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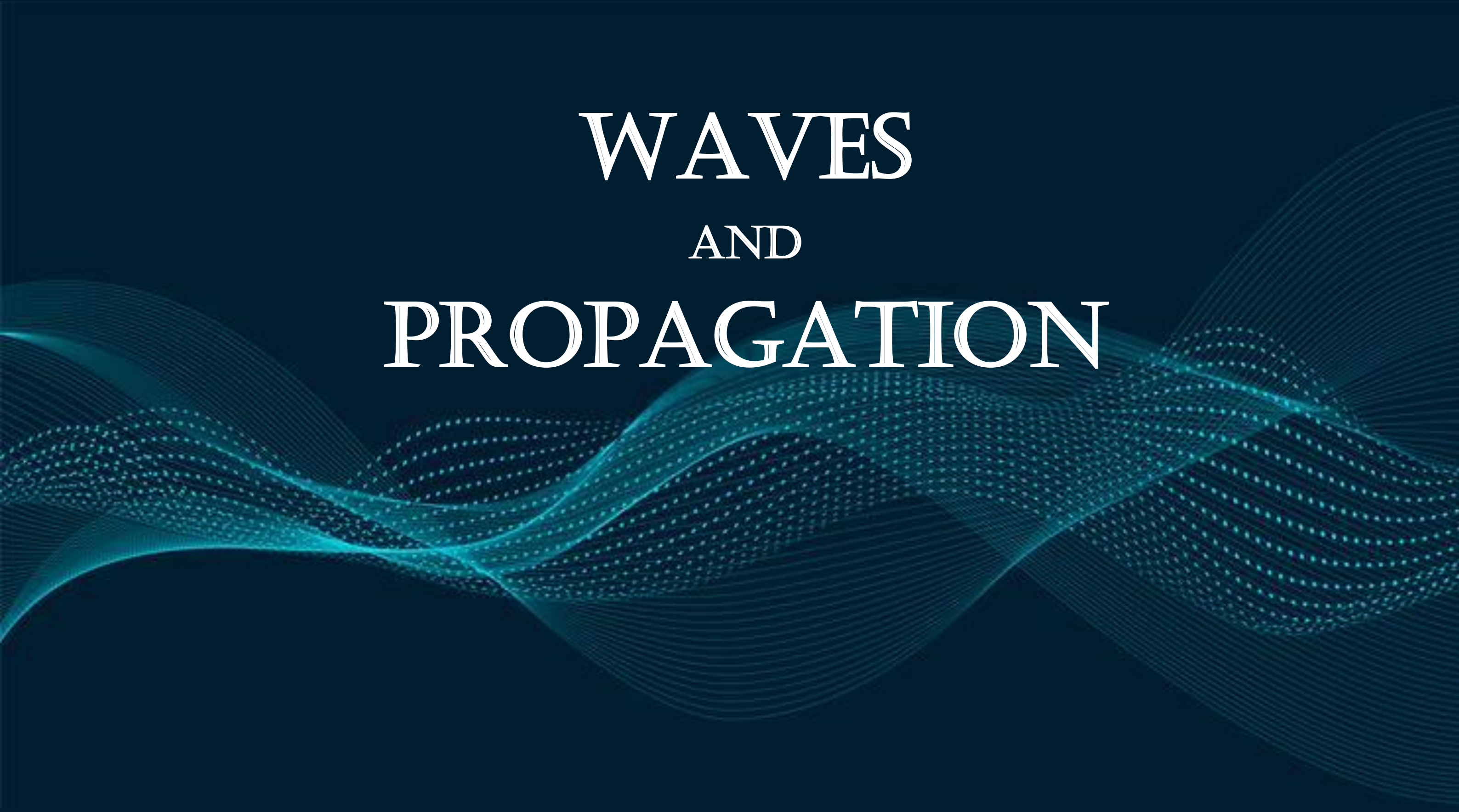

WAVES
AND
PROPAGATION

Course notes
3rd Year Telecommunications

by Dr. Djamel Sayad

September 2023

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Foreword

The study of EM wave propagation is at the heart of wireless transmission, radar systems, broadcasting or satellite communications, mastering the phenomena related to wave propagation is crucial for telecommunication engineers.

This course, intended for third-year students in "**Telecommunication**", aims to provide a solid foundation in the fundamental principles of EM wave propagation. It provides an educational support allowing students to become familiar with the general concepts of wave propagation.

We have taken care to offer a structured, clear and detailed content, and chosen the most intuitive approach possible by using numerous and colored illustrations. We also favored detailed mathematical demonstrations where many intermediate steps are deliberately kept to facilitate understanding by avoiding stumbling over secondary mathematical difficulties.

This course material is structured around six chapters :
Chapter 1 contains a brief reminder of vector analysis, where the main mathematical tools, such as the notion of complex vector field, differential operators, and complex numbers, are introduced in order to facilitate the mathematical treatment in all the following chapters.

Chapter 2 presents a brief description of Maxwell's equations, which describe how electric and magnetic fields behave and interact.

Chapter 3 treats EM waves in perfect dielectric media and vacuum. The wave equation is derived from Maxwell's equations. The characterization of the uniform plane wave, with its different types of polarization, as well as the power density via the Poynting vector, is detailed.

Chapter 4 deals with wave propagation in conductive and dissipative media. The concepts of the skin effect, which illustrates the limited penetration of waves in conductors, as well as the losses in these media, are studied.

Chapter 5 delves in the behavior of EM waves during reflection and refraction at the boundary between two dielectric media. The study of waves incident on a separation surface, for both perpendicular and parallel polarizations, is discussed based on Snell-Descartes' laws.

Chapter 6 is devoted to the propagation of radio waves in the different atmosphere layers. The effects of atmospheric refraction and reflection of waves on the ground are analyzed, as well as the propagation modes according to the frequency bands.

At the end of this course, we hope that the students will have acquired a solid knowledge of electromagnetic wave propagation phenomena allowing them to both solve various practical problems and to explore the subject further on their own if they so desire.

Dr. D. Sayad
September 2023

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Chapter 1

Vector Analysis

I. Scalars and vectors

The term ***scalar*** refers to a quantity whose value may be represented by a single (positive or negative) real number. *Temperature,* mass, density, pressure, volume, …

A ***vector*** quantity has both a *magnitude* and a *direction* in space. Force, velocity, acceleration, displacement … are examples of vectors. Each quantity is characterized by both a magnitude and a direction.

A **vector** is defined by:

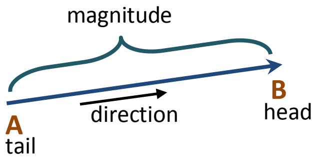


Fig.1. 1. Concept of a vector

- **support**: is the line to which the line segment AB belongs.
- **direction**: the direction of the displacement of a mobile from A to B
- **modulus** or **magnitude**: the length of the line segment AB and it is denoted by AB or $\|\overrightarrow{AB}\|$ which is always positive.

A vector is represented in an orthonormal Oxyz coordinate system (three orthogonal axes) by its three components A_x , A_y , A_z on the three axes. We write:

$$\vec{A} = (A_x , A_y , A_z) \ , \quad \text{or } \vec{A} = A_x\vec{u}_x + A_y\,\vec{u}_y + A_z\vec{u}_z \text{ , or } \vec{A} = |A|\vec{u}$$

with $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$: magnitude, $\vec{u}$: unit vector in the vector direction $\vec{A}$.

$$\vec{u} = \frac{\vec{A}}{|A|} = \frac{A_x\vec{u}_x + A_y\,\vec{u}_y + A_z\vec{u}_z}{\sqrt{A_x^2+A_y^2+A_z^2}} \text{ : is a vector having unit magnitude}$$

1. Equality and sum of two vectors

Equality : Two vectors $\vec{A} = A_x\vec{u}_x + A_y\,\vec{u}_y + A_z\vec{u}_z$ and $\vec{B} = B_x\vec{u}_x + B_y\,\vec{u}_y + B_z\vec{u}_z$ are equal if their components are equal one to one; i.e.

$$A_x = B_x,$$
$$A_y = B_y$$
$$A_z = B_z$$

Sum :

$$\vec{C} = \vec{A} + \vec{B} \Rightarrow \begin{cases} C_x = A_x + B_x \\ C_y = A_y + B_y \\ C_z = A_z + B_z \end{cases} \qquad (1.1)$$

2. Scalar product (dot product)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is the scalar defined by the product of magnitudes and the cosine of the angle between them :

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}|\cos\theta. \qquad (1.2)$$

The dot product of two perpendicular (θ=90°) vectors is zero.

The dot product can also be expressed in terms of the vectors' components:

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (1.3)$$

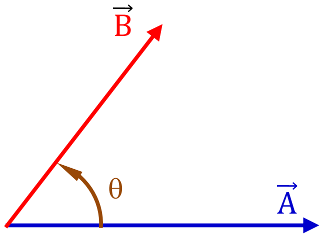


Fig.1. 2. Scalar product

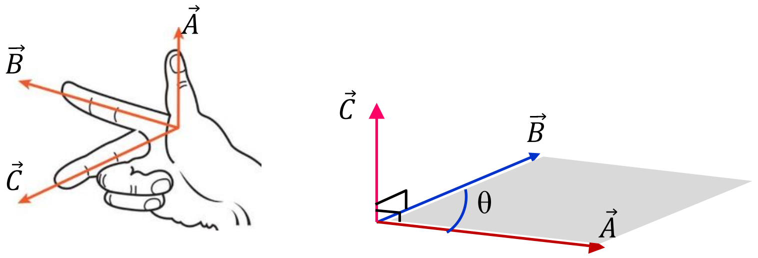
3. Vector product (cross product)

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is the **vector** denoted by $\vec{C} = \vec{A} \wedge \vec{B}$ whose magnitude is the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ and its direction is determined by the right hand rule (or screwdriver rule) as $\vec{A}$ is turned into $\vec{B}$.

$$\vec{C} = |\vec{A}| \cdot |\vec{B}|\sin(\vec{A}, \vec{B})\,\vec{u} = |\vec{A}| \cdot |\vec{B}|\sin(\theta)\,\vec{u} \qquad (1.4)$$

$\vec{C}$ is oriented such that $(\vec{A}, \vec{B}, \vec{C})$ form a right trihedron (right handed system).

the vector $\vec{C}$ is orthogonal (perpendicular) to the plane of $\vec{A}$ and $\vec{B}$ (i.e. orthogonal to of $\vec{A}$ and orthogonal to $\vec{B}$. ($\vec{u} \perp \vec{A}$ et $\vec{u} \perp \vec{B}$).



(a) (b)

Fig.1. 3. Vector product. (a) : Right hand rule for vector product, (b) : Norm of the vector product

$$\vec{C} = \vec{A} \wedge \vec{B} = \begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_x \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$= \vec{u}_x(A_yB_z - A_zB_y) - \vec{u}_y(A_xB_z - A_zB_x) + \vec{u}_z(A_xB_y - A_yB_x) \quad (1.5)$$

Vector product properties

- The cross product of two parallel vectors is zero (same direction θ=0).
- The cross product is anticommutative (antisymmetric):

$$\vec{A} \wedge \vec{B} = -\vec{B} \wedge \vec{A} \quad (1.6)$$

Table 1.1 summarizes some properties of the cross and dot products.

Table 1.1. Properties of the cross and dot products.

	Scalar product	**Vector product**
Notation	$\vec{A} \cdot \vec{B}$	$\vec{A} \wedge \vec{B}$
Nature	Scalar	Vector
Value	$\vec{A} \cdot \vec{B}$ $= \lvert\vec{A}\rvert \cdot \lvert\vec{B}\rvert \cos(\vec{A}, \vec{B})$	$\lvert\vec{A} \wedge \vec{B}\rvert = \lvert\vec{A}\rvert \cdot \lvert\vec{B}\rvert \sin(\vec{A}, \vec{B})$
Commutativity	$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$	$\vec{A} \wedge \vec{B} = -\vec{B} \wedge \vec{A}$

Distributivity	$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$	$\vec{A}\wedge(\vec{B}+\vec{C}) = \vec{A}\wedge\vec{B}+\vec{A}\wedge\vec{C}$ But : $\vec{A}\wedge(\vec{B}\wedge\vec{C}) \neq (\vec{A}\wedge\vec{B})\wedge\vec{C}$
Same vector product	$\vec{A}\cdot\vec{A} = \lvert\vec{A}\rvert^2$	$\vec{A}\wedge\vec{A} = 0$
Zero product	$\vec{A}\cdot\vec{B} = 0$ if and only if the 2 vectors are orthogonal ($\perp$) or at least one of the two vectors is zero.	$\vec{A}\wedge\vec{B} = 0$ if and only if the 2 vectors are collinear ($//$) or one of the vectors is zero
Value in cartesian coordinates	$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$	$\vec{A}\wedge\vec{B} = \begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$ $= \vec{u}_x(A_yB_z - A_zB_y) - \vec{u}_y(A_xB_z - A_zB_x) + \vec{u}_z(A_xB_y - A_yB_x)$ $= (A_yB_z - A_zB_y,\ A_zB_x - A_xB_z,\ A_xB_y - A_yB_x)$
Geometrical definition	The scalar product represents the projection of a vector onto the direction defined by the other vector	The norm of the vector product represents the area of the parallelogram formed by the two vectors

II.Vector field

A vector field is a vector function of the spatial coordinates x, y, z (and, perhaps also, time).

Examples:

$$\vec{A}(\vec{r}) = (y+2z)\vec{u}_x + 3xy\,\vec{u}_y + \vec{u}_z, \qquad (1.7.a)$$

$$\vec{B}(\vec{r},t) = (2y-2xz)\vec{u}_x + ty\,\vec{u}_y + 3t\,\vec{u}_z \qquad (1.7.b)$$

$$\vec{E}(\vec{r},t) = 2\cos(5x)\sin(\omega t)\,\vec{u}_y \qquad (1.7.c)$$

1. Circulation of a vector field

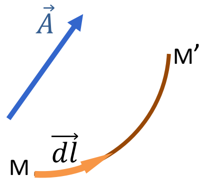


Fig.1. 4. Circulation of a vector field

Consider a vector field and an elementary displacement $\overrightarrow{MM'} = \overrightarrow{dl}$.

Elementary circulation :

$$dC = \vec{A} \cdot d\vec{l} \quad \text{(scalar)} \qquad (1.8)$$

and the total circulation of $\vec{A}$ around L is :

$$C_{MM'} = \int_{M}^{M'} \vec{A} \cdot \overrightarrow{dl} \quad \text{(line integral)} \qquad (1.9)$$

example : If field $\vec{A}$ is a force field, the circulation is the work of this force.

$$W = \int_{M}^{M'} \vec{F}\overrightarrow{dl} \qquad (1.10)$$

2. Flux of a vector field

The elementary flux $d\varphi$ of a vector field through the elementary surface $d\vec{s}$ is the scalar :

$$d\varphi = \vec{A} \cdot d\vec{s} = A.ds.\cos\alpha$$

the total flux of vector $\vec{A}$ through the finite surface S equals :

$$\Phi = \iint_S \vec{A} \cdot \overrightarrow{ds} \quad \text{(surface integral)} \qquad (1.11)$$

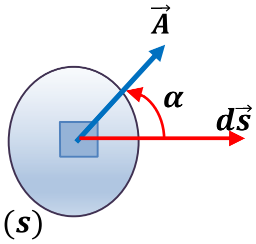


Fig.1. 5. Flux of a vector field

III. Differential operators

Most physical phenomena are described by differential equations that involve differential operators. We often encounter gradient, curl, divergence and Laplacian. A differential operator is an operator defined as a function of the differentiation operator, it transforms a function into another function.

1. Del operator

The differential operator **del,** also called **nabla,** is an important vector differential operator, **del** is defined as the derivative with respect to the three spatial dimensions :

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{u}_x + \frac{\partial}{\partial y}\vec{u}_y + \frac{\partial}{\partial z}\vec{u}_z \qquad (1.12)$$

2. Laplacian

The **Laplacian** is a definition of the second derivative with respect to the three spatial dimensions :

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad (1.13)$$

Gradient :

The gradient of a scalar field ***U(x,y,z)*** is a **vector** that points in the direction in which the scalar field *U(x,y,z)* is most rapidly increasing, with the scalar part equal to the rate of change.

It is a **vector** associated with a scalar function *U(x,y,z):*

- In Cartesian coordinates (x, y, z):

$$\overrightarrow{\mathrm{grad}}(U) = \vec{\nabla} \cdot U = \frac{\partial U}{\partial x}\vec{u}_x + \frac{\partial U}{\partial y}\vec{u}_y + \frac{\partial U}{\partial z}\vec{u}_z \qquad (1.14)$$

- In cylindrical coordinates (ρ, θ, z):

$$\overrightarrow{grad}(U) = \frac{\partial U}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho} \cdot \frac{\partial U}{\partial \theta}\vec{u}_\theta + \frac{\partial U}{\partial z}\vec{u}_z \qquad (1.15)$$

- In spherical coordinates (r, θ, φ):

$$\overrightarrow{grad}(U) = \frac{\partial U}{\partial r}\vec{u}_r + \frac{1}{r} \cdot \frac{\partial U}{\partial \theta}\vec{u}_\theta + \frac{1}{r \cdot \sin(\varphi)} \cdot \frac{\partial U}{\partial \varphi}\vec{u}_\varphi \qquad (1.16)$$

Divergence

The divergence computes a **scalar** quantity from a vector field by differentiation

It is a **scalar** associated with a vector field *U(x,y,z):*

- In Cartesian coordinates (x, y, z):

$$\mathrm{div}(\vec{A}) = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad (1.17)$$

- In cylindrical coordinates (ρ, θ, z):

$$div(\vec{A}) = \frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \qquad (1.18)$$

- In spherical coordinates (r, θ, φ):

$$div(\vec{A}) = \frac{1}{r^2} \cdot \frac{\partial(r^2 \cdot A_r)}{\partial r} + \frac{1}{r \cdot \sin(\theta)} \cdot \left[\frac{\partial(A_\theta \cdot \sin(\theta))}{\partial \theta} + \frac{\partial A_\varphi}{\partial \varphi}\right] \tag{1.19}$$

Curl (Rotation)

It is a **vector** associated with a **vector** field denoted by $\overrightarrow{\mathbf{curl}}$ or $\overrightarrow{\mathbf{rot}}$

- In Cartesian coordinates (x, y, z):

$$\begin{aligned} \overrightarrow{\mathrm{rot}}(\vec{A}) &= \vec{\nabla} \times \vec{A} \\ &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \cdot \vec{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \cdot \vec{u}_y \\ &+ \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \cdot \vec{u}_z \end{aligned} \tag{1.20}$$

- In cylindrical coordinates (ρ, θ, z):

$$\begin{aligned} \overrightarrow{\mathrm{rot}}(\vec{A}) = \left(\frac{1}{\rho} \cdot \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right) \cdot \vec{u}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) \cdot \vec{u}_\theta \\ + \frac{1}{\rho}\left(\frac{\partial \rho A_\theta}{\partial \rho} - \frac{\partial A_\rho}{\partial \theta}\right) \cdot \vec{u}_z \end{aligned} \tag{1.21}$$

- In spherical coordinates (r, θ, φ):

$$\begin{aligned} \overrightarrow{\mathrm{rot}}(\vec{A}) = \frac{1}{r \cdot \sin(\theta)} \cdot \left(\frac{\partial\big(A_\varphi \cdot \sin(\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right) \cdot \vec{u}_r \\ + \frac{1}{r}\left(\frac{1}{\sin(\theta)} \cdot \frac{\partial A_r}{\partial \varphi} - \frac{\partial\big(r A_\varphi\big)}{\partial r}\right) \cdot \vec{u}_\theta + \frac{1}{r}\left(\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right) \cdot \vec{u}_\varphi \end{aligned} \tag{1.22}$$

Scalar Laplacien

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \nabla^2 V \tag{1.23}$$

Vector Laplacien

$$\Delta \vec{A} = \nabla^2 \vec{A} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(A_x \vec{u}_x + A_y \vec{u}_y + A_z \vec{u}_z\right) \tag{1.24}$$

$$\Delta \vec{A} = (\Delta A_x)\,\vec{u}_x + (\Delta A_y)\,\vec{u}_y + (\Delta A_z)\,\vec{u}_z \tag{1.25}$$

3. Identities (Relations between operators)

$$div\left(\overrightarrow{rot}\vec{W}\right) = 0$$

$$\overrightarrow{rot}\left(\overrightarrow{grad}U\right) = \vec{0}$$

$$\Delta U = div\left(\overrightarrow{grad}U\right)$$

$$\Delta\vec{W} = \overrightarrow{grad}\left(div\vec{W}\right) - \overrightarrow{rot}\left(\overrightarrow{rot}\vec{W}\right)$$

$$\overrightarrow{grad}(U.V) = U.\overrightarrow{grad}(V) + V.\overrightarrow{grad}(U)$$

$$div\left(U.\vec{W}\right) = \vec{W}.\overrightarrow{grad}(U) + U.div\left(\vec{W}\right)$$

$$div\left(U.\vec{W}\right) = \vec{W}.\overrightarrow{grad}(U) + U.div\left(\vec{W}\right)$$

$$\overrightarrow{rot}\left(U.\vec{W}\right) = U.\overrightarrow{rot}\left(\vec{W}\right) + \overrightarrow{grad}(U) \times \left(\vec{W}\right)$$

$$div\left(\vec{W} \times \vec{V}\right) = \vec{V}.\overrightarrow{rot}\left(\vec{W}\right) - \vec{W}.\overrightarrow{rot}\left(\vec{V}\right)$$

Example

Find the gradient of the scalar field : $V = \rho^2 z \cos(2\theta)$

Solution

$$\begin{aligned}\overrightarrow{grad}(V) &= \frac{\partial V}{\partial \rho}\vec{u}_\rho + \frac{1}{\rho}\cdot\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{\partial V}{\partial z}\vec{u}_z \\ &= 2\rho z\cos(2\theta)\,\vec{u}_\rho + \frac{1}{\rho}\cdot\rho^2 z\,(-2)\sin(2\theta)\,\vec{u}_\theta \\ &\qquad + \rho^2\cos(2\theta)\,\vec{u}_z \\ &= 2\rho z\cos(2\theta)\,\vec{u}_\rho + \rho z\,(-2)\sin(2\theta)\,\vec{u}_\theta \\ &\qquad + \rho^2\cos(2\theta)\,\vec{u}_z \\ &\boxed{= 2\rho z\cos(2\theta)\,\vec{u}_\rho - 2\rho z\sin(2\theta)\,\vec{u}_\theta + \rho^2\cos(2\theta)\,\vec{u}_z}\end{aligned}$$

IV. Fundamental theorems

1. Ostrogradsky theorem (Divergence theorem)

The flux of a vector $\vec{B}$ through a closed surface S is equal to the volume integral of the divergence of $\vec{B}$ over the volume V bounded by the surface S.

$$\oiint_S \vec{B}\cdot\overrightarrow{ds} = \iiint_V div\vec{B}\cdot dv \qquad (1.26)$$

2. Stokes theorem (curl theorem)

The circulation of a vector field $\vec{A}$ on a **closed contour (C)** is equal to the flux of the curl of $\vec{A}$ through any **open surface (S)** resting on this contour.

The idea is to transform a double (surface) integral on an open surface (S) into a simple (line) integral on the contour (C) on which (S) rests.

NB. The direction of the contour (C) and the orientation of the elementary surface are determined according to the right-hand rule.

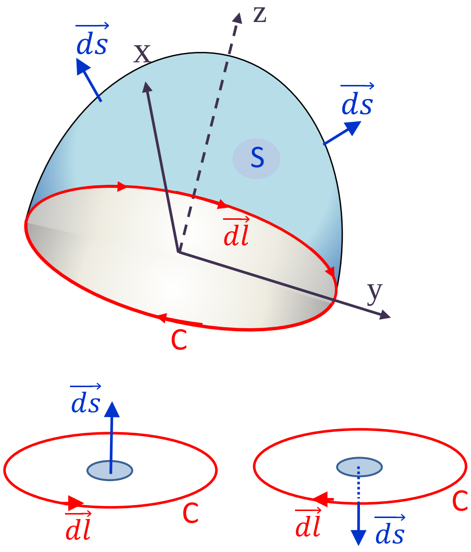


Fig.1. 6. Illustration of Stokes theorem

$$\iint_S \overrightarrow{rot}(\vec{A}) \cdot \overrightarrow{ds} = \int_c \vec{A} \cdot \overrightarrow{dl} \qquad (1.27)$$

V. Complex numbers

Complex numbers are expressions of the form a + jb, where a and b are real numbers, and $j^2 = -1$.

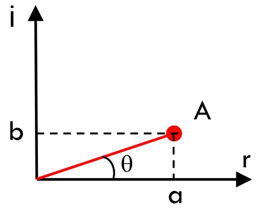


Fig.1. 7. Representation of a complex number

1. Algebraic form: *A=a+jb*

- Modulus (magnitude):

$$|A| = \sqrt{a^2 + b^2}, \qquad (1.28)$$

|A| of a complex number A= a+ jb is its distance to the origin.

- Argument (phase) : $\theta = arctg\ (\frac{b}{a})$ (1.29.a)

$$\theta = \arcsin\left(\frac{b}{\sqrt{a^2+b^2}}\right) \qquad (1.29.b)$$

or $$\theta = \arccos\left(\frac{a}{\sqrt{a^2+b^2}}\right) \qquad (1.29.c)$$

2. Polar form (Exponential form) :

$$A = |A|\angle\theta \quad \text{or } A = |A|e^{j\theta} \qquad (1.30)$$

3. Trigonometric form :

$$A = |A|(\cos\theta + j\sin\theta) \qquad (1.31)$$

4. Operations :

Product :

$$A \cdot B = |A||B|e^{j(\theta_A+\theta_B)},$$

$$A \cdot A = |A|^2 e^{2j\theta} \qquad (1.32)$$

Division :

$$A/_B = \frac{|A|}{|B|} e^{j(\theta_A-\theta_B)} \qquad (1.33)$$

Conjugate :

$$\left.\begin{array}{l} A = a + jb \rightarrow \ A^* = a - jb \\ A = |A|e^{j\theta} \rightarrow \ A^* = |A|e^{-j\theta} \end{array}\right\}$$

$$\Rightarrow AA^* = |A|^2 \rightarrow |A| = \sqrt{AA^*} \qquad (1.34)$$

Root :

$$\sqrt[n]{A} = \sqrt[n]{|A|}\ e^{j\theta/n} \qquad (1.35)$$

5. Moivre formulas:

$$(\cos\theta \pm j\sin\theta)^n = \cos n\theta \pm j\sin n\theta \qquad (1.36)$$

6. Euler Formulas :

$$\cos\theta = \frac{e^{j\theta} + \ e^{-j\theta}}{2} \qquad (1.37.a)$$

$$\sin\theta = \frac{e^{j\theta} - \ e^{-j\theta}}{2j} \qquad (1.37.b)$$

VI.Conclusion

In this chapter, we provided some fundamental mathematical tools needed to understand physical phenomena related to electromagnetic theory. Scalar, vector and vector fields concepts, as well as the operations associated with them, constitutes an essential basis for modeling complex electromagnetic phenomena. The study of differential operators, and fundamental theorems, such as Stokes and Ostrogradsky, allows us to choose adequate integral forms in the analysis of electromagnetic problems. Complex numbers and their related mathematical operations are also highlighted due to their crucial application in electromagnetic mathematics. The content of this chapter constitutes a requisite foundation for addressing topics treated along the following chapters.

Chapter 2

Maxwell’s Equations

I. Introduction

In **1865**, J. C. Maxwell published a theory unifying electric, magnetic and light phenomena. This revolutionary theory unified the laws of electricity and magnetism into a coherent set of four equations constituting the theoretical basis for the propagation of electromagnetic waves. However, it is also true that these equations were first derived experimentally by Coulomb, Ampere, Faraday, and others. Maxwell added one term in Ampere's law that connected all four equations. He predicted, the existence of electromagnetic waves propagating at the speed of light.

This theory was experimentally confirmed by Heinrich Hertz in **1887**, who managed to generate and detect these waves. Radioelectricity was born and opened the way to a multitude of applications that revolutionized communications. In **1898, G. Marconi** carried out a demonstration between the **Isle of Wight** and a boat. He then obtained a 46 km connection between **Wimereux** and the English coast in 1899.

Electromagnetics has had a great impact in science and technology. Any engineering problem that includes electromagnetic fields is solved starting from these equations. Maxwell's equations enable us to solve many practical engineering problems that deal with electromagnetic fields. This includes :

- electrical engineering,
- optics,
- wireless and optical communications,
- computers,
- remote sensing,
- bio-medical engineering,
- etc.

II. Maxwell's equations

Maxwell's equations are the basic equations of electromagnetism. Although they cannot be demonstrated, they are fully justified by experience. They are obtained by seeking a coherent modification of the equations of electrostatics and magnetostatics taking into account induction phenomena and compatibility with the current continuity equation, i.e. of the conservation of the electric charge:

$$div(\vec{J}) + \frac{\partial \rho}{\partial t} = 0 \qquad (2.1)$$

1. Maxwell-Faraday equation

This equation describes how a time-varying magnetic field can create an electric field. For example, a rotating magnet creates a changing magnetic field which generates an electric field

$$\overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \qquad (2.2)$$

2. Maxwell-Gauss equation

This local equation describes how an electric field E is generated by electric charges:

$$div(\vec{E}) = \frac{\rho}{\varepsilon_0} \qquad (2.3)$$

with :

ρ : electric charge density,

$\varepsilon = \varepsilon_r \varepsilon_0$: electrical permittivity of the medium [F/m]

ε_0: vacuum (free space) electrical permittivity $= 8.85 \times 10^{-12}\ F/m$

ε_r : relative permittivity of the medium.

3. Maxwell-Magnetic flux equation

This equation states that the magnetic field lines B are necessarily closed, and that there exists no "magnetic charge" analogous to an electric charge.

$$div(\overrightarrow{B}) = 0 \qquad (2.4)$$

4. Maxwell-Ampère equation

This equation states that magnetic fields can be generated in two ways: by electric currents (this is Ampère's theorem), or by the time-variation of an electric field (this is Maxwell's contribution on this law).

$$\overrightarrow{rot}(\vec{B}) = \mu_0\vec{J} + \underbrace{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}_{\text{Maxwell's term}} \qquad (2.5)$$

with : $\vec{J}$: electric current density [A/m²],

$\mu = \mu_r\mu_0$: magnetic permeability of the [H/m]

μ_0: magnetic permeability of vacuum $= 4\pi \times 10^{-7}\ \ H/m$

μ_r: relative permeability of the medium.

This equation, known as the Maxwell-Ampère equation, is distinguished of Ampère's theorem by the presence of the term $\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$, added by Maxwell. This additional term, called "displacement current", ensures the compatibility of the Maxwell-Ampère equation with the law of charge conservation.

$$div\,\vec{J} + \frac{\delta\rho}{\delta t} = 0 \qquad (2.6)$$

In the theory of electromagnetism, Maxwell's equations are verified by their consequences and by practice or experiments. We can add to them the expression of the Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B}) \qquad (2.7)$$

as well as the microscopic (or local) Ohm's law:

$$\vec{J} = \sigma\vec{E} \qquad (2.8)$$

with $\vec{J}$: the current density

σ : the electrical conductivity (inverse of resistivity) of the conductor

q : : electric charge density

In a conductor, there is a proportionality between the current density and the local electrostatic field.

Example: Copper has a conductivity of 58 ×10⁶ S/m,

Glass (insulator) : 10^{-11} S/m.

III. Integral forms of Maxwell's equations

1. $\overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$

$$\iint_{s} \overrightarrow{rot}(\vec{E}) \cdot \overrightarrow{ds} = -\iint_{s} \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{ds} = -\frac{\partial}{\partial t} \iint_{s} \vec{B} \cdot \overrightarrow{ds} \qquad (2.9)$$

Stokes Th .:

$$\int_{c} \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial}{\partial t} \iint_{s} \vec{B} \cdot \overrightarrow{ds} \qquad (2.10)$$

2. $div(\vec{E}) = \frac{\rho}{\varepsilon_0}$

$$\iiint_{v} div(\vec{E}) \cdot dv = \iiint_{v} \frac{\rho}{\varepsilon_0} \cdot dv \qquad (2.11)$$

Gauss + Ostro. Th. :

$$\iint_{s} \vec{E} \cdot \overrightarrow{ds} = \frac{1}{\varepsilon_0} \iiint_{v} \rho \cdot dv \qquad (2.12)$$

3. $div(\vec{B}) = 0$

$$\iiint_{v} div(\vec{B}) \cdot dv = 0 \qquad (2.11)$$

Ostro. Th.:

$$\iint_{s} \vec{B} \cdot \overrightarrow{ds} = 0 \qquad (2.12)$$

4. $\overrightarrow{rot}(\vec{B}) = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\iint_{s} \overrightarrow{rot}(\vec{B}) \cdot \overrightarrow{ds} = \iint_{s} \left(\mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot \overrightarrow{ds} = \mu_0 \iint_{s} \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot \overrightarrow{ds} \qquad (2.13)$$

Stokes :

$$\int_{c} \vec{B} . \overrightarrow{dl} = \mu_0 \iint_{s} \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \cdot \overrightarrow{ds} \qquad (2.14)$$

Table 2.1. Local and integral forms of Maxwell's equations

	Local (differential) forms	**Integral forms**
1.	$\overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$	$\int_{c} \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial}{\partial t} \iint_{s} \vec{B} \cdot \overrightarrow{ds}$
2.	$div(\vec{E}) = \frac{\rho}{\varepsilon_0}$	$\iint_{s} \vec{E} \cdot \overrightarrow{ds} = \frac{1}{\varepsilon_0} \iiint_{v} \rho \cdot dv$
3.	$div(\overrightarrow{B}) = 0$	$\iint_{s} \vec{B} \cdot \overrightarrow{ds} = 0$
4.	$\overrightarrow{rot}(\vec{B}) = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\int_{c} \vec{B} . \overrightarrow{dl} = \mu_0 \iint_{s} \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot \overrightarrow{ds}$

IV. Constitutive relations

In a given material, the electric and magnetic flux densities **D, B** are related to the field intensities **E, H** via the so-called constitutive relations, whose precise form depends on the material in which the fields exist. In vacuum, they take their simplest form:

$$\vec{B} = \mu \vec{H} \qquad (2.15)$$

$$\vec{D} = \varepsilon \vec{E} \qquad (2.16)$$

where : $\vec{E}$= electric field intensity, (V/m), $\vec{H}$= magnetic field intensity (A/m), $\vec{D}$= electric flux density (C/m^2), $\vec{B}$= magnetic flux density or magnetic induction (W/m^2).

V. Conclusion

Maxwell's equations shed light on the fundamental laws governing EM phenomena. They constitute the theoretical basis of classical electromagnetism. Their integral and local forms allow for a flexible analysis adapted to various physical situations. Electromagnetics has had a great impact in science and technology. Any engineering problem that includes EM fields is solved starting from these equations. Maxwell's equations enable us to solve many practical engineering problems that deal with EM fields. The constitutive relations establish the link between the fields and the properties of material media, making possible the realistic modeling of EM phenomena. This chapter forms a key step in the understanding of EM phenomena, such as the propagation of electromagnetic waves, the subject of the following chapter.

Chapter 3

Propagation of Electromagnetic Waves in Dielectric Media

I. Introduction

Electromagnetic (EM) waves are a form of vibrations that can travel away from their source. These waves consist of electric and magnetic oscillations that can travel through air, water, and even empty space (they do not require a medium).

EM waves come in different types, like radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and gamma rays. Each type has its own frequency and wavelength, which determine its unique characteristics and how it interacts with objects.

II. Wave equation

In regions of space far from the sources, the electromagnetic fields $\vec{E}$ and $\vec{B}$ are studied independently of the charges and currents that created them. In these regions, we assume that the sources of the electromagnetic field ρ and $\vec{J}$ equal 0. Maxwell's equations reduce to source-free equations:

$$\overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \qquad (3.1)$$

$$div(\vec{E}) = \frac{\rho}{\varepsilon} = 0 \qquad (3.2)$$

$$div(\overrightarrow{B}) = 0 \qquad (3.3)$$

$$\overrightarrow{rot}(\vec{B}) = \mu\vec{J} + \mu\varepsilon\frac{\partial \vec{E}}{\partial t} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t} \qquad (3.4)$$

taking curl operation to (3.1), we have

$$\overrightarrow{rot}\left(\overrightarrow{rot}(\vec{E})\right) = -\frac{\partial}{\partial t}\overrightarrow{rot}(\vec{B})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}\left(\overrightarrow{rot}(\vec{B})\right)$$

$$\overrightarrow{grad}\left(div(\vec{E})\right) - \Delta\vec{E} = -\frac{\partial}{\partial t}\left(\overrightarrow{rot}(\vec{B})\right) = -\frac{\partial}{\partial t}\left(\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right) \qquad \text{/ using Eq. (3.4)}$$

$$-\Delta\vec{E} = -\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad \text{/ using Eq. (3.2)}$$

$$\Delta\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0 \qquad (3.5)$$

called the ***wave equation*** of the electric field or the time-dependent ***Helmholtz*** equation where

$$\Delta\vec{E} = \nabla^2\vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(E_x\vec{u}_x + E_y\vec{u}_y + E_z\vec{u}_z\right)$$

which gives for each vector component :

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} = 0 \qquad (3.6.a)$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} = 0 \qquad (3.6.b)$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} - \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} = 0 \qquad (3.6.c)$$

III. Plane wave

If all components of *E*- and *H*-fields lie in the transverse plane, which is perpendicular to the direction of the wave propagation, such a wave is called a **plane wave.**

The fields should have no components in the longitudinal direction (the direction of wave propagation). Let's consider a wave that propagates in the +z direction. We must have E_z=H_z=0. Plane wave is also called a transverse electromagnetic wave (**TEM** wave).

IV. Uniform plane wave

For a uniform plane wave, the electromagnetic field (E, B) has the same magnitude and the same direction in the transverse plane xoy, i.e., the field does not change according to the transverse coordinates x and y. Thus, for a **uniform plane wave** propagating in the +z direction, we have:

$$\frac{\partial\vec{E}}{\partial x} = \frac{\partial\vec{E}}{\partial y} = 0 \qquad (3.7.a)$$

$$\frac{\partial\vec{B}}{\partial x} = \frac{\partial\vec{B}}{\partial y} = 0 \qquad (3.7.b)$$

For a **uniform plane wave,** Eqs. (3.6-8) reduce to :

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0 \qquad (3.8.a)$$

$$\frac{\partial^2 E_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} = 0 \qquad (3.8.b)$$

Similarly, we can derive the wave equations for the magnetic field

$$\frac{\partial^2 B_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 B_x}{\partial t^2} = 0 \qquad (3.9.a)$$

$$\frac{\partial^2 B_y}{\partial z^2} - \mu\varepsilon \frac{\partial^2 B_y}{\partial t^2} = 0 \qquad (3.9.b)$$

Note that : $\frac{1}{\sqrt{\mu\varepsilon}}$ has a unit of speed :

By examining

$$\frac{[B]}{\mathrm{L}^2} - [\mu\varepsilon]\frac{[B]}{\mathrm{T}^2} = 0$$

$$\frac{[B]}{\mathrm{L}^2} = [\mu\varepsilon]\frac{[B]}{\mathrm{T}^2}$$

$$\frac{\mathrm{T}^2}{\mathrm{L}^2} = [\mu\varepsilon]$$

$$[\sqrt{\mu\varepsilon}] = \frac{T}{\mathrm{L}}$$

$$\left[\frac{1}{\sqrt{\mu\varepsilon}}\right] = \frac{L}{\mathrm{T}} \quad : \quad [m/s]$$

T and L are time and length units

In free space :

$$v = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \cong 3 \times 10^8 \;\; m/s \qquad \text{: speed of light in vacuum}$$

V. Traveling time-harmonic uniform plane waves

For a sine wave (monochromatic), the time variation of the field is of the form $e^{j\omega t} \equiv \cos(\omega t)$, with $\omega = 2\pi f$: the angular frequency

$$\frac{\partial}{\partial t} \equiv j\omega \qquad \text{and} \qquad \frac{\partial^2}{\partial t^2} = -\omega^2$$

therefore, the wave equation

$$\frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} = 0$$

can be written as:

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu\varepsilon E_x = 0 \qquad (3.10)$$

let's put :

$$k^2 = \omega^2 \mu\varepsilon \qquad \text{or} \qquad k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v} \qquad (3.11)$$

k: wave vector (the direction of propagation is oriented by vector $\vec{k}$)

The wave equation can be written as :

$$\frac{\partial^2 E_x}{\partial z^2} + \quad k^2 E_x = 0 \qquad (3.12)$$

it is a differential equation whose general solution is:

in complex or polar form :

$$E_x(z,t) = E_{0x} e^{j(\omega t - kz)} \qquad (3.13)$$

in phasor form :

$$E_x(z) = E_{0x} e^{-jkz} \qquad (3.14)$$

in real form :

$$E_x(z,t) = real\ (E_{0x} e^{j(\omega t - kz)}) \qquad (3.15)$$

$$E_x(z,t) = E_{0x} \cos(\omega t - kz) \qquad (3.16)$$

where : E_{0x} : amplitude

ω : angular frequency

k : wave vector

z : direction of propagation

it is a forward wave which propagates along +z direction

1. Time period

$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} \quad (\mathrm{s}); \qquad f = \frac{\omega}{2\pi} \ (\mathrm{Hz}); \qquad (3.17)$$

2. Space period (Wavelength)

$$k\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{k} = vT \quad (\mathrm{m}) \qquad (3.18)$$

3. Phase velocity (speed)

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_r\varepsilon_r}}.\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{\mathrm{c}}{n} \quad (\mathrm{m/s}) \qquad (3.19)$$

$n = \sqrt{\mu_r\varepsilon_r}$: refractive index

VI. Uniform plane wave in an arbitrary direction

In this case, the wave vector has three components:

$$\vec{k} = (k_x, k_y, k_z) \qquad (3.20)$$

let $\vec{r} = \overrightarrow{OM} = (x, y, z)$: the position vector

the field at point M is given by :

$$\vec{E} = \vec{E}_0 e^{j(\omega t - \vec{k}.\vec{r})} = \vec{E}_0 e^{j(\omega t - x.k_x - y.k_y - z.k_z)} \qquad (3.21)$$

The direction of propagation is oriented by vector $\vec{k}$ and both $\vec{E}$ and $\vec{B}$ have three components :

$$\vec{E} = (E_x, E_y, E_z) \text{ and } \quad \vec{B} = (B_x, B_y, B_z)$$

1. Relationship between $\vec{\mathrm{E}}$, $\vec{\mathrm{B}}$ and $\vec{\mathrm{k}}$

For a monochromatic traveling plane wave, we can write :

$$\mathrm{div}\vec{E} = -j\vec{k} \cdot \vec{E}; \qquad \overrightarrow{rot}\vec{E} = -j\vec{k} \wedge \vec{E}$$

i.e. $\vec{\nabla} \equiv -j\vec{k}$ and $\frac{\partial \vec{E}}{\partial t} = j\omega\vec{E}$

we have :

$$div(\vec{E}) = 0$$

$$\vec{\nabla}.\vec{E} = 0$$

- $-j\vec{k}.\vec{E} = 0 \Rightarrow \vec{k} \perp \vec{E}$, the electric field $\vec{E}$ is perpendicular to the direction of propagation

and : $div(\vec{B}) = 0$

$-j\vec{k}.\vec{B} = 0 \Rightarrow \vec{k} \perp \vec{B}$, the magnetic field $\vec{B}$ is perpendicular to the direction of propagation.

We have : $\overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t}$

$$-j\vec{k} \wedge \vec{E} = -j\omega\vec{B}$$

$$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} \qquad (3.22)$$

Let : $\vec{k} = |\vec{k}|.\vec{u} = \frac{\omega}{v}\vec{u}$; $\vec{u}$: unit vector in the direction of $\vec{k}$ (direction of propagation).

$\vec{B} = \frac{\vec{u} \wedge \vec{E}}{v} \Rightarrow \vec{B} \perp \vec{u}$ and $\vec{B} \perp \vec{E}$: $\vec{u}, \vec{E}, \vec{B}$: form a direct triad

and

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{\vec{u} \wedge \vec{E}}{\mu v}$$

$$\vec{H} = \frac{\vec{u} \wedge \vec{E}}{Z} \qquad (3.23)$$

with

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$Z = Z_0\sqrt{\frac{\mu_r}{\varepsilon_r}} \qquad (3.24)$$

$$\text{with } Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$$

Z is called the intrinsic (or wave) impedance of the medium

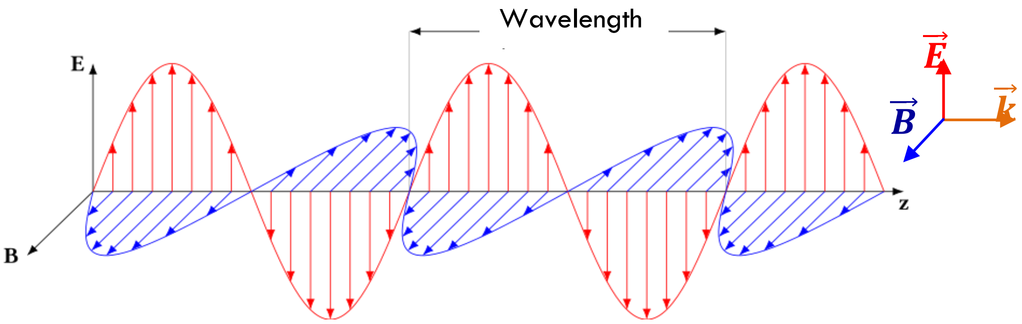


Fig.3.1. E- and H- fields of electromagnetic plane

VII. Wave polarization

We call polarization the type of the trajectory traced out by the head (tip) of the electric field E in the transverse plane (normal to the direction of propagation) over time during a period T.

Given a plane wave of the form :

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t - kx) \\ E_z = E_{0z}\cos(\omega t - kx + \varphi) \end{cases}$$

φ: the phase shift between Ey and Ez.

E_{0y} and E_{0z} : constant amplitudes

Let's put : $E_{0z} = \alpha E_{0y}$

Let's study the evolution of the electric field in a fixed transverse plane (e.g. x=0), we have :

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = \alpha E_{0y}\cos(\omega t + \varphi) \end{cases}$$

1. Linear polarization

If $\varphi = 0$ (the two E-field components are in phase)

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = \alpha E_{0y}\cos(\omega t) \end{cases}$$

$$\frac{E_z}{E_y} = \alpha \quad \text{or} \quad E_z = \alpha E_y \qquad (3.25)$$

equation of a line. The polarization is linear along the line AB. This means that the tip of the total electric field vector, for any fixed plane z, describes a line in the course of time.

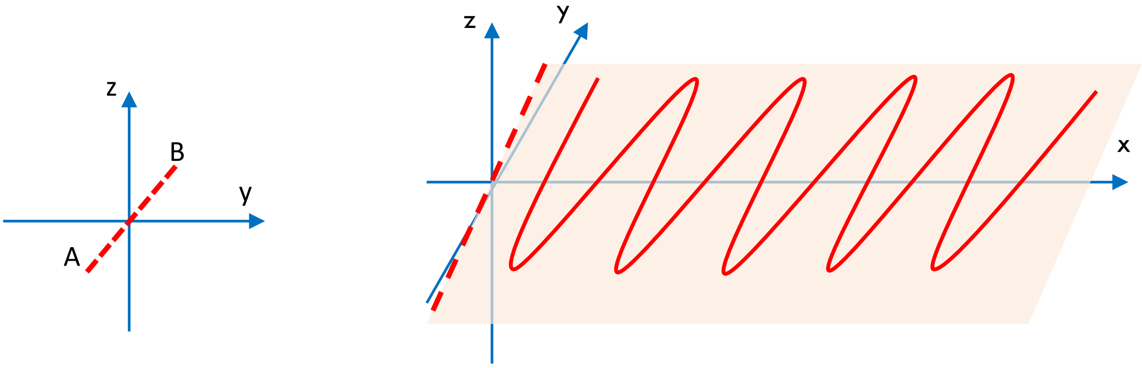


Fig. 3.2. Linear polarization

2. Circular polarization

If α =1

- $\varphi = \frac{\pi}{2}$:

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = E_{0y}\cos\left(\omega t + \frac{\pi}{2}\right) = -E_{0y}\sin(\omega t) \end{cases}$$

$$E_z^2 + E_y^2 = E_{0y}^2 \qquad (3.26)$$

equation of a circle with a radius E$_{0y}$. This means that the tip of the total electric field vector, for any fixed z, describes a circle in the course of time, and the wave is right-handed circularly polarized (**RHCP**).

- $\varphi = -\frac{\pi}{2}$:

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = E_{0y}\cos\left(\omega t - \frac{\pi}{2}\right) = E_{0y}\sin(\omega t) \end{cases}$$

$$E_z^2 + E_y^2 = E_{0y}^2 \qquad (3.27)$$

equation of a circle with a radius E$_{0y}$. It is a left-handed circularly polarized wave (**LHCP**).

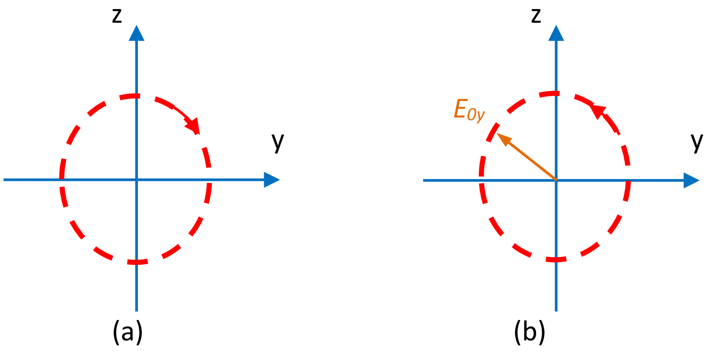


Fig. 3.3. Circular polarization. (a) RHCP and (b) LHCP

3. Elliptical polarization

If α ≠ 1

- $\varphi = \frac{\pi}{2}$:

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = \alpha\, E_{0y}\cos\left(\omega t + \frac{\pi}{2}\right) = -\alpha\, E_{0y}\sin(\omega t) \end{cases}$$

$$\left(\frac{E_z}{\alpha\, E_{0y}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = 1 \qquad (3.28)$$

equation of an *ellipse* with semi-axes E_{0y} and $\alpha\, E_{0y}$.

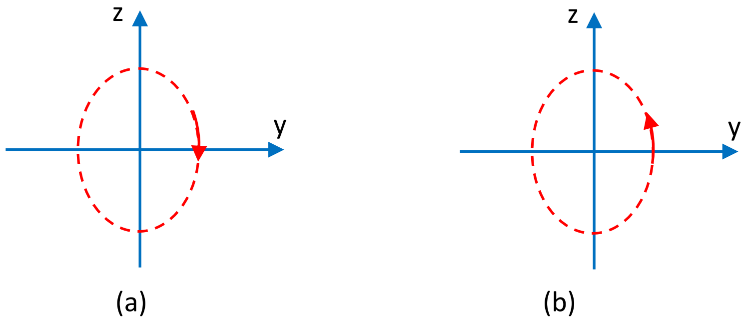
This means that the tip of the total electric field vector, for any fixed z, describes an ellipse in the course of time, and the wave is right-handed elliptically polarized (**RHEP**).

- $\varphi = -\frac{\pi}{2}$:

$$\begin{cases} E_x = 0 \\ E_y = E_{0y}\cos(\omega t) \\ E_z = \alpha\, E_{0y}\cos\left(\omega t - \frac{\pi}{2}\right) = \alpha\, E_{0y}\sin(\omega t) \end{cases}$$

$$\left(\frac{E_z}{\alpha\, E_{0y}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = 1 \qquad (3.29)$$

equation of an ellipse. It is a left-handed elliptically polarized wave (**LHEP**)



(a) (b)

Fig. 3.4. Elliptical polarization. (a) RHEP and (b) LHEP

VIII. Poynting vector and average power density

The energy transported by the electromagnetic wave is directly linked to the amplitudes of the E and H fields. We define the **Poynting vector** by :

$$\vec{R} = \frac{1}{2} real \left[\vec{E} \times \vec{H}^*\right] \qquad (3.30)$$

with $\vec{E}$ and $\vec{H}^*$ are in complex form.

The modulus of $\vec{R}$ is in $\left[\frac{V}{m}\right].\left[\frac{A}{m}\right] = \left[\frac{W}{m^2}\right]$, unit of power density. The direction is indeed the direction of propagation of the wave. Therefore, the average power is the integral of the Poynting vector on a closed surface (the flux).

$$P_{ave} = \oiint \vec{R}.\overrightarrow{ds} \qquad (3.31)$$

IX. Conclusion

This chapter explores the fundamental mechanisms of electromagnetic wave propagation in dielectric media. Starting from the wave equation, it develops a thorough understanding of plane waves, their frequency and time properties, as well as the relationships between the electric and magnetic fields and the wave vector. The notion of polarization, with its linear, circular and elliptical forms, are discussed. Finally, the introduction to the Poynting vector and the average power density allows to quantify the energy carried by these waves. This chapter thus, lays the essential foundations for the study of interactions between electromagnetic waves and media.

Chapter 4

Propagation of Electromagnetic Waves in Conductive and Dissipative Media

I. Introduction

As an electromagnetic wave propagates through space, it can pass through various media that significantly influence its behavior. These media, including earth, metals, concrete, and other materials, are generally classified into two main categories: dielectrics (or insulators) and conductors.

The interaction between an electromagnetic wave and the medium through which it travels gives rise to a multitude of complex phenomena such as absorption (loss), dispersion, reflection and refraction, which are all manifestations of the way in which the electromagnetic wave interacts with its environment.

As an electromagnetic wave propagates through a conductor, it induces an electric current due to the interaction with free electrons in the material which follows Ohm's law :

$$\vec{J} = \sigma \vec{E} \qquad (4.1)$$

with $\vec{J}$: current density,

σ : conductivity.

As a consequence, the electromagnetic energy carried by the wave is gradually converted into thermal energy, causing the conductor to heat up (Joule effect : $P=R \cdot I^2$). This process results in a loss of electromagnetic energy, and the amplitude of the electromagnetic wave is attenuated as it penetrates deeper into the conductor. The attenuation (or loss) depends on factors like frequency, material conductivity and conductor size.

In this chapter, we will explore the behavior of these waves as they propagate through conductors, such as metals.

II. Waves in a dissipative material

Let us assume that the material is characterized by a permittivity ε, a permeability μ and a conductivity σ, but not any charge or any current other than that determined by Omh's law.

Thus, Maxwell's equations :

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{4.2}$$

$$\vec{\nabla} . \vec{E} = 0 \tag{4.3}$$

$$\vec{\nabla} . \vec{B} = 0 \tag{4.4}$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{j} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t} \tag{4.5}$$

with: $\vec{j} = \sigma \vec{E}$: current of conduction

$\varepsilon \frac{\partial \vec{E}}{\partial t}$: displacement current

by taking curl of Eq. (4.2)

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{B}\right)$$

using Eq. (4.5)

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\frac{\partial}{\partial t}\left(\mu \vec{j} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

using Eq. (4.1)

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\frac{\partial}{\partial t}\left(\mu\sigma \vec{E} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla}\left(\vec{\nabla} . \vec{E}\right) - \Delta \vec{E} = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$-\Delta \vec{E} = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\Delta \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{4.6}$$

Similarly, we can find for the magnetic field :

$$\Delta \vec{H} - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

1. Traveling time-harmonic plane wave in dissipative material

$$\frac{\partial}{\partial t} \equiv j\omega \qquad \text{and} \qquad \frac{\partial^2}{\partial t^2} = -\omega^2$$

therefore, the wave equation Eq.(4.6) can be written as:

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} - j\omega\mu\sigma\vec{E} = 0 \qquad (4.7)$$

Let's consider a plane wave propagating in +z direction linearly polarized along x axis, Eq.(4.7) reduces to :

$$\frac{\partial^2 E_x}{\partial z^2} - (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_x = 0 \qquad (4.8)$$

by putting : $\gamma^2 = (j\omega\mu\sigma - \omega^2\mu\varepsilon)$

Eq.(4.8) can be rewritten as :

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \qquad (4.9)$$

which admits a solution of the form :

$$E_x(z,t) = E_{0x}e^{-\gamma z}e^{j\omega t} \qquad (4.10)$$

with :

$\gamma = \sqrt{(j\omega\mu\sigma - \omega^2\mu\varepsilon)} = \alpha + j\beta$: complex propagation constant

$\alpha : \left(\frac{Np}{m}\right)$ attenuation coefficient

$\beta : \left(\frac{rad}{m}\right)$ phase constant

2. Propagation in a good conductor

In a **good conductor,** the conduction current is typically much larger than the displacement current, and this is due to the high conductivity of the material, which allows for the easy flow of electric charges.

$$\vec{J_c} = \sigma\vec{E} \gg \vec{J_d} = \varepsilon\frac{\partial\vec{E}}{\partial t} = j\omega\varepsilon\vec{E} \quad \Rightarrow \quad \sigma \gg \omega\varepsilon$$

Or :

$$\frac{\sigma}{\omega\varepsilon} \gg 1 \qquad (4.11)$$

The propagation constant can then be adequately approximated by ignoring the displacement current term,

$$\gamma^2 = (j\omega\mu\sigma - \omega^2\mu\varepsilon) = -\omega^2\mu\varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$$

$$\cong j\omega\mu\sigma = \omega\mu\sigma e^{j\frac{\pi}{2}}$$

to give

$$\gamma = \sqrt{\omega\mu\sigma}\, e^{j\frac{\pi}{4}} \qquad (4.12)$$

$$= \sqrt{\omega\mu\sigma}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = \sqrt{\omega\mu\sigma}\left(\frac{1}{\sqrt{2}} + j\,\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{\omega\mu\sigma}{2}}\,(1 + j\,) = \alpha + j\beta$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad [m^{-1}] \qquad (4.13)$$

Remarkably, we find that $\alpha = \beta$ for a good conductor.

By replacing in Eq.(4.10)

$$E_x(z,t) = E_{0x}e^{-\alpha z}.e^{j(\boldsymbol{\omega t - \beta z})} \qquad (4.14)$$

Skin depth

The amplitude of the field in the conductor $E_{0x}e^{-\alpha z}$ will decay with z. This amplitude will decay by the factor $\frac{1}{e}$ after traveling a distance of : $\delta = \frac{1}{\alpha}$ (e : base of the Neperian logarithm =2.718)

$$e^{-\alpha\delta} = e^{-1} = \frac{1}{e} = \frac{1}{2.718} = 0.368$$

δ : Is a characteristic distance of the conductor known as the ***"skin depth"*** or ***" penetration depth "***. It represents the distance that a wave travels within the conductor before its amplitude decreases by 1/e or 36.8%. In other words, it is the distance at which 63.2% of the wave's original amplitude is lost.

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \qquad (4.15)$$

Penetration depth

is inversely proportional to frequency $\delta = f(1/\omega)$. It is considerably small, especially, at higher frequencies.

Example:

for ***Aluminum*** : $\sigma = 3.816\times10^7$ S/m

at : *f=1 MHz*, $\delta = 8.5\times10^{-5}$ *m* *(85 µm)*
at : *f=1GHz*, $\delta = 2.6\times10^{-6}$ *m* *(2.6 µm)*

The penetration depth is a very small distance, so the currents produced by the interaction of the wave with the metal remain in an extremely thin region near the surface of the conductor (**skin effect** phenomenon).

N.B. *High frequency waves (microwaves) are rapidly attenuated in good conductors. Practical application of this for RF shielding of sensitive equipment against external sources of EM waves.*

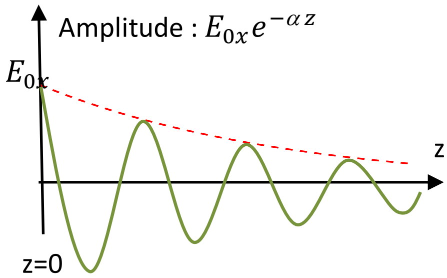


Fig. 4.1. Wave amplitude in a good conductor

Phase velocity (Speed)

$$v_p = \frac{\omega}{\beta} = \omega\,\delta = \sqrt{\frac{2\omega}{\mu\sigma}} \qquad (4.16)$$

Intrinsic impedance (Characteristic)

Given :

$$E_x = E_{0x}e^{-\gamma z}e^{j\omega t}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial\vec{H}}{\partial t} = -j\omega\mu\vec{H}$$

$$\frac{\partial E_x}{\partial z}\vec{u}_y = -j\omega\mu H_y$$

$$-\gamma E_x = -j\omega\mu H_y$$

$$\frac{E_x}{H_y} = j\frac{\omega\mu}{\gamma}$$

Using Eq.(4.12) : $\gamma = \sqrt{\omega\mu\sigma}\,e^{j\frac{\pi}{4}}$

$$\frac{E_x}{H_y} = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\frac{\pi}{4}} \quad [V/A]$$

$$Z_{cond} = \sqrt{\frac{\omega\mu}{\sigma}}\,e^{j\frac{\pi}{4}} \quad [\Omega] \qquad (4.17)$$

or in algebraic form:

$$Z_{cond} = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

Note that the impedance is now complex and that the phase of Z_{cond} is always $\approx\pi/4$

Wavelength

$$\lambda = \frac{2\pi}{\beta} = 2\pi\,\delta \quad [\mathrm{m}] \qquad (4.18)$$

Magnetic field of plane wave

The magnetic field of a plane wave in a good conductor can be found using

$$\vec{H} = \frac{\vec{u} \wedge \vec{E}}{Z_{cond}} \qquad [\mathrm{A/m}] \qquad (4.19)$$

Note that there is a phase shift of π/4 between $\vec{E}$ and $\vec{H}$ due to the phase of Z_{cond}.

Time-average power density (Poynting vector)

$$\vec{P} = \frac{1}{2} Real\left(\vec{E} \wedge \vec{H}^*\right) \quad \mathrm{W}/m^2] \qquad (4.20)$$

III. Reflection on a perfect conducting surface (Standing waves)

Considering a plane wave is incident normally on the planar interface at z = 0 between a perfect dielectric ①, of parameters ε and μ, and a perfect conductor ② (Fig.4.2). Since in a perfect conductor the skin depth is infinitely small, inside the conductor there is no field and a reflected wave propagating back in the –z direction is produced. The reflected wave has the same amplitude as the incident wave, but is π out of phase. The reflected wave combines with the incident wave to form a standing wave. A standing wave "stands" and does not travel; it consists of two traveling waves ($\vec{E_i}$ and $\vec{E_r}$) of equal amplitudes but in opposite directions.

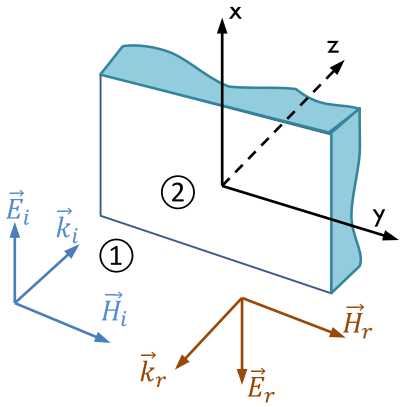


Fig. 4.2. Plane wave incident on a perfect conductor

Let the incident wave (in phasor form) be represented by :

$$\vec{E_i}(z) = E_0 e^{-jkz} \vec{u}_x$$

$$\vec{H_i}(z) = \frac{E_0}{Z_1} e^{-jkz} \vec{u}_y$$

Z_1: is the intrinsic impedance of medium ①.

The reflected wave, assuming that $\vec{k_r} = -\vec{k_i} = \vec{k_r} = k\vec{u}_z$, is then of the form :

$$\vec{E_r}(z) = -E_0 e^{+jkz} \vec{u}_x$$

The total field for z < 0 (medium ①) is obtained as a superposition of the incident and reflected waves :

$$\vec{E}_1(z) = \vec{E_i}(z) + \vec{E_r}(z) = E_0\left(e^{-jkz} - e^{+jkz}\right)\vec{u}_x = -2jE_0 sin(kz)\vec{u}_x$$

The instantaneous values of the two vectors are therefore :

$$\vec{E}_1(z,t) = \mathrm{real}\left(\vec{E}_1(z) e^{j\omega t}\right)$$

$$\vec{E}_1(z,t) = 2E_0 sin(kz) sin(\omega t)\vec{u}_x \qquad (4.21)$$

By taking similar steps, it can be shown that the magnetic field component of the wave is :

$$\vec{H}_1(z,t) = \frac{2E_0}{Z_1} cos(kz)cos(\omega t)\vec{u}_y \qquad (4.22)$$

The total wave does not contain the factor $e^{\mp jkz}$; it is not a progressive, traveling wave in either direction, but a standing wave.

A sketch of the standing wave electric field is presented in Figure 4.2. We notice that the wave does not travel but oscillates. There are planes in which $\vec{E}_1(z,t)$ is zero at all times. These planes are defined by:

$$kz_{min} = -n\pi, \qquad n = 0, 1, 2, \ldots$$

$$z_{min} = -\frac{n\pi}{k} = -n\frac{\lambda}{2}, \qquad n = 0, 1, 2, \ldots \qquad (4.23)$$

and planes in which $\vec{E}_1(z,t)$ is maximum. These planes are defined by:

$$kz_{max} = -(2n+1)\frac{\pi}{2}, \qquad n = 0, 1, 2, \ldots$$

$$z_{max} = -(2n+1)\frac{\lambda}{4}, \qquad n = 0, 1, 2, \ldots \qquad (4.24)$$

Thus, the total wave actually stays where it is, only pulsating in time according to the sine law.

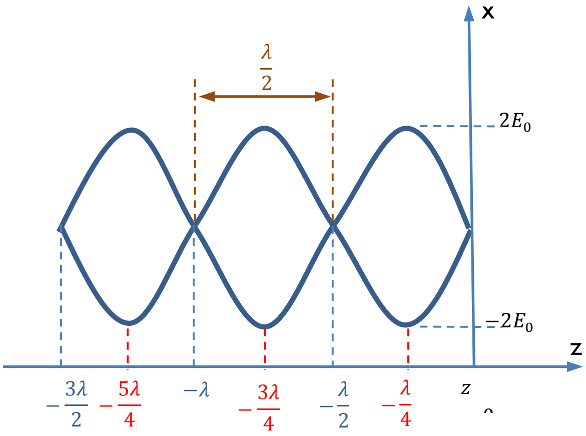


Fig. 4.3. Standing wave pattern

IV. Conclusion

In this chapter, we have examined the propagation of electromagnetic waves in conductive and dissipative media. This chapter provides an in-depth understanding of the phenomena related to wave attenuation in dissipative media. The study of good conductive materials highlights essential concepts such as skin depth, penetration depth which play a crucial role in the characterization of conductive media. Reflection on a perfect conductive surface illustrates the conditions for the formation of standing waves, a key phenomenon in resonant cavities and RF circuits.

Chapter 5

Reflection and Refraction of Electromagnetic Plane waves

I. Introduction

Plane electromagnetic waves encounter obstacles in their propagation paths: hills, buildings, metallic antennas aimed at receiving the messages the waves carry, objects from which they are supposed to partly reflect (as when the wave is a radar beam), and so on.

In reality, along its propagation path, the plane electromagnetic wave frequently crosses a series of media with different electromagnetic properties (hills, buildings, metals, earth, sea water, …), therefore, the behavior of the fields at the interface between two media must be considered. The wave at the interfaces (the boundaries) is generally partially transmitted and partially reflected with different amplitudes and directions depending on the properties of the media, the polarization and the angle of incidence.

II. Boundary conditions

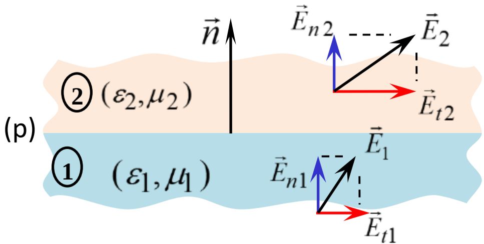


Fig. 5.1. Interface between two media

This Section discusses how Maxwell's equations strongly constrain the behavior of electromagnetic fields at boundaries between two media with different properties, where these constraint equations are called ***boundary conditions*** that have to be satisfied by the electromagnetic fields when the wave crosses the separation surface between the two media. Boundary conditions are essential in solving an electromagnetic wave problem. When an electromagnetic wave interfaces between two media, some part of the electromagnetic energy gets transmitted over the boundary and some part gets reflected. The boundary conditions of the electric and magnetic fields can be utilized to determine the directions and intensities of the transmitted and reflected waves.

Consider two media ① and ② separated by a plane (p). The discontinuity between the two media can introduce a discontinuity in the electromagnetic field.

Consider an elementary cylinder of height *dh* crossing (normal to) the separation surface (p).

- Let us **calculate the flux of the field $\vec{B}$** through the closed surface of the cylinder.

$$\oiint \vec{B}.\overrightarrow{ds} = \oiint \vec{B}_1.\overrightarrow{ds}_1 + \oiint \vec{B}_2.\overrightarrow{ds}_2 \,, \qquad \text{with } \oiint \vec{B}.\overrightarrow{ds} = 0 \text{ (Maxwell's Eq.(2.4))}$$

Now, we take the limit in which the height *dh* of the cylinder tends to zero, the flux through the side surface becomes zero and the total flux reduces to the flux through the bases of the cylinder. On the other hand, let :

$$\vec{S}_2 = S\vec{n} \text{ and } \vec{S}_1 = -S\vec{n}$$

we have

$$\oiint \vec{B}.\overrightarrow{ds} = \vec{B}_1.\vec{S}_1 + \vec{B}_2.\vec{S}_2 = -\vec{B}_1 S\vec{n} + \vec{B}_2 S\vec{n} = 0 \,, \qquad (S\neq 0)$$

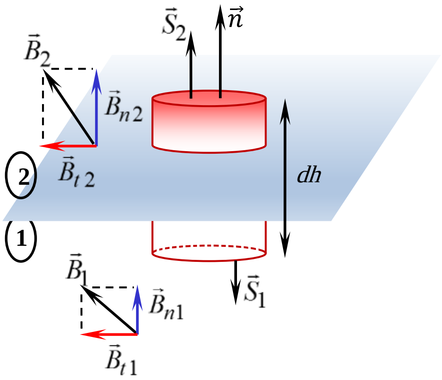


Fig. 5.2. Flux of the field $\vec{B}$

$$(\vec{B}_2 - \vec{B}_1)\vec{n} = 0$$

$$(\vec{B}_{2n} + \vec{B}_{2t} - \vec{B}_{1n} - \vec{B}_{1t})\vec{n} = 0$$

which gives :

$$B_{1n} = B_{2n} \qquad (5.1)$$

- Let us **calculate the flux of field $\vec{D}$** through the closed surface of the cylinder.

Taking into account that $\rho_{free} = 0$, with :

$$div\vec{D} = \rho = 0 \quad \text{or} \quad \oiint \vec{D}.\overrightarrow{ds} = 0$$

$$\oiint \vec{D}_1.\overrightarrow{ds}_1 + \oiint \vec{D}_2.\overrightarrow{ds}_2 = 0$$

By following the steps above, we get :

$$D_{1n} = D_{2n} \qquad (5.2)$$

Let us imagine an elementary rectangular circuit ABCD normal to the plane (p) and delimiting a surface S=ABCD.

- Let us **calculate the circulation of field $\vec{E}$** on the contour ABCD.

$$\int \vec{E}\vec{dl} = \int_{AB} \vec{E}\vec{dl} + \int_{BC} \vec{E}\vec{dl} + \int_{CD} \vec{E}\vec{dl} + \int_{DA} \vec{E}\vec{dl}$$

By making AD and BC tend towards 0, the circulation of $\vec{E}$ is reduced to the circulation on AB and CD :

$$\int_{BC} \vec{E}\vec{dl} = \int_{DA} \vec{E}\vec{dl} = 0 \qquad \text{and} \qquad \int \vec{E}\vec{dl} = \int_{AB} \vec{E}\vec{dl} + \int_{CD} \vec{E}\vec{dl}$$

$$\int \vec{E}\vec{dl} = \int_{AB} \vec{E}_1\vec{dl}_1 + \int_{CD} \vec{E}_2\vec{dl}_2 = 0$$

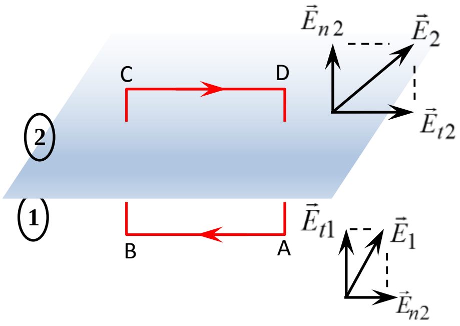


Fig.5.3. Circulation of field $\vec{E}$

On the other hand, we have :

$$\int \vec{E}\vec{dl} = \oiint \overrightarrow{rot}\vec{E}\,.\overrightarrow{ds} \overset{Stokes}{\overbrace{=}} \oiint -\frac{\partial\vec{B}}{\partial t}\,.\overrightarrow{ds} = 0 \qquad (ds\rightarrow 0)$$

Let : $\vec{dl}_1 = -\vec{dl}_2 = \overrightarrow{AB}$

$$\vec{E}_1\,\overrightarrow{AB} + \vec{E}_2\,\overrightarrow{CD} = 0$$

$$\vec{E}_1\,\overrightarrow{AB} - \vec{E}_2\,\overrightarrow{AB} = 0$$

which gives :

$$E_{1t} = E_{2t} \qquad (5.3)$$

or : $\vec{n}\wedge\vec{E}_1 = \vec{n}\wedge\vec{E}_2$

- Let us **calculate the circulation of the field $\vec{H}$** (with J = 0).

$$\int \vec{H}\vec{dl} = \oiint \overrightarrow{rot}\vec{H}\,.\overrightarrow{ds} = \oiint \varepsilon\frac{\partial\vec{E}}{\partial t}\,.\overrightarrow{ds} = 0 \qquad (ds\rightarrow 0)$$

which gives :

$$H_{1t} = H_{2t} \qquad (5.4)$$

or $\vec{n}\wedge\vec{H}_1 = \vec{n}\wedge\vec{H}_2$

From (5.2) : $D_{1n} = D_{2n} \Rightarrow \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

which gives

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n} \qquad (5.5)$$

From (5.4) :

$$H_{1t} = H_{2t} \quad \Rightarrow \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

which gives :

$$B_{2t} = \frac{\mu_2}{\mu_1} B_{1t} \qquad (5.6)$$

Hence, we can conclude that, across a dielectric interface :

1. **Normal** components of $\vec{D}$ and $\vec{B}$ are **continuous** (Eqs. (5.1) and (5.2)).
2. **Tangential** components of $\vec{E}$ and $\vec{H}$ are **continuous** (Eqs. (5.3) and (5.4)).
3. **Normal** components of $\vec{E}$ are **discontinuous** (Eq. (5.5)).
4. **Tangential** components of $\vec{B}$ are **discontinuous** (Eq. (5.6)).

III. Reflection and refraction

Electromagnetic waves can be reflected and refracted when they encounter a boundary between two different media, such as air and glass or air and water. Reflection occurs when the wave bounces off the surface of the boundary, while refraction occurs when the wave bends as it passes through the boundary.

The reflection and refraction principles describe how EM waves behave when encountering boundaries between different mediums, explaining phenomena such as the formation of images by mirrors, the bending of light through lenses, and the appearance of objects in water.

1. Snell-Descartes Laws

When a uniform plane wave (ω, k_1) traveling in medium ① falls on the plane of separation (p) with medium ②, it gives rise to:

- A reflected wave (ω, k_1') in medium ①.
- A refracted or transmitted wave (ω, k_2) in medium ②.

So :

✓ in medium ① : there will be a superposition of two waves: **incident** and **reflected**,

✓ in medium ② : a **refracted** wave.

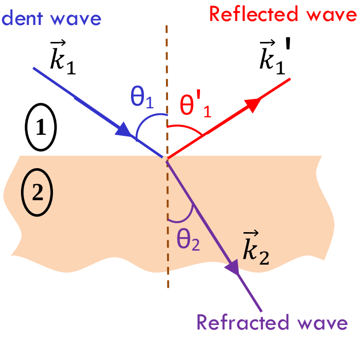


Fig.5.4. Reflection and refraction

Snell-Descartes First Law

It states that the angle of reflection θ_1' is equal to the angle of incidence θ_1.

$$\theta_1' = \theta_1 \qquad (5.7)$$

This means that the incoming wave and the reflected wave make the same angle with the normal to the surface at the point of incidence.

Snell-Descartes Second Law

When a wave passes from one medium to another, its speed changes, causing a change in direction :

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \qquad (5.8)$$

with : n_1 and n_2 are the refractive indices of the first and second mediums, respectively, and θ_2 is the angle of refraction.

$$n = \frac{c}{v} \qquad (5.9)$$

or :

$$n = \sqrt{\varepsilon_r \mu_r}$$

This means that the amount of bending that the wave undergoes depends on the difference in the refractive indices of the two media.

IV.Reflection and transmission coefficients

1. Normal incidence $(\theta_1 = 0)$

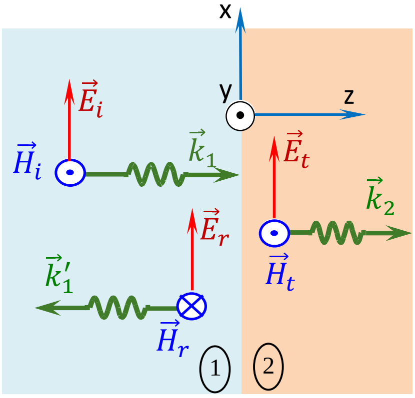


Fig. 5.5. Normal incidence

Incident fields :

$$\vec{E}_i = E_0 e^{-jk_1 z}\,\vec{u}_x$$

$$\vec{B}_i = \frac{E_0}{v_1} e^{-jk_1 z}\,\vec{u}_y$$

$$\vec{H}_i = \frac{E_0}{Z_1} e^{-jk_1 z}\,\vec{u}_y$$

Reflected fields :

$$\vec{E}_r = \Gamma E_0 e^{+jk_1 z}\,\vec{u}_x$$

$$\vec{H}_r = -\frac{\Gamma E_0}{Z_1} e^{+jk_1 z}\,\vec{u}_y$$

where :

$\Gamma = \frac{E_{0r}}{E_0}$: the reflection coefficient

k_1 : wave vector in medium ①

Z_1 : wave impedance of medium ①

Transmitted (refracted) fields :

$$\vec{E}_t = T E_0 e^{-jk_2 z}\,\vec{u}_x$$

$$\vec{H}_t = \frac{T E_0}{Z_2} e^{-jk_2 z}\,\vec{u}_y$$

where :

$T = \frac{E_{0t}}{E_0}$: transmission coefficient.

k_2 : wave vector in medium ②

Z_2 : wave impedance of medium ②

By applying the boundary conditions in the separation plane P (z=0),

- The continuity of E-field tangential component gives :

$$\underbrace{\vec{E}_i(z=0)+\vec{E}_r(z=0)}_{\textcircled{1}} = \underbrace{\vec{E}_t(z=0)}_{\textcircled{2}}$$

$$E_0 \quad + \quad \Gamma E_0 \quad = \quad T E_0$$

which gives :

$$1+\Gamma \;=\; T \qquad (5.10)$$

- The continuity of H-field tangential component gives :

$$\underbrace{\vec{H}_i(z=0)+\vec{H}_r(z=0)}_{\textcircled{1}} = \underbrace{\vec{H}_t(z=0)}_{\textcircled{2}}$$

$$\frac{E_0}{Z_1} - \frac{\Gamma E_0}{Z_1} \quad = \quad \frac{T E_0}{Z_2}$$

$$\frac{1}{Z_1}(1-\Gamma) \quad = \quad \frac{T}{Z_2}$$

which gives :

$$\Gamma = \frac{z_2 - z_1}{z_2 + z_1} \qquad (5.11)$$

and by using Eq.(5.9), we get :

$$T = \frac{2z_2}{z_2 + z_1} \qquad (5.12)$$

2. Oblique incidence $(\theta_1 \neq 0)$

The equations depend on the polarization of the wave, i.e. the orientation of the electric field with respect to the plane of incidence. Two special cases are to be considered : one with the E field parallel to the plane of incidence, the other with the E field perpendicular to it. Any other polarization may be considered as a linear combination of these two cases.

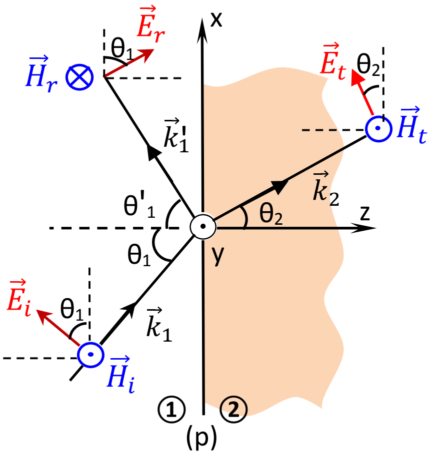


Fig. 5.6. Oblique incidence : Parallel polarization

Parallel polarization

E-field parallel to the plane of incidence (transverse magnetic (TM) case). Given an incident E-field contained in the xz-plane, the plane of incidence.

In medium ①, we have :

$$\vec{E_i} = \overrightarrow{E_0} e^{-j\overrightarrow{k_1}\vec{r}}$$

or :

$$\vec{E_i} = \begin{cases} E_0 cos\theta_1 e^{-jk_1(xsin\theta_1 + zcos\theta_1)} \\ 0 \\ -E_0 sin\theta_1 e^{-jk_1(xsin\theta_1 + zcos\theta_1)} \end{cases}$$

and

$$\vec{H_i} = \begin{cases} 0 \\ \frac{E_0}{Z_1} e^{-jk_1(xsin\theta_1 + zcos\theta_1)} \\ 0 \end{cases}$$

Reflected and transmitted fields

The reflected fields exist in medium ① and are given by

$$\vec{E_r} = \begin{cases} \Gamma E_0 cos\theta_1 e^{-jk_1(xsin\theta_1 - zcos\theta_1)} \\ 0 \\ \Gamma E_0 sin\theta_1 e^{-jk_1(xsin\theta_1 - zcos\theta_1)} \end{cases}$$

$$\vec{H_r} = \frac{-\Gamma E_0}{Z_1} e^{-jk_1(xsin\theta_1 - zcos\theta_1)} \vec{u_y}$$

The transmitted fields exist in medium ② and are given by

$$\vec{E_t} = \begin{cases} T\, E_0 cos\theta_2 e^{-jk_2(xsin\theta_2 + zcos\theta_2)} \\ 0 \\ -T\, E_0 sin\theta_2 e^{-jk_2(xsin\theta_2 + zcos\theta_2)} \end{cases}$$

$$\vec{H_t} = \frac{T\, E_0}{Z_2} e^{-jk_2(xsin\theta_2 + zcos\theta_2)} \vec{u_y}$$

Determination of Γ and T :

Considering the continuity of the tangential components of the total E and H fields (x- and y-components) at the boundary z=0.

For E field, we obtain

$$E_{ix}(0) + E_{rx}(0) = E_{tx}(0)$$

$$cos\theta_1 e^{-jk_1 xsin\theta_1} + \Gamma cos\theta_1 e^{-jk_1 xsin\theta_1} = Tcos\theta_2 e^{-jk_2 xsin\theta_2}$$

We have from Descartes law : $n_1 \sin\theta_1 = n_2 \sin\theta_2$

or

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 \qquad (5.13)$$

which gives : $cos\theta_1 + \Gamma cos\theta_1 = Tcos\theta_2$

and $T = (1+\Gamma)\frac{cos\theta_1}{cos\theta_2}$ (5.14)

For H field, we obtain

$$H_{iy}(0) + H_{ry}(0) = H_{ty}(0)$$

$$\frac{1}{Z_1} e^{-jk_1 xsin\theta_1} - \frac{\Gamma}{Z_1} e^{-jk_1 xsin\theta_1} = \frac{T}{Z_2} e^{-jk_2 xsin\theta_2}$$

which gives : $\frac{1}{Z_1} - \frac{\Gamma}{Z_1} = \frac{T}{Z_2}$ (5.15)

(5.13) in (5.14) : $\frac{1}{Z_1} - \frac{\Gamma}{Z_1} = \frac{(1+\Gamma)}{Z_2}\frac{cos\theta_1}{cos\theta_2}$

which gives :

$$\Gamma_{\parallel} = \frac{Z_2 cos\theta_2 - Z_1 cos\theta_1}{Z_1 cos\theta_1 + Z_2 cos\theta_2} \qquad (5.16)$$

and

$$T_{\parallel} = \frac{2\, Z_2 cos\theta_1}{Z_1 cos\theta_1 + Z_2 cos\theta_2} \qquad (5.17)$$

Perpendicular Polarization

E-field normal to the plane of incidence (transverse electric (TE) case)

$$\vec{E_i} = E_0 e^{-jk_1 (xsin\theta_1 + zcos\theta_1)} \vec{u}_y$$

and $$\vec{H}_i = \begin{cases} -\frac{E_0}{Z_1} cos\theta_1 e^{-jk_1(xsin\theta_1+zcos\theta_1)} \\ 0 \\ \frac{E_0}{Z_1} sin\theta_1 e^{-jk_1(xsin\theta_1+zcos\theta_1)} \end{cases}$$

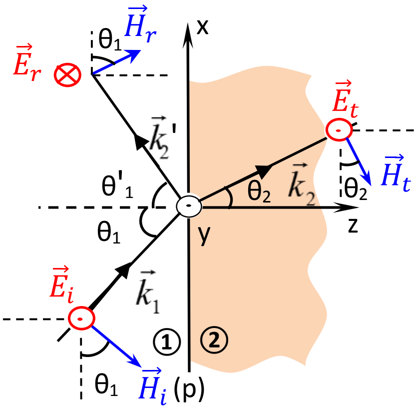


Fig. 5.7. Oblique incidence : Perpendicular polarization

Reflected and transmitted fields

$$\vec{E}_r = \Gamma E_0 e^{-jk_1(xsin\theta_1-zcos\theta_1)} \vec{u}_y$$

$$\vec{H}_r = \begin{cases} \frac{\Gamma E_0}{Z_1} cos\theta_1 e^{-jk_1(xsin\theta_1-zcos\theta_1)} \\ 0 \\ \frac{\Gamma E_0}{Z_1} sin\theta_1 e^{-jk_1(xsin\theta_1-zcos\theta_1)} \end{cases}$$

$$\vec{E}_t = T\, E_0 e^{-jk_2(xsin\theta_2+zcos\theta_2)} \vec{u}_y$$

$$\vec{H}_t = \begin{cases} -\frac{T\, E_0}{Z_2} cos\theta_2 e^{-jk_2(xsin\theta_2+zcos\theta_2)} \\ 0 \\ \frac{T\, E_0}{Z_2} sin\theta_2 e^{-jk_2(xsin\theta_2+zcos\theta_2)} \end{cases}$$

Determination of Γ and T :

Again, requiring that the tangential components of E and H be continuous at $z = 0$.

For E-field, we get:

$$E_{iy}(0) + E_{ry}(0) = E_{ty}(0)$$

$$E_0 e^{-jk_1 xsin\theta_1} + \Gamma E_0 e^{-jk_1 xsin\theta_1} = T E_0 e^{-jk_2 xsin\theta_2}$$

$$E_0 + \Gamma E_0 = T E_0$$

$$1 + \Gamma = T \qquad (5.18)$$

and for H-field, we get:

$$H_{ix}(0) + H_{rx}(0) = H_{tx}(0)$$

$$-\frac{E_0}{Z_1} cos\theta_1 e^{-jk_1 xsin\theta_1} + \frac{\Gamma E_0}{Z_1} cos\theta_1 e^{-jk_1 xsin\theta_1} = -\frac{T\, E_0}{Z_2} cos\theta_2 e^{-jk_2 xsin\theta_2}$$

By considering (5.13), we get :

$$-\frac{cos\theta_1}{Z_1}+\frac{\Gamma}{Z_1}cos\theta_1=-\frac{T}{Z_2}cos\theta_2 \qquad (5.19)$$

(5.18) in (5.19)

$$(1-\Gamma)\frac{cos\theta_1}{Z_1}=\frac{1+\Gamma}{Z_2}cos\theta_2$$

which gives :

$$\Gamma_\perp=\frac{Z_2cos\theta_1-Z_1cos\theta_2}{Z_2cos\theta_1+Z_1cos\theta_2} \qquad (5.20)$$

and

$$T_\perp=\frac{2\,Z_2cos\theta_1}{Z_2cos\theta_1+Z_1cos\theta_2} \qquad (5.21)$$

V. Conclusion

This chapter explores the phenomena of reflection and refraction of plane EM waves, fundamental elements of electromagnetism. By introducing boundary conditions, it highlights the interactions between waves and interfaces between two media, establishing the basis for understanding wave transmission and reflection. Snell-Descartes' laws provide a geometric description of these phenomena, while the analysis of reflection and transmission coefficients, for normal and oblique incidences, quantitatively details the energy transferred or reflected as a function of polarization.

Chapter 6

Radio Waves Propagation

I. Introduction

In the exposition of electromagnetic (EM) waves propagation undertaken in the preceding chapters, the emphasis was on the computation of the EM field. The environment was assumed to be infinite free space in all cases and the effects of the media and the discontinuities on the propagation of radio waves were ignored. However, this ideal situation is not met in practice. EM waves on their propagation path are exposed to various environmental influences. There exist a number of factors which affect the propagation of radio waves in a real environment. The most important of these are : the **Earth** with different kinds of obstacles on the ground (buildings, trees, hills, valleys, water, bodies, etc) and the **atmosphere** with its layered structure causing phenomena such as **reflection**, **refraction**, **diffraction**, **absorption**, **interference** and **attenuation** which greatly affect the quality of information transmission.

II. Electromagnetic spectrum

The EM waves have different characteristics depending upon how they are produced, how they interact with matter and their practical applications. Maxwell's equations predicted the existence of an infinite number of frequencies of EM waves, all travelling with the speed of light. This is the first indication of the existence of the entire EM spectrum. Nonetheless, the main significance of the EM spectrum is that it can be used to classify EM waves and arrange them according to their different frequencies or wavelengths. The EM spectrum extends from gamma γ rays to very low frequencies.

Since the EM spectrum is a common resource, which is open for access by anyone, several national and international agreements have been drawn regarding the usage of the different frequency bands within the spectrum. The individual national governments allocate spectrum for applications such as AM/FM radio broadcasting, television broadcasting, mobile telephony, military communication, and government usage. Worldwide, the International Telecommunications Union Radio Communication (ITU-R) tries to coordinate the spectrum allocation by the various national governments, so that communication devices that can work in multiple countries can be manufactured.

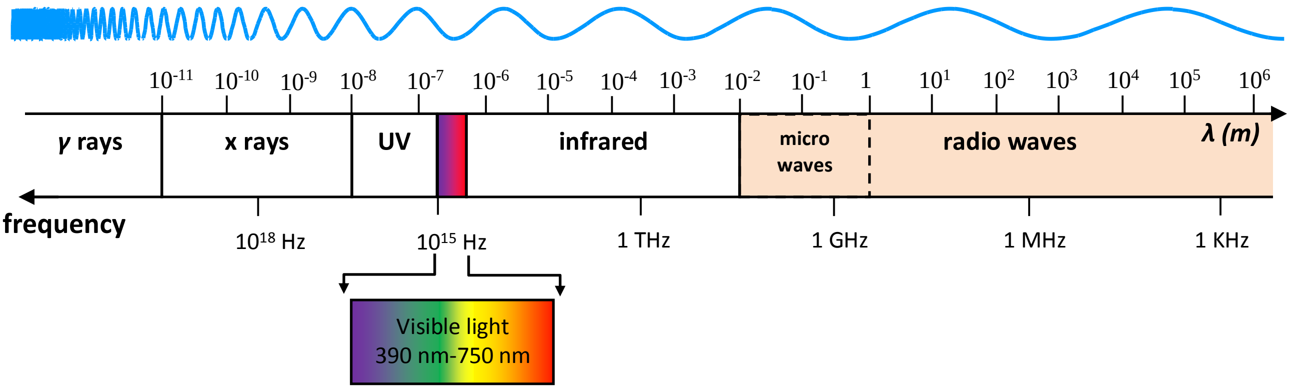


Fig. 6.1. Radio wave spectrum

1. Classification of radio waves by frequency bands

Each particular radio frequency belongs to the radio spectrum, which is a wide range of frequencies from several hertz up to 3 THz. The entire radio spectrum is divided into frequency bands based on decimal division. The range for each frequency band extends from 0.3×10^N to 3×10^N Hz, where N is a band number given in the first column of Table 6.1 [9].

There are many subdivisions within each band based on allocations to services (applications) and across continents. A subdivision of microwave bands, shown in Table 6.2, is widely used in radar and satellite applications.

Table 6.1. ITU Classification of the radio waves by frequencies [9,19]

Band Number, N		Frequency Band Name by ITU-R	Acronym	Frequency Range, Hz	Wavelength in meters	Descriptive Name
—		Extremely low frequency	ELF	$< 3\times10^3$	$>10^5$	-
4		Very low frequency	VLF	$(3 \text{ to } 30)\times10^3$	10^4 to 10^5	Miriameter waves
5		Low frequency	LF	$(30 \text{ to } 300)\times10^3$	10^3 to 10^4	Kilometer waves
6		Medium frequency	MF	$(0.3 \text{ to } 3)\times10^6$	10^2 to 10^3	Hectometer waves
7		High frequency	HF	$(3 \text{ to } 30)\times10^6$	10 to 10^2	Decameter waves
8		Very high frequency	VHF	$(30 \text{ to } 300)\times10^6$	1 to 10	Meter waves
9	**Microwaves**	Ultra high frequency	UHF	$(0.3 \text{ to } 3)\times10^9$	10^{-1} to 1	Decimeter waves
10		Super high frequency	SHF	$(3 \text{ to } 30)\times10^9$	10^{-2} to 10^{-1}	Centimeter waves
11		Extremely high frequency	EHF	$(30 \text{ to } 300)\times10^9$	10^{-3} to 10^{-2}	Millimeter waves

Table 6.2. Microwave Bands Subdivision for Radar and Satellite Applications [9,19]

Band Name	L	S	C	X	Ku	K	Ka
Frequency range (GHz)	1–2	2–4	4–8	8–12	12–18	18–27	27–40

III. Earth atmosphere and its layers

The real Earth's atmosphere has a complex structure, which significantly impacts radio wave propagation, causing effects such as smooth refraction, scatter, and energy absorption of the radio wave. Variations of the EM parameters of the atmospheric air are highly dependent on its gaseous composition, pressure, humidity and ionization. The earth's atmosphere extends all the way up to a height of about 600 km.

The atmosphere plays a crucial role in the propagation of EM waves due to its varying composition and structure across different layers. It consists of several layers, including the **troposphere**, **stratosphere** and **ionosphere**, each with distinct properties namely variation of its dielectric constant (refractive index) as a function of the altitude, time of day, solar activity, … and the ionization density of its highest regions. The atmosphere absorbs EM waves due to the presence of gases and water vapor. Different gases absorb different frequencies of EM waves, leading to frequency-dependent attenuation. Water vapor, especially in the lower atmosphere, can cause significant absorption in certain frequency bands, such as in the microwave region. Rain can cause additional attenuation due to scattering and absorption, particularly affecting higher frequency bands like those used in satellite communications and radar systems.

1. Troposphere: is the lowest layer of the atmosphere of Earth, extending from the Earth's surface up to about 8 to 15 kilometers depending on the location (its height is lower at Earth's poles and higher at the equator). It contains 80% of the total mass of the planetary atmosphere, and is where most weather phenomena occur such as clouds, rain, storms, and wind. In this environment, temperature, humidity and pressure vary significantly depending on time and location and cause variations in the refractive index of the air.

2. Stratosphere: extends from 10km to 50km in altitude. Like the troposphere, it is less ionized than the other layers. It contains most of the ozone layer. The refractive index in the stratosphere is more because of the relatively uniform temperature and lower humidity and air density. The stratosphere does not alter the propagation of radio waves significantly.

3. Ionosphere: The ionosphere is the upper part of the Earth's atmosphere, which extends from 60 km upwards to 400 km. It is exposed to intensive cosmic rays causing the air to be ionized (i.e., the atoms and molecules split into positively charged ions and free electrons). The ionosphere is typically divided into four main layers:

D layer: Exists between 60-90 km and is present only during the daytime. It primarily affects low-frequency (LF) radio waves, causing significant absorption.

E layer: Located between 90-150 km. It is present during the day and partially at night. It affects medium-frequency (MF) radio waves and supports some radio wave reflection.

F1 layer: Located between 150-220 km and exists mainly during the daytime. It merges with the F2 layer at night.

F2 layer: The most important layer for long-distance communication, located from about 250 km upward. It has the highest concentration of free electrons and remains partially ionized even at night.

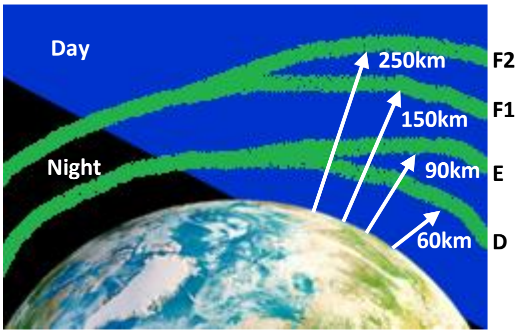


Fig.6.2. Ionospheric layer

IV.Different Modes of atmospheric propagation

In a homogeneous medium, EM waves travel in a straight line. However, when the antennas are located near the Earth, various phenomena will alter the wave path: the presence of the Earth, the variation in atmospheric density with altitude, and the presence of ionized layers in the upper atmosphere.

When an EM wave is produced by an antenna it moves from the transmitter to the receiver in several propagating modes:

1. Ground waves : A part of the wave travels along or near the surface of the earth. This wave is called the ground wave or surface wave. They are effective for frequencies below 3 MHz (Medium Frequency and below) and can also operate in the range of 3 to 30 MHz for shorter distances. They are used mainly for AM radio broadcasting and maritime communication.

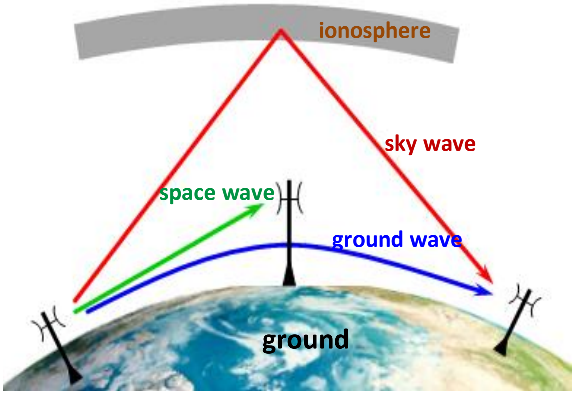


Fig.6.3. Atmospheric modes of propagation

Examples: AM radio (530 kHz - 1710 kHz), Long wave radio (30 kHz - 300 kHz). They are frequencies up to around 3 MHz.

2. Sky waves : Some waves travel upwards into space towards the sky and get reflected back to the receiver by the ionospheric layer allowing for long distance communication. These waves are called sky waves or ionospheric waves. This mode of propagation is effective for frequencies ranging from approximately 2 MHz to 30 MHz. They are widely used in international broadcasting and amateur radio. Examples: Shortwave radio (3 MHz - 30 MHz), Amateur radio.

3. Space waves : Some waves travel in a straight line (line-of-sight) through the atmosphere. They do not follow the earth and are not reflected by the sky. These waves are called direct waves. They operate at higher frequencies, typically between 30 MHz and 300 MHz or more. They are used in applications requiring line-of-sight transmission, such as TV broadcasting, FM radio, mobile communication, and radar. Examples: FM radio (88 MHz - 108 MHz), Television broadcasting, Wi-Fi (2.4 GHz or 5 GHz).

V. Atmospheric refraction

The atmosphere consists of gas molecules and water vapor with height-dependent densities. As a result, the dielectric constant and hence refractive index of air changes with height. The refractive index decreases with height. This variation gives rise to different phenomena like refraction, reflection, scattering, duct propagation and fading.

By definition, the refractive index, n is the square root of the dielectric constant $n = \sqrt{\varepsilon_r}$.

1. Tropospheric refraction

The troposphere is the region of the atmosphere extending above the highest mountains. It is the atmospheric layer of meteorological phenomena involving water: fog, clouds, rain, snow, etc. Its altitude varies between 7 and 14 km depending on location and atmospheric conditions. The temperature, humidity and pressure vary significantly resulting in variations in the refractive index of the air. A major consideration in space wave propagation is refraction by the atmosphere. Measurements of the index showed that these variations were, approximately, a linear function of altitude. If n is the index at altitude h and n_0 the index at sea level [11]:

$$n = n_0(1 + Bh) \qquad (6.1)$$

where B is given the Booker formula

$$B = \frac{1}{R_0}\left(-0.2 + 30\frac{dp}{dh} - 6\frac{dT}{dh}\right)$$

R_0 : radius of the Earth: $R_0 = 6400 \text{ km}$;

$\frac{dp}{dh}$: variation of water vapor pressure with altitude, expressed in mbars/m;

$\frac{dT}{dh}$: variation of temperature with altitude in °C/m.

Since n_0 is close to 1 and $Bh \ll 1$:

$$n = n_0 + Bh \qquad (6.2)$$

For typical atmospheric conditions, we define the **Standard atmosphere** [11] :

$$n = n_0 - \frac{h}{4R_0} \qquad (6.3)$$

The law governing the variations of the standard atmosphere is not valid throughout the troposphere. This is why the CCIR (Comité Consultatif International des Radiocommunications) decided to define a fundamental **Reference atmosphere** for which the refractive index is given by:

$$n(h) = 1 + 315 \times 10^{-6} \times e^{-0.136 \times 10^{-6} h}$$

where h is the height above the earth's surface.

The phenomenon of refraction in the stratified troposphere due to changes in refractive index is shown in Fig. 6.4. The refractive index variation creates super refraction or duct (waveguide effect) phenomena as shown in Fig.6.5.

Duct propagation *: is a* waveguide effect between the ground and the atmospheric duct region. It exists between two levels where the variation of modified refractive index with height is minimum. The duct region exists between two levels where the variation of modified refractive index with height is minimum. The ray travels round the earth in a series of hops with successive reflections from the earth.

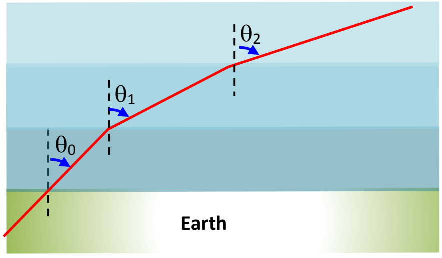


Fig.6.4. Ray path in stratified troposphere

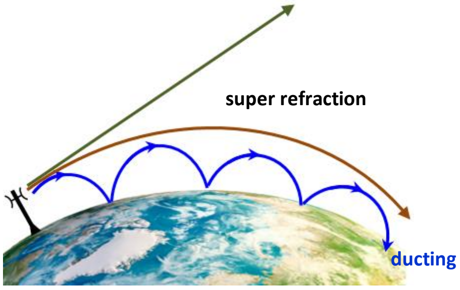


Fig.6.5. Cases of propagation due to changes in refractive index

2. Ionospheric refraction

When an EM ray enters a layer in the ionosphere, the free electrons move in response to the wave electric field. Because of variations in ionospheric constituents and the wavelength range of the incident sunlight, typical night and day time ionosphere sublayers appear as shown in Fig. 6.6.

The effective dielectric constant in the ionosphere with ionisation density of N electrons per cubic metre is governed by the following relation [9,11,23]:

$$\varepsilon_r = 1 - \frac{81\,N}{f^2} \qquad (6.4)$$

Interaction of the EM field with the ionised layer leads to refraction of the wave, hence, the refraction phenomena in the ionosphere is governed by the approximate value of the refractive index :

$$n = \sqrt{1 - \frac{81\,N}{f^2}} \qquad (6.5)$$

where f is the frequency of the incident wave.

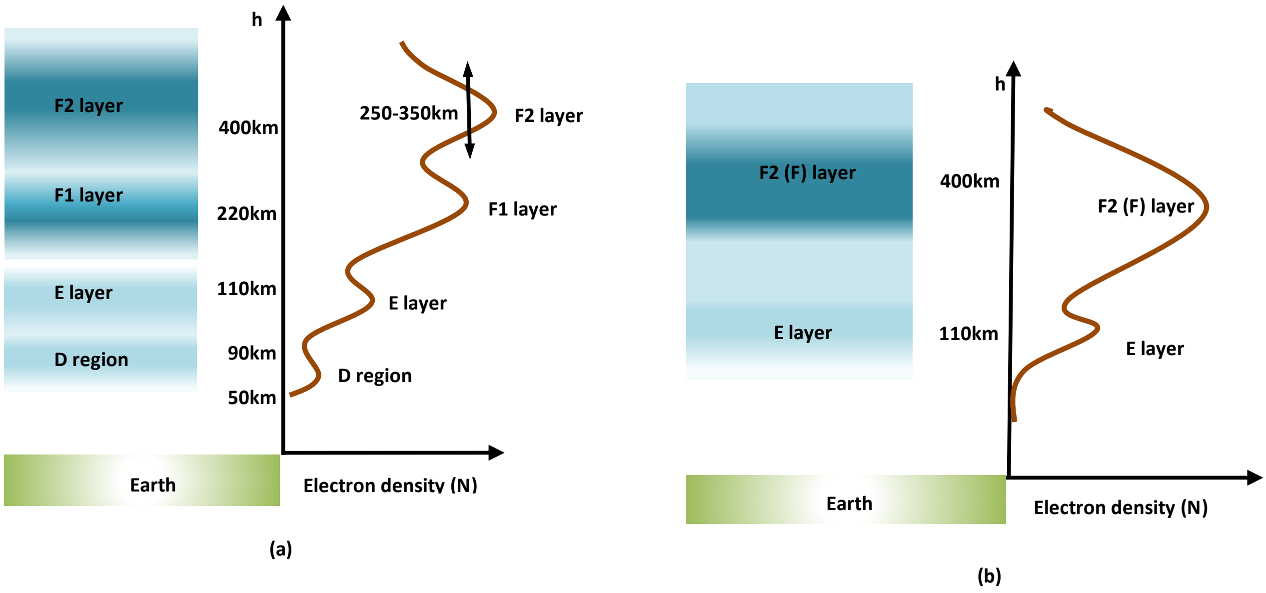


Fig.6.6. Variation of electron density with height (a) : Typical daytime ionosphere and (b) : Typical night time ionosphere.

In order to analyse the effect a layer has on the passage of a radio wave we assume that the ionosphere can be modelled as plane stratified media (Fig.6.7).

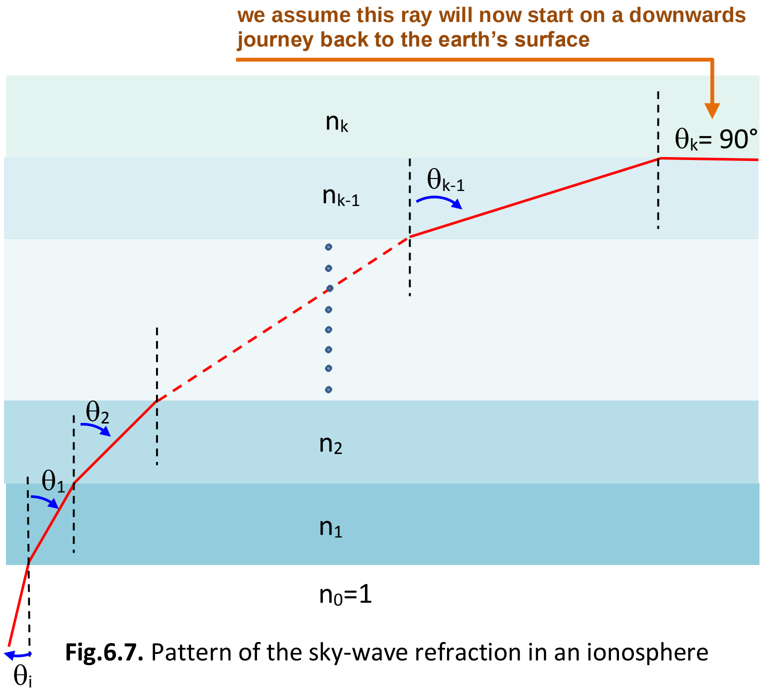


Fig.6.7. Pattern of the sky-wave refraction in an ionosphere

The wave path can be predicted using Snell's law :

$$n_0 \sin\,\theta_i = n_1 \sin\,\theta_1 = \cdots\ n_{k-1} \sin\,\theta_{k-1} = n_k \sin\,\theta_k$$

where θ_i is the angle of incidence with respect to the normal and θ_1 is the angle of refraction at the lower edge of the ionosphere. At the next interface between layers, the angle of incidence is θ_1 and the angle of refraction is θ_2. At the lower edge of the ionosphere the electron density is zero and hence $n_0 = 1$. Therefore, the equation representing Snell's law reduces to

$$\sin\,\theta_i = n_k \sin\,\theta_k$$

The condition for the wave to return to earth is to have total internal reflection, which begins when the refracted angle is $\theta_k = 90°$ (the wave becomes horizontal).

$$\sin\,\theta_i = n_k \sin 90° = n_k$$

From this, it follows that for a given electron density N and frequency f_{ob} (oblique incidence), the angle of incidence θ_i required to achieve total internal reflection is [9]:

$$\sin\,\theta_i = \sqrt{1 - \frac{81\,N}{f_{ob}^2}} \qquad (6.6)$$

If the incidence angle is greater than θ_i, the wave returns to the earth.

For a given angle of incidence, higher frequency EM waves are reflected from the region having a higher value of N. For example, a 5 MHz EM wave incident at 45° gets reflected by the ionospheric layer having $N = 1.54\times10^{11} m^{-3}$, which is at a height of about 100 km from the surface of the earth. A 10 MHz wave (incident at the same angle) is reflected by the ionospheric layer having an electron density of $6.17\times10^{11} m^{-3}$. This layer is found further away at about 170 km from the surface of the earth. Therefore, higher the frequency, the wave has to travel deeper into the ionosphere before it can be reflected.

Critical frequency

The critical frequency is the highest frequency of the vertically launched EM wave that can be reflected from the ionosphere having a certain maximum electron density.

Consider an EM wave launched vertically into the ionosphere having a maximum electron density N. Substituting $\theta_i = 0$

$$sin(0) = \sqrt{1 - \frac{81\,N}{f_{ob}^2}} = 0 \qquad (6.7)$$

the critical frequency that gets reflected is given by [9,11] :

$$f_{cr} = 9\sqrt{N} \qquad (6.8)$$

For example, if the highest electron density in the ionosphere is 10^{12} m^{-3}, the critical frequency is 9 MHz. For any other angle of incidence, the highest frequency that can be reflected from the ionosphere will be greater than the critical frequency.

Maximum usable frequency (MUF)

The highest frequency that gets reflected by the ionosphere for a given value of angle of incidence (θ_m) is known as the maximum usable frequency f_{MUF}.

$$sin\ \theta_m = \sqrt{1 - \frac{81\,N}{f_{MUF}^2}}$$

By introducing f_{cr} Eq. (6.8),we get :

$$\sin\theta_m = \sqrt{1 - \frac{f_{cr}^2}{f_{MUF}^2}} \qquad (6.9)$$

which gives :

$$f_{MUF} = \frac{f_{cr}}{\cos\theta_m} = f_{cr}\sec\theta_m \qquad (6.10)$$

For example, if the critical frequency is 9 MHz, the maximum usable frequency corresponding to an angle of incidence of 45° is 12.73 MHz.

Skip Distance : The minimum distance at which the wave returns to the ground at a critical angle θ_m is termed the skip distance d_{skip} (Fig. 8). For launch angles $\theta_i < \theta_m$ the ionosphere cannot reflect the waves back, it is not possible to establish a communication link by sky waves (reflected from the ionosphere). It is given by [9]:

$$d_{skip} = 2h\sqrt{\frac{f_{MUF}^2}{f_{cr}^2} - 1} \qquad (6.11)$$

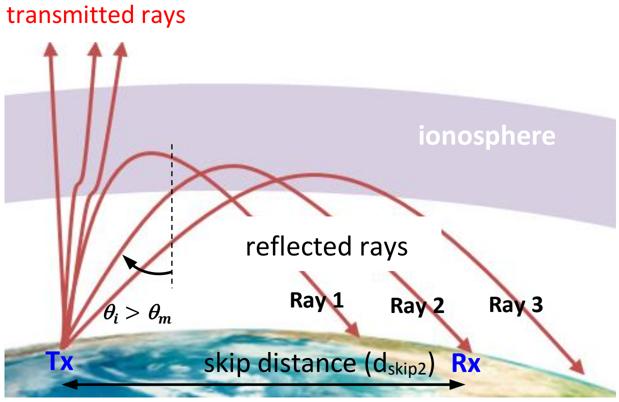


Fig.6.8. Ray paths for different angles of incidence and skip distance

3. Ground Reflection

Earth a partially conducting medium, where its conductivity σ leads to the dissipation of EM energy by Joule effect resulting in wave attenuation and hence its relative dielectric constant is complex:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0}$$

The imaginary part of the complex relative permittivity, εr″, affects the absorption of energy and is called the "attenuation factor".

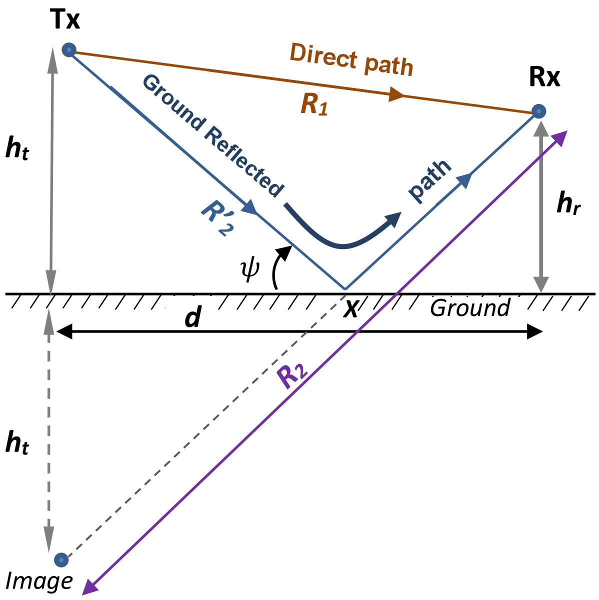


Fig. 6.9. Direct and ground-reflected paths of wave

When two vertically polarized antennas are situated close to the ground, the ground's influence becomes significant at low frequencies. It contributes to a reflected wave, whose characteristics depend on factors like the angle of incidence, wave polarization, ground's electrical properties (conductivity and dielectric constant), and the wave's frequency. The wave received at the receiving antenna is therefore composed of the **direct path** wave and the **ground reflected path** wave. Fig. 9 illustrates the principle.

For convenience, let's assume that the transmit antenna and the field point are located in the y-z plane. Let us also assume that the transmit antenna is an infinitesimal dipole oriented along the x-axis. The electric field of an infinitesimal dipole oriented along the x-axis is given by:

$$\vec{E} = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR}}{R}\left(-cos\theta\ cos\varphi\,\vec{u}_\theta + sin\,\varphi\ \vec{u}_\varphi\right)$$

where R is the distance from the antenna to the field point. In the y-z plane, ϕ = 90◦. Since cos 90◦ = 0, the θ-component of the electric field is zero. The ϕ-component of the electric field at Rx (receiver point) due to the direct wave is given by

$$E_1 = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR_1}}{R_1}$$

The field at Rx also has a contribution from the wave that travels via the ground reflected path. At X the incident and the reflected rays satisfy Snell's law of reflection. The incident field at X is given by

$$E_i = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR_2'}}{R_2'}$$

where R_2' is the distance from the transmitter to X and the incident E field vector is perpendicular to the plane of incidence.

Perpendicular oblique incidence:

For a perpendicular oblique incidence, the reflection coefficient is given by

$$\Gamma_\perp = \frac{E_r}{E_i} = \frac{\sin\psi - \sqrt{\varepsilon_r' - \cos^2\psi}}{\sin\psi + \sqrt{\varepsilon_r' - \cos^2\psi}} \qquad (6.12)$$

where ψ is the *grazing angle* of incidence with respect to the horizontal. The electric field at the receiver due the reflected wave is

$$E_2 = jkZ\Gamma_\perp\frac{I_0 dl}{4\pi}\frac{e^{-jkR_2}}{R_2}$$

This is the field at Rx due to an equivalent (image) dipole having a strength of $I_0 dl$ located at (0, 0, −ht). The total electric field at Rx is given by :

$$E = E_1 + E_2 = jkZ\frac{I_0 dl}{4\pi}\left(\frac{e^{-jkR_1}}{R_1} + \Gamma_\perp\frac{e^{-jkR_2}}{R_2}\right)$$

If the field point Rx is far away from the transmitter Tx, we can use the approximation $R_2 \approx R_1$ for $R_1$and R_2 appearing in the denominator. Thus, the total electric field is given by :

$$E = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR_1}}{R_1}\left[1 + \Gamma_\perp e^{-jk(R_2 - R_1)}\right]$$

$$E = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR_1}}{R_1}F_\perp \qquad (6.13)$$

This can be thought of as a product of the free-space field and an environmental factor $\Gamma_\perp$, given by :

$$F_\perp = \left[1 + \Gamma_\perp e^{-jk(R_2 - R_1)}\right] \qquad (6.14)$$

Parallel oblique incidence

Let us now derive an expression for the total field at Rx due to an infinitesimal dipole at (0, 0, ht) oriented along the z-direction. The electric field of a z-directed infinitesimal dipole is :

$$\vec{E} = jkZ\frac{I_0 dl \sin\theta}{4\pi}\frac{e^{-jkR_1}}{R_1} \quad \vec{u}_\theta$$

where R_1 is the distance from the antenna to the field point. The electric field is parallel to the plane of incidence and the reflection coefficient, $\Gamma_\parallel$, at X is given by :

$$\Gamma_\parallel = \frac{\varepsilon_r' \sin\psi - \sqrt{\varepsilon_r' - \cos^2\psi}}{\varepsilon_r' \sin\psi + \sqrt{\varepsilon_r' - \cos^2\psi}} \qquad (6.15)$$

The total field at point Rx is given by :

$$E = jkZ\frac{I_0 dl}{4\pi}\frac{e^{-jkR_1}}{R_1} F_\parallel$$

where

$$F_\parallel = \left[1 + \Gamma_\parallel e^{-jk(R_2 - R_1)}\right] \qquad (6.16)$$

From Fig. 9

$$R_1 = \sqrt{d^2 + (h_r - h_t)^2} = d\sqrt{1 + \left(\frac{h_r - h_t}{d}\right)^2}$$

$d \gg h_r$ and $d \gg h_t$, using the first two significant terms in the binomial expansion of $\sqrt{(1+x)}$:

$$\sqrt{(1+x)} \approx 1 + \frac{x}{2} \quad \text{for} \quad x \ll 1$$

we can write :

$$R_1 \approx d\left[1 + \frac{1}{2}\left(\frac{h_r - h_t}{d}\right)^2\right]$$

Similarly, R_2 can be approximated to :

$$R_2 \approx d\left[1+\frac{1}{2}\left(\frac{h_r+h_t}{d}\right)^2\right]$$

The path difference, $R_2 - R_1$, is given as :

$$R_2 - R_1 = \frac{2h_r h_t}{d}$$

For

$$\frac{h_r h_t}{d} \ll \lambda$$

$$\varDelta\theta = k(R_2 - R_1) = \frac{4\pi h_r h_t}{d\lambda}$$

is small so that $\sin x = x$ and $\cos x = 1$, and we can write :

$$e^{-jk(R_2-R_1)} = \cos(\varDelta\theta) - j\sin(\varDelta\theta) \simeq 1 - j\frac{2kh_r h_t}{d}$$

For small *grazing angles* of incidence ψ, we get :

$$\varGamma_\perp \approx \varGamma_\parallel \approx -1$$

and hence, we can set :

$$F = F_\perp = F_\parallel \approx j\frac{2kh_r h_t}{d}$$

Taking into account the ground reflection, the received power can be written as :

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R_1}\right)^2 |F|^2 \qquad (6.17)$$

For h_r and h_t small compared to d,

$$R_1 \approx d\left[1+\frac{1}{2}\left(\frac{h_r-h_t}{d}\right)^2\right] \approx d$$

Therefore, the received power is approximately given by :

$$P_r \approx \frac{P_t G_t G_r (h_r h_t)^2}{d^4} \qquad (6.18)$$

For large d, the received power decreases as d^4. This rate of change of power with distance is much faster than that observed in the free space propagation condition.

VI. Propagation modes by frequency band

Radio waves propagation behavior varies depending on the frequency of the radio waves. They interact with the environment in different ways. Each of these frequency bands has unique characteristics that make them suitable for specific types of communication and applications. Table 6.3 summarizes the frequency bands allocation and their propagation methods.

Table 6.3. Radio wave frequency allocation

Acronym	Frequency range	Designation	Propagation method
VLF	3-30kHz	Very Low Frequencies	Ground wave and ionospheric reflection Guided between the Earth and the ionosphere.
LF	30-300kHz	Low Frequencies	
MF	300k-3MHz	Medium Frequencies	
HF	3 – 30MHz	High Frequencies	Ionospheric refraction
VHF	30 – 300MHz	Very High Frequencies	Line-of-sight propagation. (Direct wave)
UHF	300MHz –3GHz	Ultra High Frequencies	
SHF	3GHz – 30GHz	Super High Frequencies	
EHF	30 – 300GHz	Extremely High Frequencies	Line-of-sight propagation limited by atmospheric absorption to a few km

1. Very low frequency (VLF) band : covers 3–30kHz. The low frequency dictates that large antennas are required to achieve a reasonable efficiency. The primarily mode of propagation is ground-wave propagation. VLF has been successfully used with underground antennas for submarine communication. These waves follow the curvature of the Earth and can travel long distances through the ground and seawater mostly used up to 1000 km.

2. Low-(LF) and medium-frequency (MF) bands : cover the range from 30kHz to 3MHz. Both bands use ground-wave propagation and some sky wave at night. While the wavelengths are smaller than the VLF band, these bands still require very large antennas. Uses include AM radiobroadcast.

3. High-frequency (HF) band : covers 3–30MHz. These frequencies support some ground-wave propagation, but most HF communication is via sky wave. The advantages of the HF band include reasonably sized antennas. Several segments of the HF band are still used for amateur radio, maritime communication, military ground and over-the-horizon communication.

4. Very high frequency (VHF) and ultra-high frequency (UHF) bands : cover frequencies from 30MHz to 3GHz. In these ranges, there is very little ionospheric propagation, which makes them ideal for frequency reuse. For the most part, VHF and UHF waves travel by LOS and ground-bounce propagation. VHF and UHF employ moderately sized antennas, show minimal sensitivity to weather and moderate building penetration, making them a good choice for mobile communications. Applications of these frequencies include broadcast FM radio, television broadcasting, aircraft radio, public service radio such as police and fire departments, and the Global Positioning System. These bands are the region where satellite communication begins since the signals can penetrate the ionosphere with minimal loss.

5. Super-high-frequency (SHF) band : include 3–30GHz. These waves are highly directional and require line-of-sight (LOS) propagation. In this band, moderately sized directional antennas with high gain are used. **SHF** are used for radar, satellite communication, direct broadcast satellite television, and microwave links. Precipitation and gaseous absorption can be an issue in these frequency ranges, particularly near the higher end of the range and at longer distances.

6. Extra-high-frequency (EHF) band : covers 30–300GHz and is often called *millimeter wave*. In this region, much greater bandwidths are available. Propagation is strictly LOS, and precipitation and gaseous absorption are a significant issue. EHF waves are used in advanced communication systems like millimeter-wave radar and some satellite communication. They are highly directional and can be absorbed by atmospheric moisture, limiting their range.

VII. Effect of earth curvature on space wave propagation

When the distance between the transmitting and receiving antennas is considerably large, curvature of the earth has considerable effect on space wave propagation.

1. Optical horizon (Line-of-Sight)

In line-of-sight (LOS) communication, the transmitter and receiver must have a direct visual path to each other, without any obstacles blocking the signal. The optical horizon refers to the farthest point an observer can see over the Earth's surface, assuming no obstructions like mountains or buildings.

Assuming that the maximum range is determined by the tangent to the earth (Fig. 10), as :

$$(R+h_t)^2 = R^2 + d_1^2$$

$$R^2 + 2Rh_t + h_t^2 = R^2 + d_1^2$$

We have $h_t << 2R$, which gives :

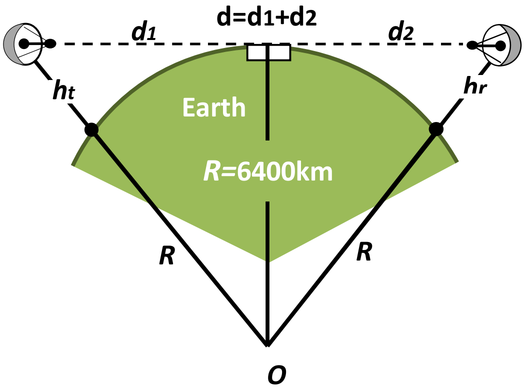


Fig.6.10. LOS propagation geometry over

$$d_1^2 \approx 2Rh_t$$

and $d_1 \approx \sqrt{2Rh_t}$

Similarly :

$$d_2 \approx \sqrt{2Rh_r}$$

and the total range will be :

$$d = d_1 + d_2 \approx \sqrt{2Rh_t} + \sqrt{2Rh_r}$$

and :

$$d \approx 3570\left(\sqrt{h_t} + \sqrt{h_r}\right) \qquad (6.19)$$

2. Radio horizon

The atmospheric refraction causes the radio waves to bend pursuing (following) the earth curvature, extending slightly (~+15%) the range of communication beyond the optical horizon.

$$d \approx 3570\left(\sqrt{kh_t} + \sqrt{kh_r}\right) \qquad (6.20)$$

The radio horizon can be approximated by modifying the optical horizon formula with a correction factor (typically about $k = 4/3$).

$$d \approx 4122\left(\sqrt{h_t} + \sqrt{h_r}\right) \qquad (6.21)$$

3. Effect of obstacles close to the direct path (Fresnel zone)

A Fresnel zone is an ellipsoidal region of space between and around a transmitter and a receiver. The primary wave will travel in a relative straight line from the transmitter to the receiver. Any obstacle inside the Fresnel ellipsoid reflects part of the wave arriving at the receiver out of phase with the direct signal due to the different path lengths and affects the received signal. Depending on the magnitude of the phase difference between the two waves, the waves can interfere constructively or destructively.

The concept of Fresnel zone clearance may be used to analyze interference by obstacles near the path of a radio beam. The first zone must be kept largely free from obstructions to avoid interfering with the radio reception. Most of the wave energy (60%) is contained inside this ellipsoid.

The size of the Fresnel zone is determined by the frequency of operation and the distance between the two sites. The higher the frequency or the longer the distance, the larger the Fresnel zone.

In the simplistic case of a single reflected path, the receiver combines the two waves:

$$E_o \cos(wt - kd)$$

and $$E_o \cos(wt - kr)$$

with : $d = d_1 + d_2$: direct path

$r = r_1 + r_2$: reflection path

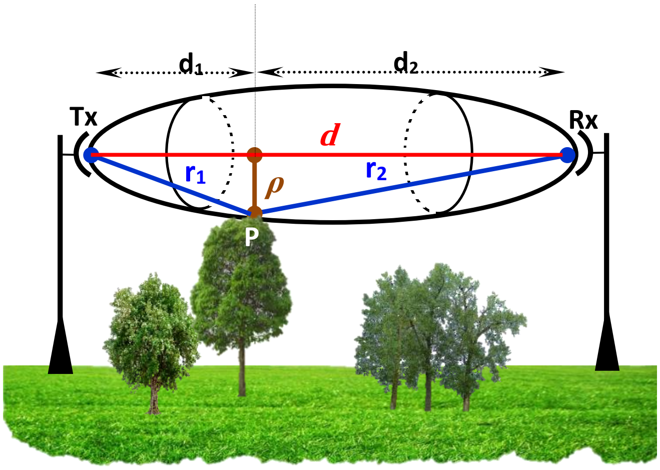


Fig.6.11. Fresnel zone

If the two signals arrive in phase opposition, the received field is zero (destructive interference), which occurs when the phase difference is half an odd integer multiple of the period :

$$r = d + \frac{\lambda}{2}$$

The space limited by the relation defines an ellipsoid of revolution with Tx and Rx two focal points.

$$r_1 + r_2 = r = d_1 + d_2 + \frac{\lambda}{2} = d + \frac{\lambda}{2} = constant$$

Using Pythagoras, we immediately find that the radius of the ellipsoid ρ is :

$$\rho = \sqrt{\frac{d_1 d_2}{d_1 + d_2}\lambda} \qquad (6.22)$$

N.B : ρ is maximum for $d_1 = d_2$.

- ρ corresponds to the strictly minimum value for which the space surrounding the direct ray joining the two antennas is clear of any obstacle.

- As long as the Fresnel ellipsoid does not contain any obstacle, the laws of free space apply.
- This characteristic is specific to microwave links and only applies to links at frequencies of several GHz.
- The lower the frequency, the larger the ellipsoid, for example, for a distance of 10 km with a frequency of 20 GHz, the maximum radius of the ellipsoid is 6.12 m and for 5 GHz it is 2.25 m.

VIII. Wave propagation in free space

Free space implies an idealized infinite space without any object that can interact with the EM waves. Communication is purely dependent on having a clear line of sight between the antennas.

1. Free space path loss (FSPL)

The distance between the antennas must be large enough to fulfill the far field region condition.

The free-space path loss is the loss factor that is due to distance and frequency (wavelength). The power received, P_r, at a distance R is given by the Friis formula :

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 \qquad (6.23)$$

where :

- P_t is the transmitted power.
- G_t is the gain of the transmitting antenna.
- G_r is the gain of the receiving antenna.

The transfer of EM energy from the transmit antenna to the receive antenna takes place in a straight-line path and, hence, such a communication link is called a line-of-sight (LOS) link.

The factor $\left(\frac{\lambda}{4\pi R}\right)^2$ is due to the propagation and is called the free space path loss. It represents the attenuation of the signal due to the spreading of the power as a function of distance. In decibel, the FSPL is expressed as :

$$P_L(R) = 10\log_{10}\left(\frac{4\pi R}{\lambda}\right)^2 \text{ dB} \qquad (6.24)$$

IX. Wave Propagation in complex environments

In real-world environments consisting of hills, trees, buildings, vehicles, etc. the propagation of waves is far more complex due to various factors that can reflect, refract, or diffract the signal. Such a situation is generally encountered in mobile phone and wireless local area network applications. These effects lead to additional **path loss**, often much higher than in free space.

1. Path loss in complex environments

Since it is generally difficult to obtain accurate electrical properties of the environment, researchers have come up with empirical models to predict the propagation in complex environments. These models are based on extensive measurements carried out in the corresponding environments. Path loss in complex environments is typically much higher than in free space and depends on multiple factors like frequency, distance, environment characteristics, and mobility.

Log-distance path loss model: This model introduces a **path loss exponent** n to account for the complexity of the environment:

$$P_{Ln}(R) = P_{Ln}(R_0) + 10\log_{10}\left(\frac{R}{R_0}\right)^n \text{ dB} \qquad (6.25)$$

where $P_{Ln}(R_0)$ is the path loss at a reference point R_0 (within the far-field zone) obtained by measurements. The value of n depends on the environment. For example, at 900 MHz, n is in the range of 2.7 to 3.5 for an urban environment, and 4 to 6 in the case of obstructed buildings.

X. Conclusion

This chapter provides a comprehensive overview of radio wave propagation mechanisms, relating the characteristics of electromagnetic waves to the properties of the Earth’s atmosphere and environmental interactions. The classification of frequency bands and their relationship to propagation modes (ground, sky, and space waves) helps to understand how waves travel through different atmospheric media and layers. The analysis of refraction, reflection, and limitations imposed by the Earth’s curvature, as well as key parameters such as critical frequency and skip distance, illustrates the complexity of propagation in real-world environments. This chapter provides an essential foundation for understanding practical challenges in wireless networks, satellite systems and emerging new technologies.

General Conclusion

The study of electromagnetic wave propagation is a fundamental pillar in telecommunications engineering. This course aims to provide students with a solid theoretical foundation allowing them to understand the complex phenomena related to wave propagation in various media.

Starting from Maxwell's equations, we explored the fundamental properties of electromagnetic waves, particularly in vacuum and in dielectric and conductive media. The concepts of plane wave, polarization, Poynting vector and skin effect were developed in detail.

The study of reflection and refraction phenomena at the interface of two media made it possible to understand how waves interact with objects and surfaces. These concepts are essential for the design of wireless communication systems.

Finally, we addressed the specificities of radio wave propagation in the atmosphere, taking into account the effects of refraction and reflection on the ground. This part of the course is particularly important for the design of long-range radiocommunication systems.

Students are now able to model, analyze, design and solve problems related to wave propagation in various communication systems. This course provides a solid foundation for the study of more advanced topics such as antenna theory, propagation in complex environments (urban, indoor) and radar systems.

In conclusion, mastering wave propagation phenomena is essential for any engineer wishing to contribute to the evolution of wireless communication systems.

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