in a straight line or as a ray.

- **Geometric Optics** focuses on studying the formation of images by optical systems, including their position, magnification, image quality, and nature. It utilizes the concept of light rays and the laws of reflection and refraction. This branch disregards phenomena such as interference, diffraction, polarization, and scattering and does not consider processes of light emission and absorption.
- **Light Sources** are any objects or systems that emits electromagnetic radiation in the visible spectrum, making it possible for humans to perceive light. Light sources can be **natural** (occurring in nature) or **artificial** (created by humans). They can also emit light through various mechanisms, such as thermal radiation, chemical reactions, or quantum processes.

Light sources can be also classified as **primary** source, which emits light itself (Sun, Lamp, Candle, etc.), and **secondary** source which reflects light (Mirror, Moon, etc.).

- **Environments** are the media from which light propagates; these later can be divided to three main media:

Transparent medium: Allows us to see objects clearly. Example: Air, Water, Glass.

Opaque environment: We cannot see the objects; like wood.

Translucent medium: allows light to pass through but vision through these mediums is blurry (not sharp) like frosted glass.

1.1. Ray of light and light Beam

1.1.1. Ray of light

A ray of light is an idealized, one-dimensional representation of the path along which light energy travels. It is used in geometric optics to model the direction and propagation of light. A ray is a **theoretical construct** that simplifies the study of optics by treating light as traveling in straight lines.

The main characteristics of Ray of light are:

Straight-Line Propagation: In a homogeneous medium, light travels in straight lines.

Directionality: A light ray indicates the direction of energy transfer in the medium.

Idealization: A light ray has no thickness and is used for simplifying problems in geometric optics.

1.1.2. Light Beam

A light beam is a collection of light rays traveling together in the same direction. It is essentially a stream of light that can vary in width, intensity, and divergence. A light beam is typically described as a **macroscopic representation** of multiple rays.

There are three types of light beams:

Parallel Beam: The light rays in the beam are parallel to each other.

Convergent Beam: The light rays converge toward a single point.

Divergent Beam: The light rays spread out as they travel.

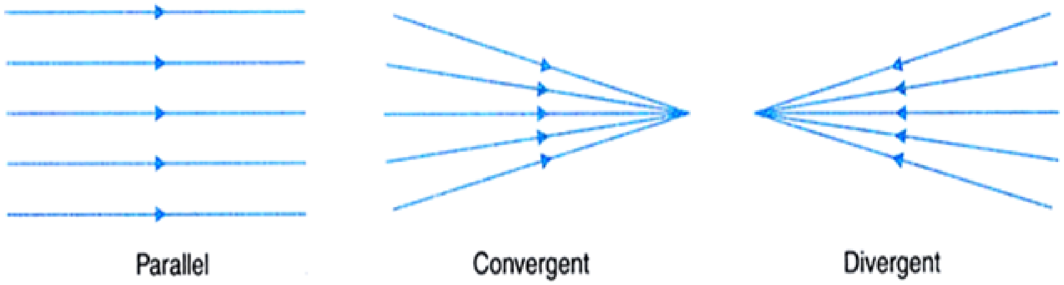


Figure III.1: *Types of light beams.*

Table III.1: Comparison between a light beam and a light ray.

Aspect	Light Beam	Ray of Light
Definition	A collection of light rays traveling together.	A single, idealized representation of light's path.
Structure	Macroscopic; includes multiple rays.	One-dimensional, theoretical construct.
Types	Parallel, convergent, divergent.	Not classified into types.
Practical Use	Describes real-world light propagation (e.g., laser beams).	Simplifies the study of light behavior in optics.
Thickness	Can vary in width.	Has no thickness (idealized).

1.2. Refractive Index

The refractive index (**n**) of a material is a dimensionless number that quantifies how light propagates through that material. It represents the ratio of the speed of light in a vacuum (c) to the speed of light in the medium (*v*):

$$n = \frac{c}{v}$$

Where:

- n: Refractive index (dimensionless),
- c: Speed of light in a vacuum (3×10^8 m/s),
- *v*: Speed of light in the medium (m/s).

The refractive index describes how much a material slows down light compared to its speed in a vacuum.

- The **relative refractive index** between two media (n_{12}):

$$n_{12} = \frac{n_1}{n_2}$$

where n_1 and n_2 are the absolute indices of the two media in contact.

1.2.1. Applications of refractive index

Optical Instruments: Designing lenses for cameras, microscopes, and telescopes.

Fiber Optics: Used in telecommunications to guide light through optical fibers via total internal reflection.

Material Identification: Determining the type or purity of a material by measuring its refractive index.

Vision Correction: Glasses and contact lenses rely on refractive index to correct vision by bending light properly.

Metrology: Measuring liquid concentrations (e.g., sugar solutions in refractometers).

Table III.2: Typical Refractive Index Values.

Material	Refractive Index (n)
Vacuum	1.0
Air	1.0003
Water	1.33
Glass	1.5
Diamond	2.42

The refractive index is a fundamental property in optics, providing insight into how light interacts with materials and enabling a wide range of technological applications.

1.2.2. Wavelength (λ)

Wavelength is the distance between two consecutive points in phase on a wave, such as two adjacent crests or troughs in a transverse wave, or two compressions or rarefactions in a longitudinal wave. It represents the spatial periodicity of the wave and is a fundamental property of wave motion.

The wavelength (expressed in **m**) of a light ray is given by the relation:

$$\lambda = c.T = \frac{c}{f}$$

Where: T is the period and f the frequency.

For light traveling in a medium with refractive index n:

$$\lambda_{\mathbf{medium}} = \frac{\lambda_0}{n}$$

Where: λ_0 is the wavelength in a vacuum.

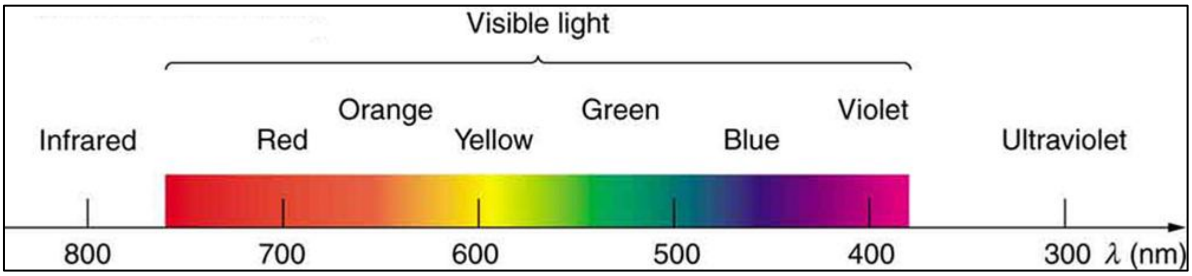


Figure III.2: *Wavelength spectrum of electromagnetic waves.*

1.3. Diopter

In geometric optics, a diopter refers to the interface between two transparent media with different refractive indices, where light undergoes refraction (bending) according to Snell's law. A diopter can be either planar (flat) or spherical, depending on the shape of the interface.

Additionally, in optometry, the diopter (D) is a unit of measure used to express the optical power of lenses or curved mirrors, defined as the reciprocal of the focal length in meters:

$$P = \frac{1}{f}$$

Where: P: is the optical power in diopters (D), and f is the focal length of the lens in meters (m).

1.4. Optical system

An optical system consists of **components** such as lenses, mirrors, prisms, and apertures, **that work together to manipulate light for a specific purpose**. Optical systems are used to focus, magnify, reflect, refract, or otherwise modify light to form images, transfer light, or analyze optical properties.

Engineers and scientists use these systems in various applications, including telecommunications, scientific research, imaging, and sensing.

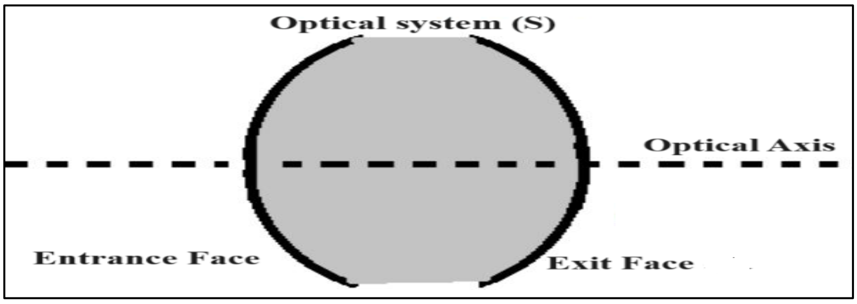


Figure III.3: *Simple illustration of optical system.*

There are various types of optical systems, including the following:

- **Reflective systems:** Designed to minimize optical aberrations and provide a wider field of view than equivalent refractive systems. They can also be more durable and less prone to thermal expansion, making them well-suited for harsh environments. One notable example of a reflective system is the Hubble Space Telescope, which uses a series of mirrors to capture stunning images of space.
- **Diffraction systems**: These use diffraction elements, such as gratings or prisms, to manipulate light. Diffraction systems are used in spectrometers, laser beam shaping, and other applications.
- **Lens systems:** These are a combination of one or more lenses arranged in a specific way to control and manipulate light propagation. They are commonly used in cameras, microscopes, telescopes, and eyeglasses, among other applications.

- **Fiber optic systems**: Fiber-optic communication systems use thin strands of glass or plastic fibers to transmit data and information over long distances. These fibers use the principle of total internal reflection to guide light signals without loss of signal strength or quality.

2. Fundamental hypothesis

2.1. Fermat's principle

In a homogeneous and isotropic medium, light traveling between two points (A and B) follows a path such that the travel time is stationary (minimum or maximum).

Consequences of Fermat's principle

- A medium is **homogeneous** if it has the same physical properties at every point.
- A medium is **isotropic** if its physical properties are the same in all directions.
- Light propagates in straight lines: the shortest path between two points is a straight line.
- **Reversibility Principle**: The path taken by light is independent of its direction.
- **Independence of Light Rays**: The trajectory of one light ray is not affected by others.

2.2. Snell-Descartes laws

When a light ray, called an incident ray, falls at a point **I** on the surface of a diopter separating two optical media of indices n_1 and n_2. A part of the light, called ***Reflected Ray***, reflected in the medium of index n_1 and a part, called ***Refracted Ray***, penetrates into the medium of index n_2.

The angle formed by the incident ray and the normal to the diopter $\mathbf{i_1}$ called the **angle of incidence**. **i'** is the angle formed by the reflected ray and the normal called the **reflection angle**. The angle $\mathbf{i_2}$ is that formed by the refracted ray and the normal called the **angle of refraction**.

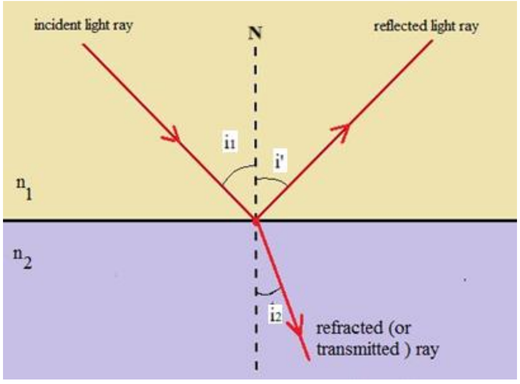
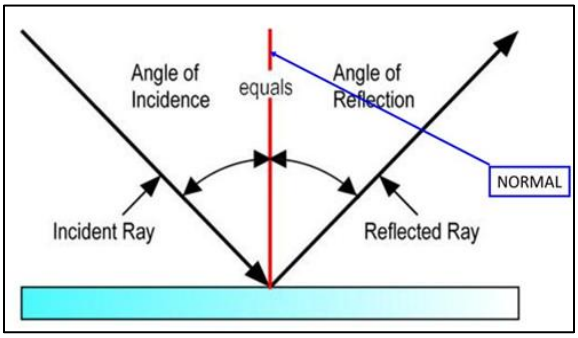


Figure III.3: *Different deviation of an incident light ray that falls on the surface of a diopter*.

A. For Reflection

- **First law:** The incident ray (I_R), the reflected ray (R_R), and the normal (N) lie in the same plane called the plane of incidence.
- **Second law (Law of reflection):** The angle of incidence (i_1) and reflection angle (i') are equal: $\mathbf{i_1 = i'}$.



B. For Refraction

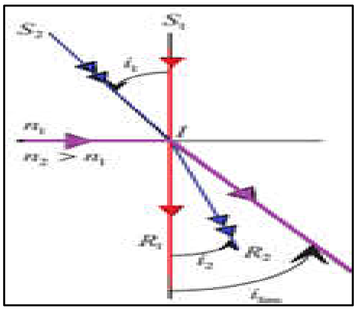
- **Third law (Law of refraction):** The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction. This variation is giving by: $\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1} = cst \Rightarrow \mathbf{n_1 \sin i_1 = n_2 \sin i_2}$.

Remark: The angles are always measured relative to the perpendicular to the surface (normal), at the point where the light ray strikes.

2.3. Discussion of the law of refraction

A. When the second medium is more refractive $n_2 > n_1$:

The light passes from the less refractive medium to the more refractive medium (e.g. air→ water or water → glass): the refracted ray approaches the normal. There are three ways of incidence:

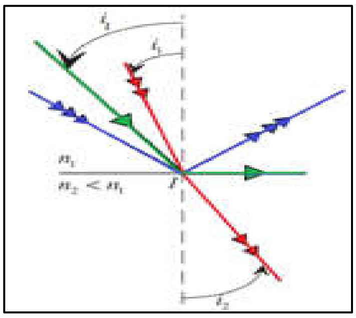


- For : $i_1 = 0$ (normal incidence) → $i_2 = 0$.
- For : $0 < i_1 < \pi/2$ → $i_1 > i_2$ the refracted ray therefore approaches the normal.
- For : $i_1 = \pi/2$ → $i_2 = i_{lim}$: $n_1 \sin i_1 = n_2 \sin i_2 \Rightarrow n_1 \sin \pi/2 = n_2 \sin i_{lim}$, So:

$$\boldsymbol{\sin i_{lim} = \frac{n_1}{n_2}} \text{ and } \boldsymbol{i_{lim} = \arcsin\frac{n_1}{n_2}}.$$

Then the corresponding angle of refraction is the limiting angle of refraction.

B. When the incident medium is more refractive than the refracted medium $n_1 > n_2$:



The light passes from the more refractive medium to the less refractive medium: the refracted ray moves away from the normal and the law of refraction implies that $i_1 < i_2$. For an angle of incidence limit there the angle of refraction will be maximum ($i_2 = \pi/2$), therefore $\boldsymbol{i_{lim}} = \frac{\boldsymbol{n_2}}{\boldsymbol{n_1}}$.

Beyond this angle, the incident ray is totally reflected (total reflection) and the diopter behaves like a mirror.

Example : A light ray traveling in air strikes the surface of a liquid, making an angle α = 60° with the horizontal plane. The deviation between the incident ray and the refracted ray is θ = 15°.

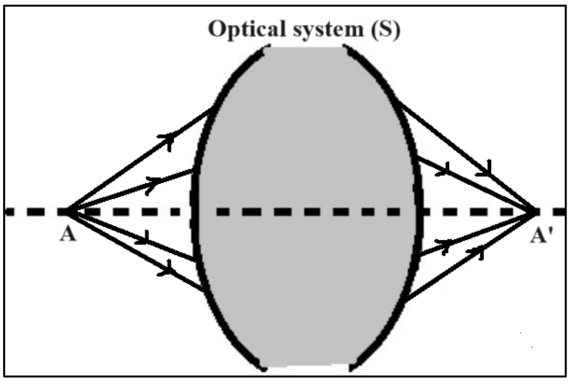
- What is the refractive index of the liquid?

2.4. Image and Object Concepts

2.4.1. Stigmatism

In geometric optics, stigmatism refers to the image-formation property of an optical system which focuses a single point source in one phase optics space into a single point in image space. Two such points are called a stigmatic pair of the optical system.

A point A has an image A′ if all light rays originating from A converge at A′. A system satisfying this property is said to be **Stigmatic**.

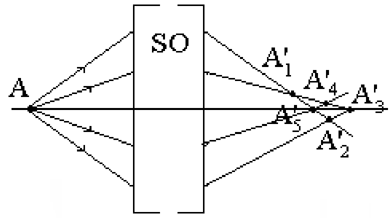


We say that A and A' are conjugated with respect to the optical system (S)

2.4.2. Approximate Stigmatism – Gaussian Conditions

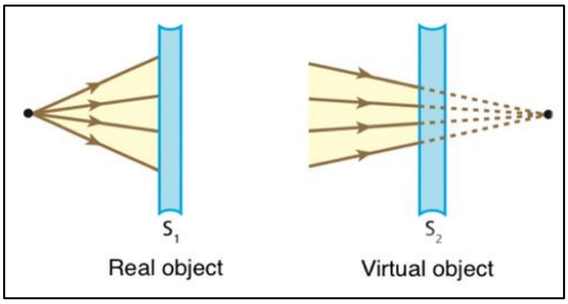
Exact stigmatism is rare. Approximate stigmatism occurs when the image of a point is sufficiently small to be considered a point.

- Gaussian Conditions for Approximate Stigmatism:
  - Narrow and **paraxial rays** (rays making small angles with the optical axis).
  - Small angles of incidence.



2.4.3. Object (Objective)

For an optical system (S), the point source **A** is considered as an ***object*** for it, if it is at the intersection of the light rays' incident on the system, or their extensions. **A'** is a point imaged by the optical system if it is at the intersection of the light rays emerging from the system or their extensions.



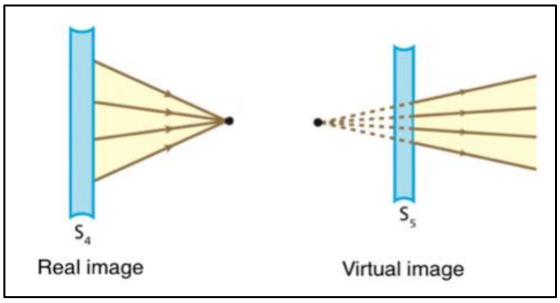
- Real objects are **points from which light diverges**.
- Virtual objects are points **towards which light converges**.

The object plane is the plane that lies in front of the input face of the optical system **S**. If the object is in the object plane then it is a real object.

2.4.4. Image

In optics, an image is defined as the collection of focus points of light rays coming from an object.

- A real image is the collection of focus points actually made by converging/diverging rays.
- Virtual image is the collection of focus points made by extensions of diverging or converging rays.



The image plane is the plane that lies after the output face of the optical system.

2.4.5. Conjugation Relationship

In **optics**, the **conjugation relationship** describes the mathematical link between the positions of an **object** and its **image** relative to an optical system (e.g., a lens, mirror, or diopter). It establishes how light rays originating from one point (object) converge or diverge to form another point (image), based on the geometry and optical properties of the system.

This relationship is fundamental in geometric optics and allows for determining the position, size, and orientation of the image formed by an optical element.

2.4.6. Magnification in optics

Magnification (γ) is a measure of how much larger or smaller an image is compared to the object. It is the ratio of the height (or size) of the image **p'** to the height (or size) of the object **p**. Magnification is used to describe the optical properties of lenses, mirrors, and optical systems.

Linear magnification is given by the relation:

$$\gamma = \frac{\overline{AB'}}{\overline{AB}} = \frac{p'}{p}$$

- Sign Convention for Magnification

Positive Magnification (M > 0):

- The image is **upright** with respect to the object.
- This typically occurs for virtual images.

Negative Magnification (M < 0):

- The image is **inverted** with respect to the object.
- This typically occurs for real images.

|M| > 1:

- The image is larger than the object (magnified).

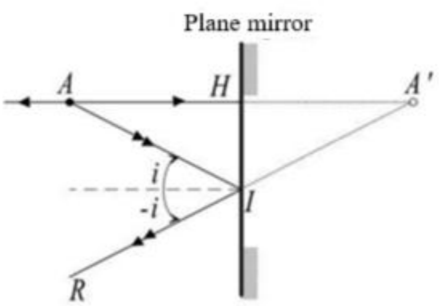
|M| < 1:

- The image is smaller than the object (reduced).

3. Elements with flat faces

3.1. Plane Mirrors

A mirror is a perfectly reflective smooth surface (reflection of 98% of incident rays). A mirror, too, is an optical system by reflection (catadioptric).



- the real object space located before the mirror and the virtual object space located after the mirror.
- the real image space located before the mirror and the virtual image space located after the mirror

3.1.1. Conjugation relationship

- The position of the image A' relative to the mirror is equal to the position of the object A relative to the mirror
- The image is symmetrical of the object in relation to the mirror SA' = SA
- The object and the image are of different nature: Real object - Virtual image
- Image size is equal to object size A'B' = AB

3.1.2. Moving an image by moving a plane mirror

A. Translation: let's move a mirror M from position M_1 to position M_2 in a direction normal to its surface.

When the mirror moves by d, the corresponding image moves by 2d.

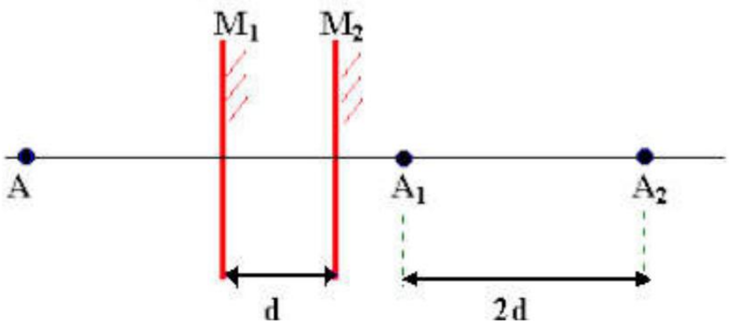


Figure III.4: *Plane mirror translation.*

B. Rotation: Let us rotate the mirror M around an axis, passing through O and belonging to its plane, by an angle α from position M_1 to position M_2. When a mirror rotates by an angle α around an axis, the image rotates around this axis and in the same direction by a double angle 2α.

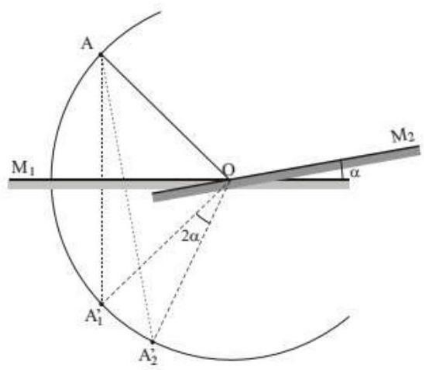


Figure III.5: *Plane mirror rotation.*

C. Association of plane mirrors

➢ **Parallel mirrors:** When two **plane mirrors** are arranged parallel to each other, they create multiple reflections of light rays or objects placed between them. This setup is commonly used in optical devices, experiments, and decorative purposes.

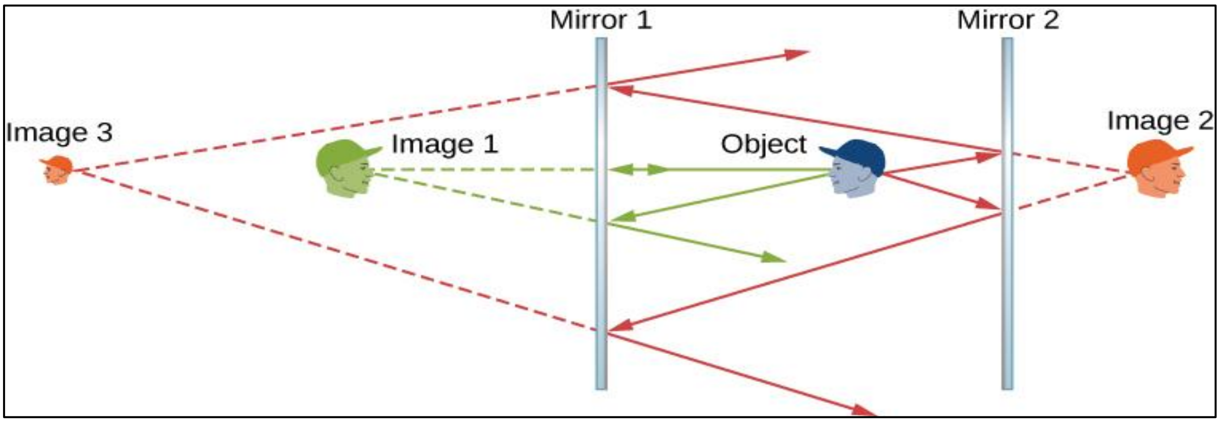


Figure III.6: *Association of plane parallel mirrors*.

- When two plane mirrors are perfectly parallel, a light ray (or image of an object) reflects back and **forth infinitely** between the two mirrors.
- In practice, the number of visible reflections is limited by factors such as the absorption of light by the mirrors, imperfections, or slight misalignments
- An object placed between parallel mirrors appears to produce an **infinite sequence of images** that seem to recede into the distance.

➢ **Mirrors making an angle α between them:** When two mirrors are placed at an angle **α** relative to each other, they create multiple reflections of an object or light ray placed between them. The behavior of the reflections depends on the angle **α**, and the system has many applications in optical devices and experiments.

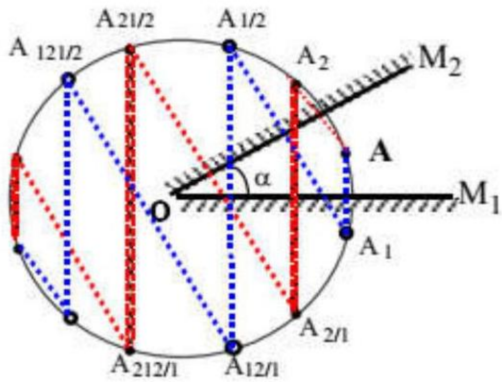


Figure III.7: *Association of plane mirrors making an angle α between them*.

The number of images **N** formed by the two mirrors depends on the angle **α** between them and is given by:

$$N = \frac{360°}{\alpha} - 1$$

- **Two mirrors at right angles:** A single object reflecting from two mirrors at a right angle can produce three images, as shown by the green, purple, and red images.

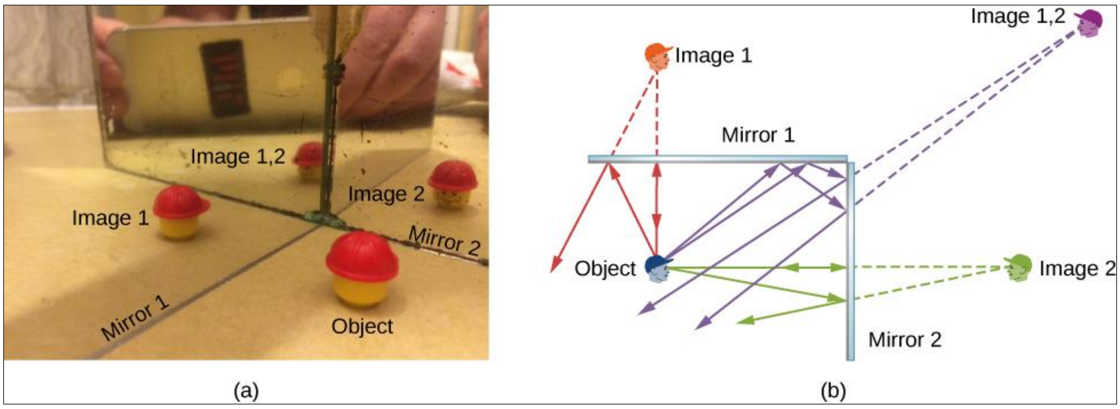


Figure III.8: *Association of two plane mirrors making a right angle between them.*

3.2. Prism

3.2.1. Definition

The prism is a refractive, transparent, homogeneous and isotropic medium of index n limited by two plane diopters form between them an angle A called the angle of the prism. It is typically shaped like a triangular block of glass or other transparent material. Prisms are used to disperse light into its component colors, bend light, or reflect light in optical systems.

3.2.2. Structure of a Prism

Base: The flat surface on which the prism rests.

Apex Angle (A): The angle between the two refracting surfaces of the prism.

Refracting Surfaces: The two polished, flat surfaces through which light enters and exits the prism.

Material: The refractive index (**n**) of the material determines how the prism bends or disperses light.

3.2.3. Relationships of the prism

We designate by i the angle of incidence, i' the angle of the outgoing ray with the normal, r and r' the angles of internal refraction and D the angle of deviation of the light ray.

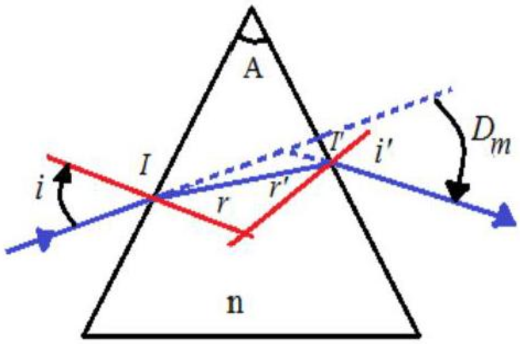
These angles are linked by the relations:

$$\sin i = n \sin r$$

$$\sin i' = n \sin r'$$

$$A = r + r'$$

$$D = i + i' - A$$



3.2.4. Properties of a Prism

Refraction: When light passes through the prism, it is refracted (bent) at the entry and exit surfaces.

Dispersion: Prisms separate white light into its component colors due to the wavelength dependence of the refractive index. Shorter wavelengths (e.g., blue light) are bent more than longer wavelengths (e.g., red light).

Deviation Angle (D_m): The angle by which the light ray is deviated from its original path after passing through the prism.

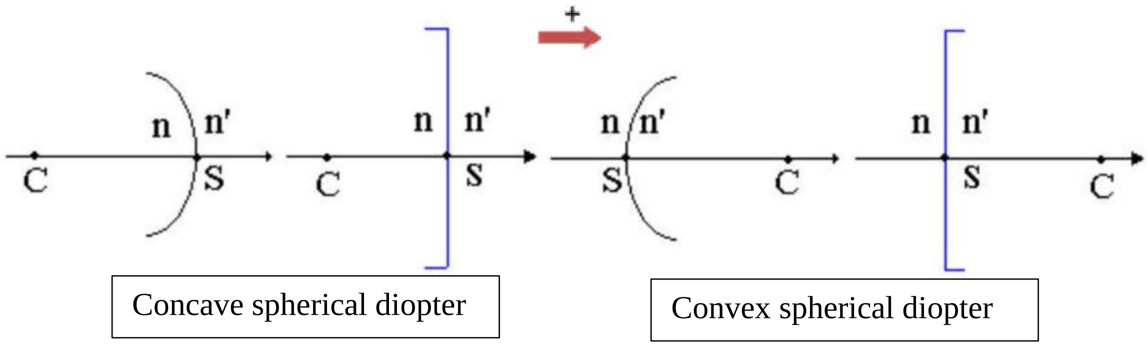
Example: A prism has an apex angle of A = 60° and a refractive index of n=1.5. If a light ray passes symmetrically through the prism, find the angle of minimum deviation.

4. Elements with spherical faces

4.1. Spherical diopter

4.1.1. Definition: A spherical diopter is a set of two homogeneous and transparent media of different indices (n_1 and n_2) separated by a refractive spherical surface. It is characterized by:

- The center C of the sphere called the diopter center.
- The point S called the top of the diopter.
- The optical axis, the axis of symmetry of revolution of the diopter, passing through points C and S.
- The radius of the sphere R= SC, called the radius of curvature, an algebraic quantity which is negative for a concave spherical diopter SC < 0 and positive for a convex spherical diopter R = SC.
- The concave spherical diopter is convergent if $n_1 > n_2$ otherwise it is divergent.
- The convex spherical diopter is convergent if $n_1 < n_2$ otherwise it is divergent.



4.1.2. Conjugation relationship

$$\frac{n_1}{\overline{SA}} - \frac{n_2}{\overline{SA'}} = \frac{n_1 - n_2}{\overline{SC}} = V$$

V is the **vergence** of the diopter.

- The diopter is convergent if V> 0.
- The diopter is divergent if V< 0.

4.1.3. Construction of the image by spherical diopter

To obtain the image produced by this optical system we need to draw two of the three following rays:

- A ray passing through the center C and reflecting on itself.
- A ray parallel to the optical axis and reflecting passing through F.
- A ray passing through F and reflecting parallel to the optical axis.

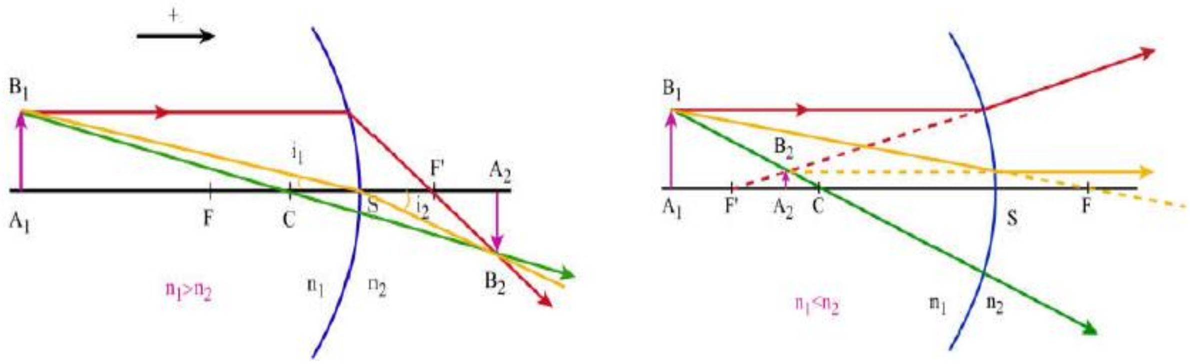


Figure III.9: *Image construction by spherical diopter*.

4.2. Focal Distance and Focus

4.2.1. Focus

In optics, the **focus** (or focal point) is the point where light rays originating from a point source, parallel to the optical axis, converge (for converging systems like convex lenses) or appear to diverge (for diverging systems like concave lenses or convex mirrors) after passing through or reflecting from an optical element.

- For a **lens**: The focus is the point where rays parallel to the optical axis meet after refraction.
- For a **mirror**: The focus is the point where rays parallel to the optical axis converge after reflection.

4.2.2. Focal Distance

The **focal distance** (or **focal length**) is the distance between the optical center (or vertex for mirrors) of an optical system and its focus. It is denoted by ***f*** and depends on the curvature and refractive properties of the optical element.

4.3. Spherical mirrors

A **spherical mirror** is a mirror with a reflecting surface that forms part of a sphere. Depending on the orientation of the reflective surface, spherical mirrors are classified as:

Concave Mirror: The reflective surface is on the **inner side** of the sphere.

Convex Mirror: The reflective surface is on the **outer side** of the sphere.

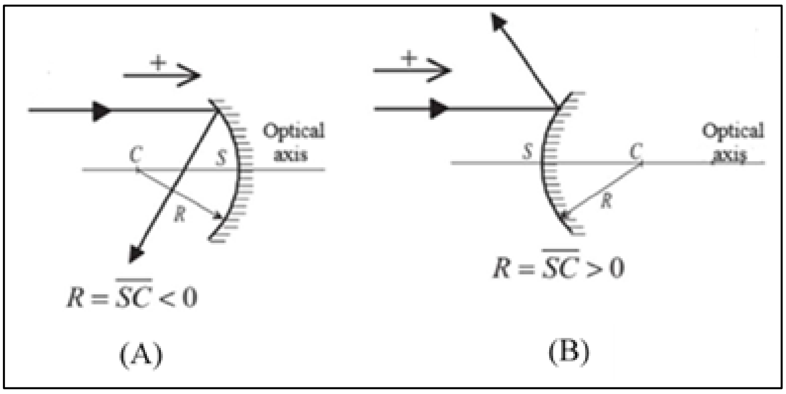


Figure III.10: *(A) Concave spherical mirror, (B) Convex spherical mirror.*

4.3.1. Terms and components

Center of Curvature (C): The center of the sphere from which the mirror is a segment.

Radius of Curvature (R): The radius of the sphere. It is the distance from the center of curvature (C) to any point on the mirror's surface.

Pole (S): The geometric center of the mirror.

Principal Axis: A straight line passing through the pole (S) and the center of curvature (C).

Focus (*F*): The point where light rays parallel to the principal axis converge (for concave mirrors) or appear to diverge (for convex mirrors).

The focal length (*f*) is the distance between the pole (P) and the focus (F).

4.3.2. Focal length

- The object F and image F' foci are combined.
- **Focal length *f*:** is the distance between the focal point and the vertex (top) of the defined mirror:

$$f = \overline{SF} = \frac{\overline{SC}}{2} = \frac{R}{2}$$

- For a concave mirror F is real (SF< 0), while for a convex mirror F is virtual (SF> 0).

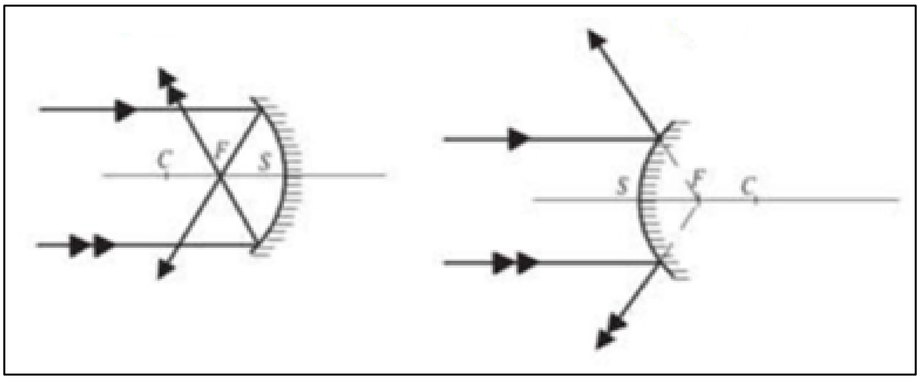


Figure III.11: *Focus F for concave and convex spherical mirrors.*

4.3.3. Construction of the image by spherical mirror

To construct the image A'B' of an object AB, two rays are enough. In practice, we choose these two rays from the following particular rays:

- A ray passing through the center C and reflecting on itself.

- A ray parallel to the optical axis and reflecting passing through F.

- A ray passing through F and reflecting parallel to the axis.

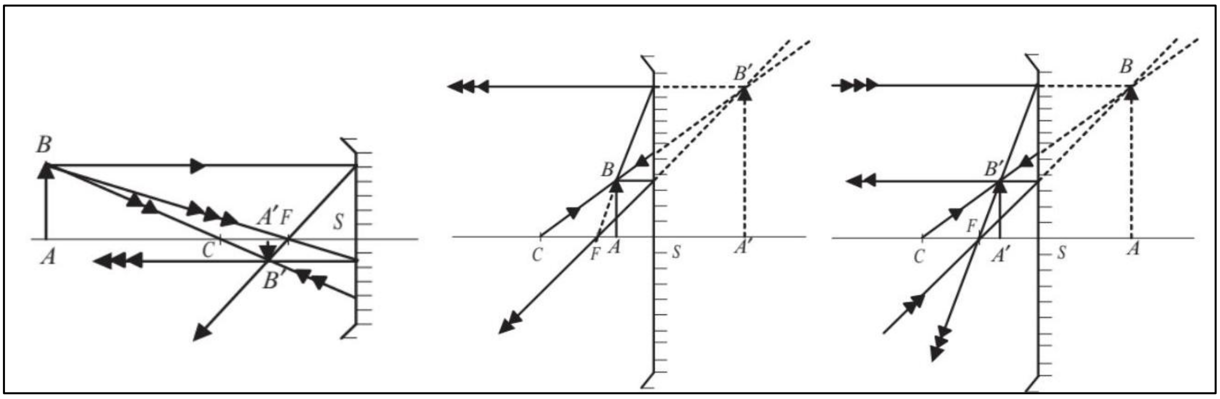


Figure III.12: *Construction of the image by concave spherical mirror*.

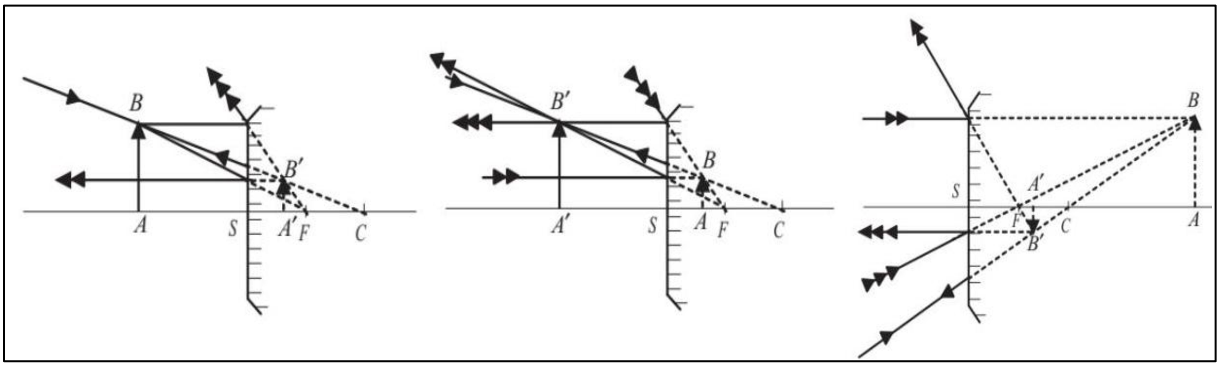


Figure III.13: *Construction of the image by convex spherical mirror*

4.3.4. Conjugation relation

Under Gaussian conditions, the conjugation formula of a spherical mirror is:

Origin at the top:

$$\frac{1}{\overline{SA}} + \frac{1}{\overline{SA'}} = \frac{2}{\overline{SC}}$$

Origin at center:

$$\frac{1}{\overline{CA}} + \frac{1}{\overline{CA'}} = \frac{2}{\overline{SC}}$$

5. Thin lenses

A thin lens is a combination of two spherical diopters whose vertices are practically merged into a single O center. The optical axis of the lens is the axis passing through the centers of the two spherical diopters. N is the index of the medium constituting the lens (n>1).

There are two types of thin lenses: The converging lens and the diverging lens.

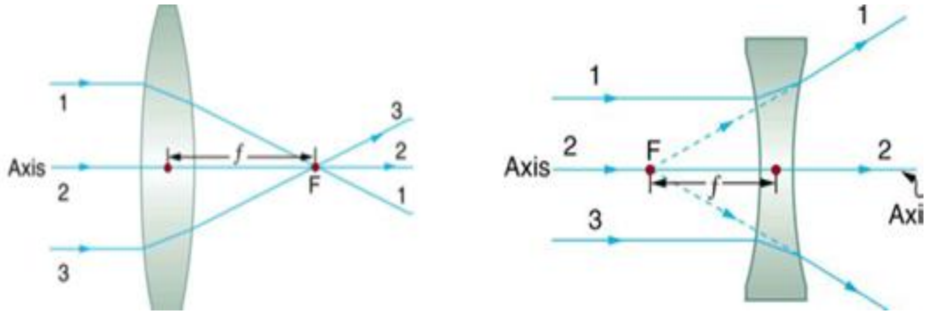


Figure III.14: *The converging lens and the diverging lens respectively*.

- **Converging or Convex lens:** Converging lenses transform a beam of light rays parallel to the optical axis into a converging beam.
- **Diverging Lens:** Divergent lenses transform a beam of light rays parallel to the optical axis into a divergent beam.
- **Focal Length f :** The distance from the center of the lens to its focal point is called focal length *f*.

5.1. The conjugation relation

The position of the image A' obtained by the lens is related to the position of the object A by the relation :

$$\frac{1}{\overline{OA'}} - \frac{1}{\overline{OA}} = \frac{1}{\overline{f'}}, or\ \frac{1}{\overline{p'}} - \frac{1}{\overline{p}} = \frac{1}{\overline{f'}}$$

Where : P’ = OA, P = OA and *f*’ = OF’

5.2. Image Construction

- Rays parallel to the optical axis (from the left side of lens) are deflected through the right focal point.

- Rays passing through the left focal point becomes parallel to the optical axis;

- Rays passing through the center of the lens remain in the same direction.

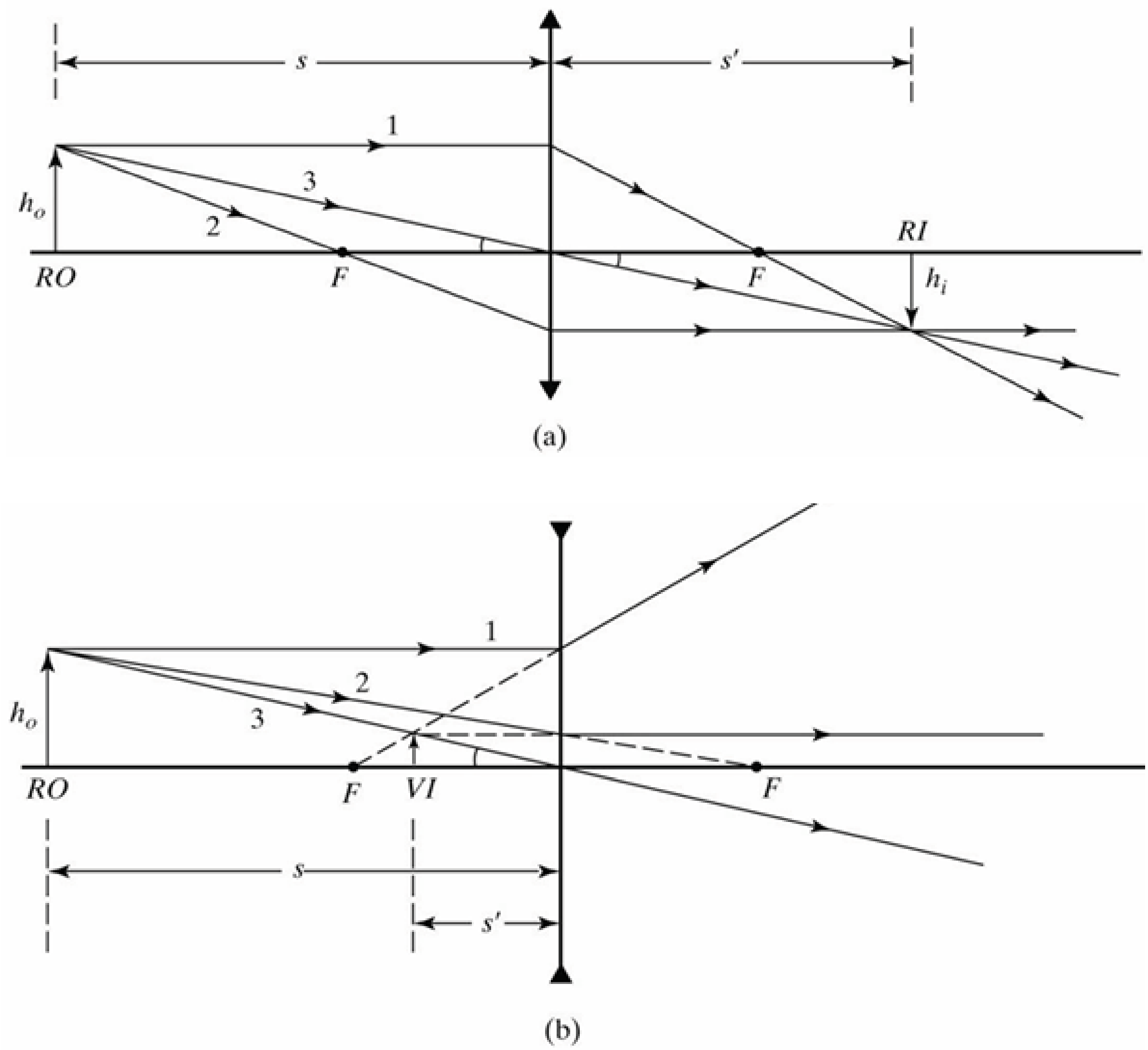


Figure III.15: *Construction of the image by (a) conerging lens, (b) diverging lens*.

6. The eye

The structures and functions of the eyes are complex. Each eye constantly adjusts the amount of light it lets in, focuses on objects near and far, and produces continuous images that are instantly transmitted to the brain.

The inner structures of the eye all work together to produce an image that the brain can understand.

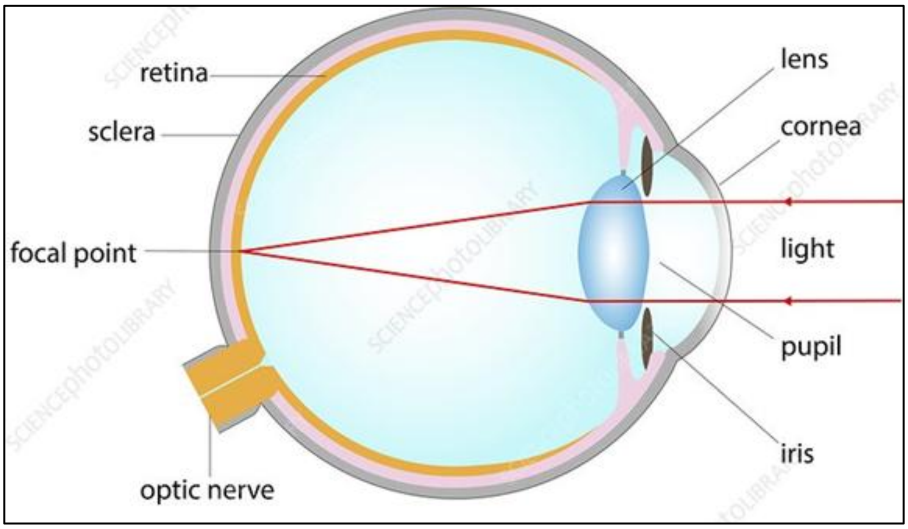


Figure III.16: *The strucrture of the human eye.*

6.1. Parts of the eye

The eye contains over two million working parts and is considered the second most complex organ in the body; the most complex is the brain. The principal parts of the human eye are:

Cornea: The clear dome-like structure that covers the front of the eye and is responsible for bending light as it enters the eye.

Pupil: The dark opening in the center of the eye that opens and closes in response to light intensity.

Iris: The colored part of the eye that is made up of muscles that control the pupil—contracting the pupil in bright light and expanding the pupil in low light.

Sclera: The white part of the eye that surrounds the iris. This structure is made up of fibrous tissue that protects the inner structures of the eye.

Lens: Located behind the pupil, this transparent structure focuses light onto the retina.

Ciliary body: Located behind the iris, this structure contains a muscle that helps to focus the lens.

Vitreous humor: The clear jelly-like substance that fills the central cavity of the eye.

Retina: The light-sensitive membrane that lines the back of the eye; responsible for transforming light signals into electrical impulses to be sent through the optic nerve to the brain.

Rods and Cones: Photoreceptors located in the retina, responsible for processing light signals. Rods allow you to see shapes, while cones allow you to see colors.

Macula: The center of the retina responsible for central vision, and vision for fine details.

Optic nerve: A bundle of nerve fibers that contains more than one million nerve cells. Located in the back of the eye, this nerve is responsible for carrying visual information from the retina to the brain.

6.2. Image production

In order to produce a clear image, the eyes must complete a five step process:

Step 1: ***Light enters the eye through the cornea***

When we look at an object, the light that is reflected off of the object enters the eye through the clear front layer of the eye, called the **cornea**. The cornea bends the light before it passes through a watery substance that fills the area behind the cornea, called the **aqueous humor**.

Step 2: ***The pupil adjusts in response to the light***

The light continues to travel through the black opening in the center of the iris, called the **pupil**. The **iris** is the colorful part of your eye that gives it its blue, green, hazel, brown or dark appearance.

The pupil then automatically gets bigger or smaller, depending on the intensity of the light.

How does the pupil expand and contract?

The iris is actually made up of muscles that expand and contract to control the pupil and adjust its size. So when you see your pupil getting bigger or smaller, it is really the iris that is controlling the pupil opening in response to the intensity of light entering the eye.

Step 3: ***The lens focuses the light onto the retina***

The light passes through the pupil to the **lens** behind it. The lens adjusts its shape to bend and focus the light a second time, to ensure that you have a clear image of what you are looking at.

At this point, the light has been bent twice as it moved from the cornea through the lens, and then from the lens to the retina. This “double bending” has actually flipped the image upside down.

Step 4: ***The light is focused onto the retina***

The light then passes from the lens to the back of the eye which is filled with a clear, gelatinous substance called the **vitreous** until it reaches the **retina**, the light-sensitive layer at the back of the eye.

The light is then focused throughout the retina which contains nerves called **photoreceptors**. The photoreceptors are made up of rods and cones, and are responsible for transforming the light rays into electrical impulses. While the light is focused throughout the retina, most of the light entering the eye is focused onto the **focal point** on the retina, known as the **macula**.

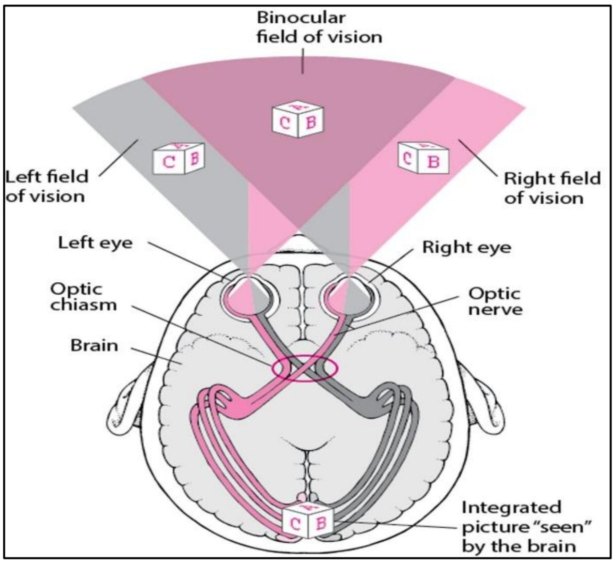
Step 5: ***The optic nerve transmits visual information to the brain***

The nerves of the retina collect all of the electrical impulses, which then travel through the **optic nerve** at the very back of the eye up to the occipital lobe in the back of the **brain**. At this point, the light then passes from the lens to the back of the eye which is filled with a clear, gelatinous substance called the **vitreous** until it reaches the **retina**, the light-sensitive layer at the back of the eye.

6.3. Tracing the visual pathways

Vision is dependent on the connections between the eyes and the brain. The light that enters the eye is required to go through a specific process in order to focus properly on the retina.

Nerve signals travel from each eye along the corresponding optic nerve and other nerve fibers (called the visual pathway) to the back of the brain, where vision is sensed and interpreted. The two optic nerves meet at the optic chiasm, which is an area behind the eyes immediately in front of the pituitary gland and just below the front portion of the brain (cerebrum). There, the optic nerve from each eye divides, and half of the nerve fibers from each side cross to the other side and continue to the back of the brain. Thus, the right side of the brain receives information through both optic nerves for the left field of vision, and the left side of the brain receives information through both optic nerves for the right field of vision. The middle of these fields of vision overlaps. It is seen by both eyes (called binocular vision).



An object is seen from slightly different angles by each eye so the information the brain receives from each eye is different, although it overlaps. The brain integrates the information to produce a complete picture.

6.4. Range of vision of human eye

The human eye is a very incredible instrument that allows us to see at infinite distances till the light from that object can reach our eyes. Thus the far range of the human eye is infinity. We see the stars in the night sky that are very far away from us and the light from them reaches our eyes and thus we see them.

For the near point of the eye, it is the point till which the human eye can see distinctly and the near point of the human eye is, 25 cm, i.e. any object till 25 cm can be distinctly viewed by the human eye.

6.5. Lens of eye

A lens placed behind the cornea of the human eye is called the eye lens. It is an optical lens made of proteins and other organic materials. It is situated exactly behind the Iris that allow light to pass through the lens and the eye lens then converges the light to the Retina of the eye.

It is ellipsoidal in shape and is roughly 10 mm long and 4 mm wide. It is made up of translucent protein molecule and thus allow light to pass through it.

6.6. Defects in eyes

If the connections between the eye and brain are not well developed, the visual information that is sent to the brain will not be interpreted properly, and the image will be difficult to see.

The process of seeing is dependent on the perfection of the eye and all of its components, including:

- Eyeball shape,
- Corneal shape and integrity,
- Lens clarity and curvature,
- Retinal health.

If any of these components do not function properly, or are irregularly shaped, vision problems can occur - most commonly, blurry vision will develop.

When this happens, corrective lenses in the form of eyeglasses or contact lenses are prescribed to help the light focus accurately onto the retina and enable clear vision.

There are various defects that develop in our eyes because of aging or carelessness. Some of them are mentioned below:

- Myopia,

- Hypermetropia,
- Presbyopia,
- Cataract,
- Glaucoma,
- Astigmatism.

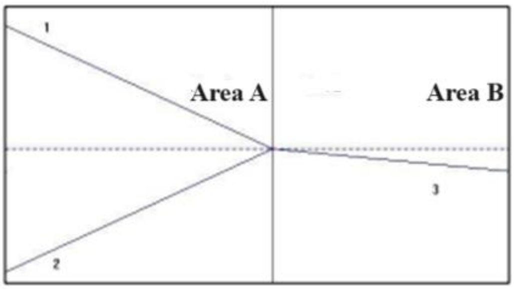
Exercises

Exercise 1:

1. A wave propagates in a vacuum at a speed of $3{,}10^8$ m/s. Is it a light wave?
2. A ray of light is sent from the air towards a piece of flint (flint is a type of optical glass made from lead). We give the refractive index of the flint n = 1.585 for radiation of wavelength λ = 486 nm. What happens to the following quantities: frequency and speed of the wave when light passes from air to flint (we compare air to vacuum)?

Exercise 2:

A slim luminous brush arrives on a flat diopter separating water from air. We give: n_{water} =1.33. We represent the rays observed in the figure below:



By justifying your answers:

1. Identify the different rays.
2. Indicate the direction of light propagation.
3. In what area is the water located?
4. Calculate the limiting angle of refraction.
5. Generalize the result by specifying the zone where the limiting angle is located according to the difference in refraction of the media present and the consequences on the propagation of light from one medium to another.

Exercise 3:

Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of 45.0°, and you observe the angle of refraction to be 40.3°.

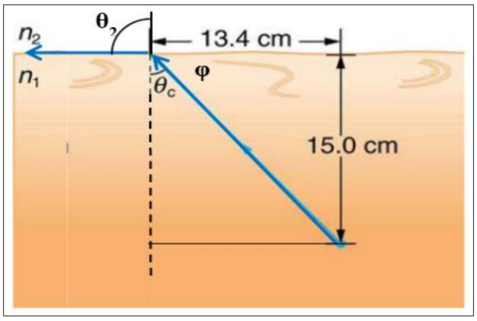
- What is the index of refraction of the substance and its likely identity?

The index of refraction of water is 1.33.

Exercise 4:

A ray of light, emitted beneath the surface of an unknown liquid with air above it, undergoes total internal reflection as shown in figure.

- What is the index of refraction for the liquid and its likely identification?



Exercise 5:

A fisherman sees a fish located 1 m below the surface of the water, on the same vertical. Considering that, his eyes are 1.40 m above the water.

1. At what distance does the fisherman see the fish?
2. How far from the fish's eye is the image of the fisherman?
3. At what depth must the fish be so that the image seen by the fisherman is shifted by 15 cm from its actual position?

We give the water index n=1.33.

Exercise 6:

A glass prism of index n = 1.6 and angle A = 30° is crossed by a monochromatic light ray. The incident ray falls on the prism at an angle i = 30°.

- Determine the angle of refraction r on the first face, the angle of incidence r' on the second face, the angle of emergence i' and the total deviation D created by this prism.

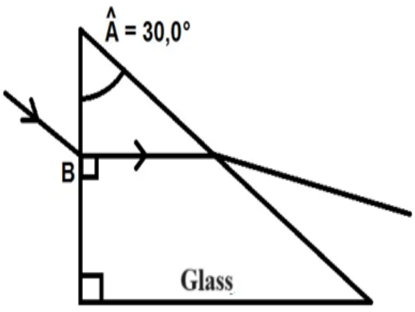
Exercise 7:

A prism of angle A and index n=1.5 is illuminated by an incident ray perpendicular to the entry face of the prism.

- Trace the path of the light ray and calculate the deviation D in the following two cases: A = 30° and A = 60°.

Exercise 8:

We consider a glass prism with glass index n_{Glass}= 1.50, having a right angle and an angle A of 30°. A ray arrives at point B on the prism, passes through it then exits according to the following diagram:



1. Calculate the angle of incidence at B.
2. Calculate the total deviation.

We now consider that angle A is unknown.

3. Calculate the limiting angle A, noted A_{lim}, corresponding to the total reflection

Exercise 9:

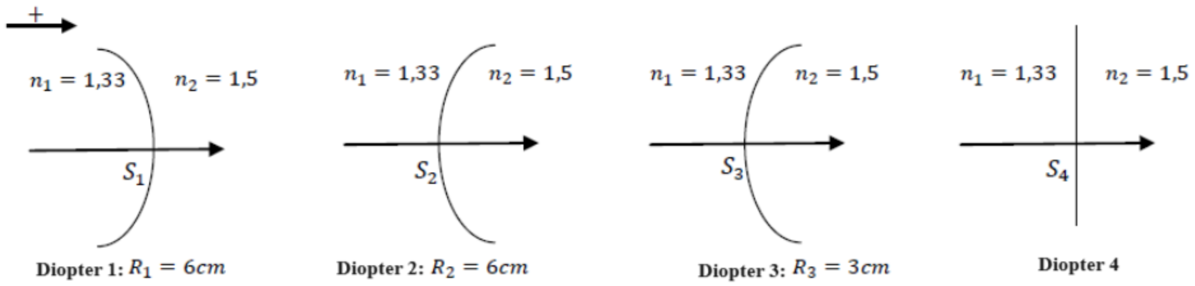
Consider a prism with a vertex angle of 30° and index n=1.5.
- Give the values of the angles of incidence, emergence and the angle of total deviation in the following cases:

1. Grazing incidence, normal incidence and minimum deviations.
2. Grazing emergence and normal emergence.

- Make a diagram corresponding to each case.

Exercise 10:

Calculate in diopters the vergence of the different spherical diopters below, deduce their



natures.

Exercise 11:

I. We have a thin converging lens with a focal length of 6 cm.

1. What is the position of the image of a real object placed 18 cm from the lens? What is the nature of the image (real or virtual)? Is it upright or upside down? Enlarged or reduced?
2. How much is growth worth?
3. Same questions for a real object placed 3 cm from the lens.
4. Same questions for a virtual object placed 12 cm from the lens.

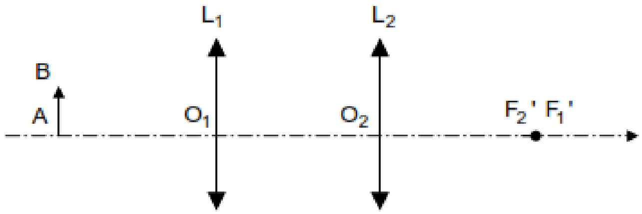
II. We have a thin divergent lens with a focal length of 6 cm.

1. What is the image position of a real object placed 12 cm from the lens? What is the nature of the image (real or virtual)? Is it upright or upside down? Enlarged or reduced?
2. How much is growth worth?
3. Same questions for a virtual object placed 3 cm from the lens.
4. Same questions for a virtual object placed 18 cm from the lens.

Exercise 12:

I. Association of two converging lenses:

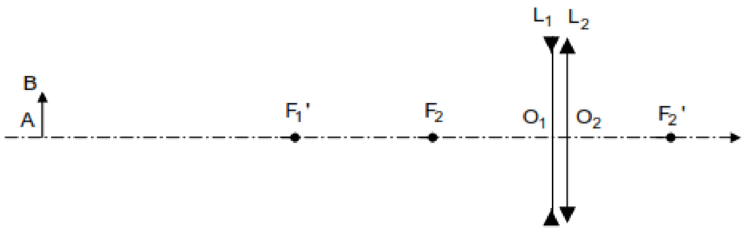
1. Construct geometrically the image A'B' of the object AB (of height l = 1 cm and placed 4 cm in front of O_1) through the optical system described in the figure below, where $f_1' = 2f_2' = 8$cm and $O_1O_2 = 4$cm.



2. Repeat the same approach but by calculation.
3. Express the growth of this montage.

II. Association of a divergent lens and a converging lens:

1. Construct geometrically the image A'B' of the object AB (placed 16 cm in front of O_1) through the optical system described in the figure, where $f_1' = -2f_2' = -8$cm and $O_1O_2 = 0$cm.



2. Repeat the same approach but by calculation.
3. Express the growth of this montage.

Exercise 13:

I. Consider a thin converging lens, with optical center O, with foci F and F'.
1. Recall the conjugation and magnification formulas with origin at the optical center.
2. Construct the image A'B' of an object AB perpendicular to the principal axis located between $-\infty$ and the object focus F.
3. Find growth formulas with origins in homes.
4. Deduce Newton's formula.
The small object AB moves from $-\infty$ to $+\infty$.
5. The object space can be broken down into 3 zones, construct the images corresponding to an object placed successively in each of these zones. Deduce the corresponding areas of the image space.
6. Indicate in each case the nature of the image.
II. Repeat this study in the case of a divergent lens.

Chapter-IV:

Concept of Crystallography

CHAPTER IV : *Concept of Crystallography*

Crystallography is the branch of science that studies the structure, properties, and formation of crystals. It involves analyzing the atomic and molecular arrangement in crystalline solids using techniques such as X-ray diffraction (XRD), electron diffraction, and neutron diffraction.

Crystallography is essential in various fields, including materials science, chemistry, physics, and biology, as it helps determine the internal structure of minerals, metals, semiconductors, and biomolecules like proteins and DNA.

1. Definitions

1.1. Definition of Crystal

A crystal is a solid material whose component atoms, ions, or molecules are arranged in a highly ordered, repeating pattern extending in all three spatial dimensions. This regular arrangement, called a crystal lattice, gives crystals their characteristic geometric shapes and distinct physical properties, such as cleavage, symmetry, anisotropy and reflects its internal symmetry.

Examples of crystals include quartz, diamond, salt (NaCl), and ice. Crystals are studied in crystallography and are widely used in electronics, optics, and materials science.

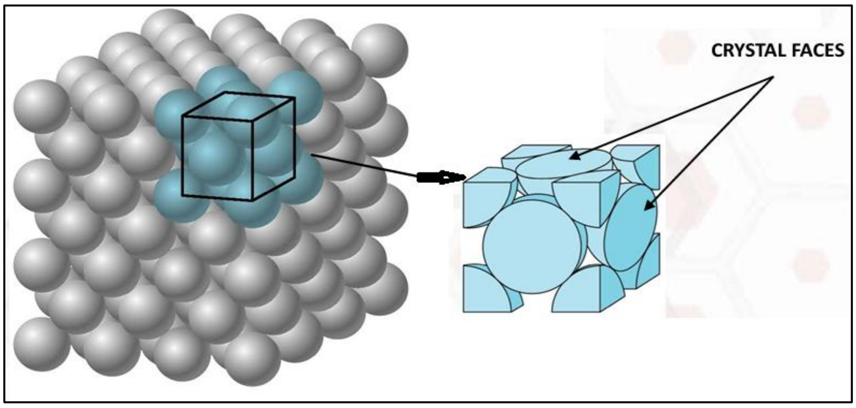


Figure IV.1: *Crystalline solid.*

1.2. Unit cell

The unit cell (or mesh) is the smallest repeating unit of a crystal lattice that, when repeated in three dimensions, forms the entire crystal. It defines the symmetry and structure of the crystal.

- Defined by lattice parameters (edge lengths a, b, c and angles α, β, γ).
- Can be primitive (smallest possible) or conventional (for better visualization of symmetry).

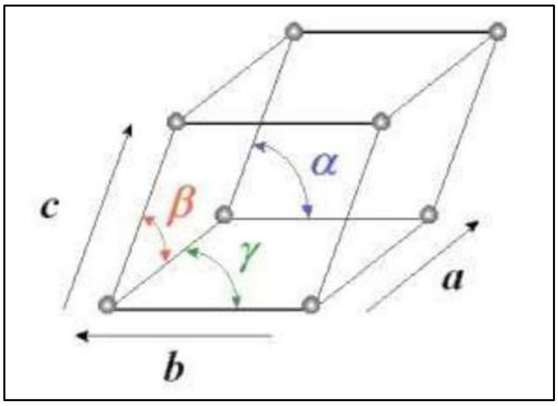


Figure IV.2: *Unit cell*.

According to Hauy, any crystal can be constructed by periodic translation in the three directions of space of an elementary parallelepiped unit called unit of repetition or elementary mesh.

This character of periodic repetition by translation is one of the important properties of crystals.

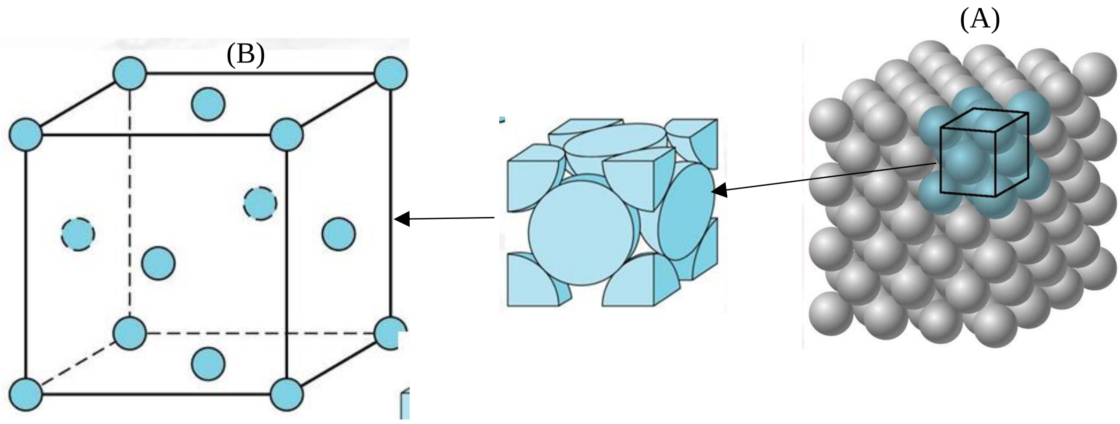


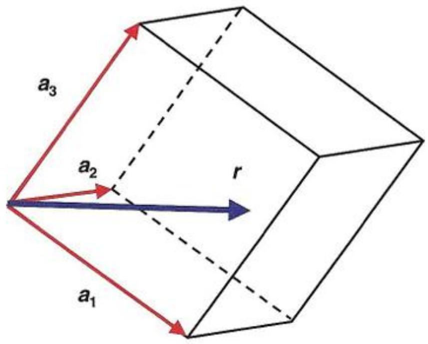
Figure IV.3: *(A) Atomic hard sphere model, (B) Lattice points*.

- In 3D space the unit cells are replicated by three noncoplanar translation vectors: $\overrightarrow{a_1}$, $\overrightarrow{a_2}$, $\overrightarrow{a_3}$ and the latter are typically used as the axes of coordinate system.

In this case the unit cell is a parallelepiped that is defined by length of vectors: $\overrightarrow{a_1}$, $\overrightarrow{a_2}$, $\overrightarrow{a_3}$ and angles between them.

The volume of the parallelepiped is given by the mixed scalar-vector product of translation vectors:

$$\mathbf{V} = \overrightarrow{\mathbf{a_1}}.(\overrightarrow{\mathbf{a_2}} \times \overrightarrow{\mathbf{a_3}})$$



Any point, r, within a unit cell is defined by three fractional coordinates, x, y, z:

$$\vec{r} = x\overrightarrow{a_1} + y\overrightarrow{a_2} + z\overrightarrow{a_3}$$

1.3. Motif (Basis)

The motif (or basis) is the group of atoms, ions, or molecules attached to each lattice point. It represents the actual atomic arrangement within the unit cell.

2. Crystalline network

The crystal lattice is generated by the translation of the mesh by the vectors of base. All nodes in the network are defined by this translation, in other words, The crystal lattice is a purely geometric concept. It is made up of the set of points, extremities of all translation vectors possible.

In a simpler way: ***Crystal structure = Network + Motif***.

3. Primitive and non-primitive unit cell

Concerning the unit cells, there are two types of unit cells

- Primitive Unit Cell
- Non-Primitive Unit Cell

3.1. Primitive Unit Cell

Primitive Unit Cell are those in which atoms are at corners of the unit cell. One such example is simple cubic.

- It has the simplest structure, with lattice points located at the corners of a cube.
- Atoms are only present at the corners.

3.2. Non-Primitive Unit Cell

In non-primitive unit cell, the atoms are present at corners as well as other parts of unit cell. There are three types of non-primitive unit cell

- Body Centered
- Face Centered
- Edge Centered

3.2.1. Body-Centered

In addition to the lattice points at the corners there is one additional lattice point at the body center. This structure is denser than the simple and is commonly found in some metals, such as iron and chromium.

3.2.2. Face-Centered

In addition to the lattice points at the corners of the unit cell, there is one lattice point at the center of each face of the unit cell

This structure is even denser than the body-centered and is found in many metallic elements, including aluminum, copper, and gold.

3.2.3. Edge Centered

In addition to the lattice point at the corners of the unit cell, the lattice points are present on the center of each edge

3.2.4. End Centered

In addition to lattice point at the corned of the unit cell, lattice points are present on the center of a pair of opposite face of the unit cell

4. Crystal Systems

The different parameters of unit cell generate different types of crystal structures called Crystal System. There are seven possible crystal with different configuration of unit cell parameters. The seven crystal systems are:

- Cubic,
- Tetragonal,
- Orthorhombic,
- Monoclinic,
- Triclinic,
- Rhombohedral
- Hexagonal

4.1. Cubic crystal system

In cubic crystals, all three crystallographic axes are of equal length and intersect each other at right angles (90 degrees). Examples include common substances like salt (sodium chloride) and diamonds.

4.2. Tetragonal crystal system

In tetragonal crystals, two crystallographic axes are of equal length, while the third axis is different and perpendicular to the other two. This creates a rectangular prism shape. Rutile is an example of a mineral with a tetragonal crystal structure.

4.3. Orthorhombic crystal system

Orthorhombic crystals have three axes of different lengths, all of which intersect at right angles. The resulting shape is a rectangular prism, but the sides are not necessarily equal in length. Sulfur and bismuth are examples of minerals with orthorhombic crystal structures.

4.4. Monoclinic crystal system

Monoclinic crystals have three axes of different lengths, two of which intersect at oblique angles (not 90 degrees), while the third axis is perpendicular to the other two. This creates a prism shape with one angle that is not a right angle. Gypsum is an example of a mineral with a monoclinic crystal structure.

4.5. Triclinic

In triclinic crystals, all three crystallographic axes are of different lengths and intersect at oblique angles (not 90 degrees). This results in a crystal shape that lacks any form of symmetry. An example of a mineral with a triclinic crystal structure is microcline.

4.6. Rhombohedral

Rhombohedral crystals have three axes of equal length, all of which intersect at oblique angles (not 90 degrees). The resulting shape is a rhombohedron, which is like a cube that has been distorted into a shape with no right angles. Calcite is an example of a mineral with a rhombohedral crystal structure.

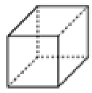
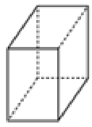
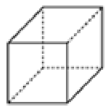
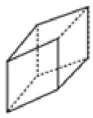
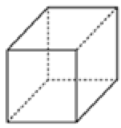
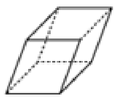
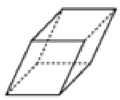
4.7. Hexagonal

Hexagonal crystals have four axes, three of which are of equal length and intersect at angles of 120 degrees, while the fourth axis is perpendicular to the other three. This creates a prism shape with a hexagonal base. Quartz and graphite are examples of minerals with hexagonal crystal structures.

These crystal systems help scientists classify and understand the structures of various minerals and materials based on their symmetry and atomic arrangement.

These seven crystal structure and their existence of various unit cell lead to fourteen possible lattice structure called ***Bravais Lattice***. A lattice is arrangement of points in space. When atoms are placed on these lattice points it forms crystal. The crystal system chart is tabulated below:

Table IV.1: Characteristics of the seventh crystal systems.

System	Axial length	Axial Angle	Unit Cell Geometry
Cubic	a = b = c	α = β = γ = 90°	
Tetragonal	a = b ≠ c	α = β = γ = 90°	
Orthorhombic	a ≠ b ≠ c	α = β = γ = 90°	
Rhombohedral	a = b = c	α = β = γ ≠ 90°	
Hexagonal	a = b ≠ c	α = β = 90°, γ=120°	
Monoclinic	a ≠ b ≠ c	α = γ = 90°, β≠90°	
Triclinic	a ≠ b ≠ c	α ≠ β ≠ γ	

5. Bravais Networks (Lattices)

Bravais networks, more commonly known as **Bravais lattices**, are fundamental concepts in crystallography and solid-state physics. They describe all the possible distinct, periodic arrangements of points (or atoms) in space that can be generated through translational symmetry.

- **Definition**

A Bravais lattice is an infinite array of discrete points generated by a set of discrete translation operations. Each point has an identical environment.

- **3D Bravais Lattices**

In three dimensions, there are **14 distinct Bravais lattices**, which are grouped into **7 crystal systems**:

- **Cubic:** Simple, Body-Centered, Face-Centered
- **Tetragonal:** Simple, Body-Centered
- **Orthorhombic:** Simple, Base-Centered, Body-Centered, Face-Centered
- **Hexagonal:** (includes the trigonal or rhombohedral system)
- **Monoclinic:** Simple, Base-Centered
- **Triclinic:** Simple

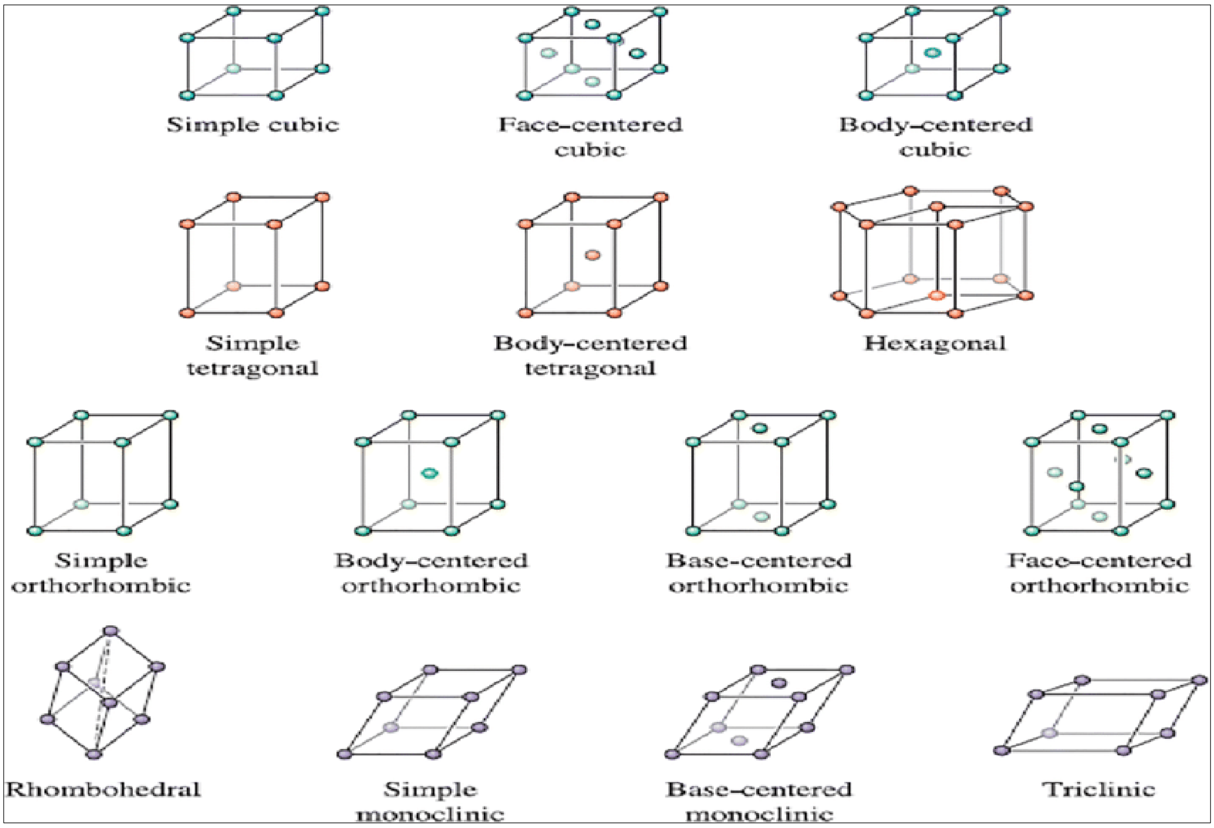


Figure IV.4: *The 14 Bravais networks*.

- **2D Bravais Lattices**

In two dimensions, there are **5 distinct Bravais lattices**:

- Oblique
- Rectangular
- Centered Rectangular

- Square
- Hexagonal

- **Importance**
- **Crystallography:** Understanding Bravais lattices is essential for describing the structure of crystals. They help determine the possible symmetry operations (like rotations and reflections) that a crystal can have.
- **Physical Properties:** The lattice type influences many properties of materials, such as their electronic structure, optical behavior, and mechanical strength.

***Note:** Different Bravais lattices have unique geometric arrangements which can be visualized by their unit cells and lattice points.*

6. Close packing in solids

Close packing in solids is like arranging a bunch of marbles as tightly as possible in a box. You try to stack them in layers so that they fill up the space efficiently. In close packing, the marbles are packed in such a way that they are as close together as possible, leaving very little empty space between them. This arrangement is important because it helps determine the properties of the material, such as its density and strength.

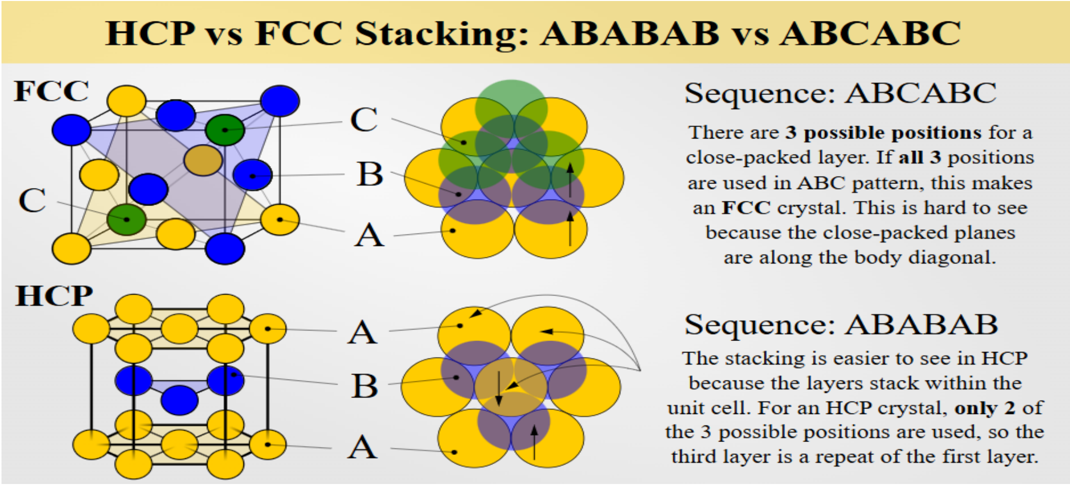
There are two common types of close packing: cubic close packing (CCP) and hexagonal close packing (HCP). Both types involve stacking layers of spheres in a way that minimizes empty space between them, resulting in a densely packed structure. Close packing is important in materials science because it influences the physical and mechanical properties of solid materials.

6.1. Types of Close Packing

There are two main types of close packing in solids:

6.1.1. Cubic Close Packing (CCP): It's like arranging marbles in layers where the overall 3D arrangement appears to be a cubic structure. Each marble touches six other marbles. Then, the layers are stacked so that the marbles in the second layer fit into the spaces between the marbles in the first layer. This creates a structure with marbles at the corners and centers of each face of a cube.

6.1.2. Hexagonal Close Packing (HCP): It's similar to CCP, but the arrangement of marbles in each layer forms a hexagon shape. The layers are stacked in a way that the marbles in the second layer fit into the spaces between the marbles in the first layer, following a repeating ABAB... stacking pattern. This creates a structure with a hexagonal shape.



7. Symmetry in crystals

Symmetry in crystals refers to the repetition of structural features in a crystal when subjected to specific symmetry operations. It describes how the atomic arrangement in a crystal remains unchanged under various transformations, such as rotations, reflections, and translations. Symmetry is a fundamental characteristic of crystals and plays a crucial role in determining their physical properties, such as optical behavior, cleavage, and anisotropy.

7.1. Symmetry elements

Haüy's method of building crystals from stacked parallelepipeds has been replaced in modern crystallography by three-dimensional lattices (Bravais lattices). The 32 crystallographic point groups combine the following symmetry elements:

A. *Axis of symmetry*: If a crystal has an axis of symmetry through its centre, such that the crystal can be rotated around the axis into a position where it appears identical to the starting position, then it has an axis of symmetry. A crystal may have zero, one, or multiple axes of symmetry but, by the crystallographic restriction theorem, the order of rotation may only be 2-fold, 3-fold, 4-fold, or 6-fold for each axis. An axis of symmetry is also known as a proper rotation.

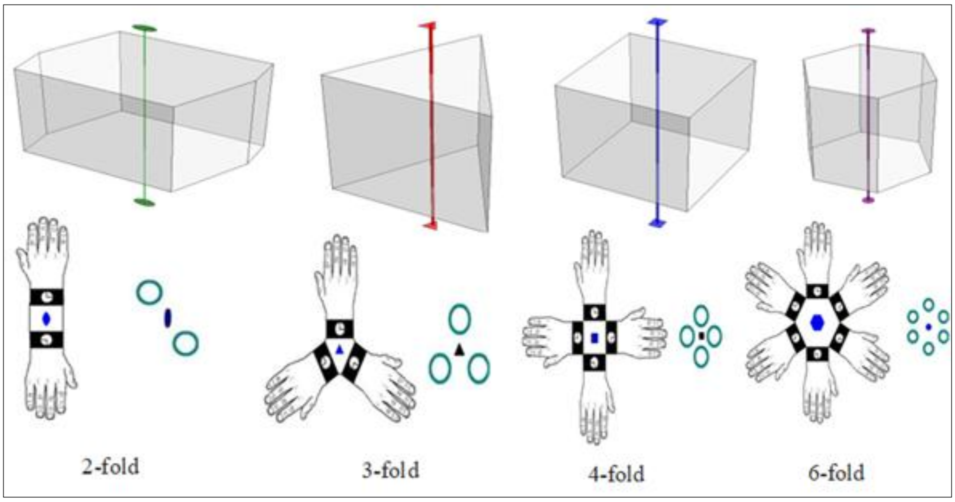


Figure IV.5: *Axis of symmetry.*

The Symbol of the axes rotation is **n** (e.g., 2, 3, 4, 6). An **n**-fold rotation axis will rotate the object by 360/n°.

B. ***Plane of symmetry*****:** If a crystal can be divided by a plane into two mirror-image halves, then the plane is a plane of symmetry. A crystal may have zero, one, or multiple planes of symmetry. For example, a cube has nine planes of symmetry. A plane of symmetry is also known as reflection symmetry or mirror symmetry.

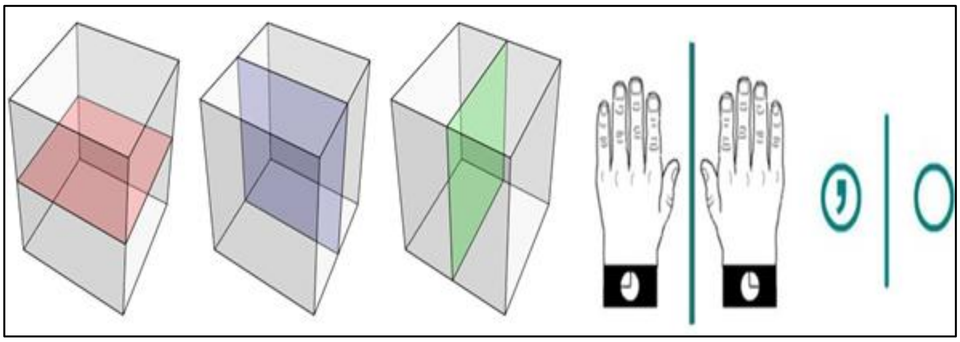
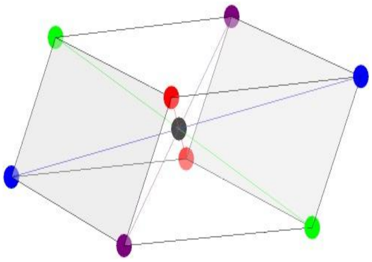


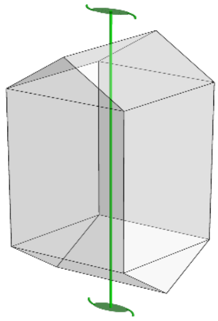
Figure IV.6: *Plane of symmetry*

A mirror plane changes the handedness of the object it is operating on. The ***mirror planes*** are represented by the letter **m**.

C. ***Centre of symmetry*****:** If every face of a crystal has another identical face at an equal distance from a central point, then this point is called the centre of symmetry symbolised as i. A crystal can only have one centre of symmetry. A centre of symmetry is also known as point reflection, inversion symmetry, or centrosymmetry.



D. *Rotoinversion symmetry*: A rotoinversion, symbolised as ($\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$ or $\bar{6}$), is a combination of a rotation about an axis and a reflection in a plane perpendicular to that axis. As an example, a two-fold rotoinversion ($\bar{2}$) is illustrated in the figure. Rotoinversion is also known as improper rotation, rotoreflection, or rotation-reflection.



7.2. Types of symmetry in crystals

In crystals, the symmetry axes (rotation axes) can only be ***two-fold*** (**2**), ***three-fold*** (**3**), ***four-fold*** (**4**) or ***six-fold*** (**6**), depending on the number of times (***order of rotation***) that a motif can be repeated by a rotation operation, being transformed into a new state indistinguishable from its starting state. Thus, a rotation axis of order **3** (3-fold) produces **3** repetitions (copies) of the motif, one every 120 degrees (= 360/**3**) of rotation. The main types of symmetry in crystals are:

A. Translational Symmetry

- The crystal structure is repeated at regular intervals in space.
- Defined by the **unit cell**, which, when repeated, forms the entire crystal lattice.

B. Rotational Symmetry

- The crystal appears unchanged after rotation by a specific angle about an axis.
- Possible **rotation axes**:
  - **2-fold (180°)**
  - **3-fold (120°)**
  - **4-fold (90°)**
  - **6-fold (60°)**
- Example: A cubic crystal remains unchanged after a 90° rotation.

C. Reflection Symmetry (Mirror Planes)

- The crystal structure is mirrored across a **plane of symmetry**.
- Example: Salt (NaCl) has mirror symmetry along certain planes.

D. Inversion Symmetry

- Every point at (x, y, z) has an equivalent point at (-x, -y, -z).
- Example: Some center-symmetric structures like diamond exhibit this property.

E. Glide and Screw Symmetry

- **Glide Plane**: A combination of reflection and translation.
- **Screw Axis**: A combination of rotation and translation.

The ***glide planes*** (mirror planes involving reflexion and a translation parallel to the plane) are represented by the letters **a**, **b**, **c**, **n** or **d**, depending if the translation associated with the reflection is parallel to the reticular translations (***a***, ***b***, ***c***), parallel to the diagonal of a reticular plane (**n**), or parallel to a diagonal of the unit cell (**d**).

The ***screw axes*** (or helicoidal axes, ie, symmetry axes involving rotation followed by a translation along the axis) are represented by the order of rotation, with an added subindex that quantifies the translation along the axis. Thus, a screw axis of type $\mathbf{6_2}$ means that in each of the six rotations an associated translation occurs of **2**/**6** of the axis of the elementary cell in that direction.

The letters and numbers that are used to represent the symmetry elements also have an equivalence with some **graphic symbols**.

Exercises:

Exercise 1:

1. Discuss in detail how a crystal structure is built from a lattice (the network of points) and a motif (the basis).
2. In your answer, explain the roles of lattice parameters (edge lengths and angles) and describe how variations in these parameters could influence the physical properties of the crystal.
3. Provide examples where appropriate.

Exercise 2:

1. Explain the differences between primitive and non-primitive unit cells.
2. List and describe the types of non-primitive unit cells (body-centered, face-centered, edge-centered, and end-centered) and give examples of materials or metals where each type might be found.

Exercise 3:

1. Identify the seven crystal systems and explain how these systems lead to the 14 distinct Bravais lattices in three dimensions.
2. Discuss one key geometric feature of each crystal system that distinguishes it from the others.

Exercise 4:

List the five distinct Bravais lattices found in two dimensions. For each 2D lattice type (oblique, rectangular, centered rectangular, square, and hexagonal), provide a brief description of its geometric arrangement.

Exercise 5:

1. Compare and contrast cubic close packing (CCP) and hexagonal close packing (HCP).
2. Describe the stacking patterns for each type and discuss how these arrangements affect the density and properties of the solid.

Exercise 6:

True or False: "A primitive unit cell contains atoms only at its corners, while a non-primitive unit cell includes additional atoms located at positions such as the center, edges, or faces." Justify your answer with reference to the definitions provided in the chapter.

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